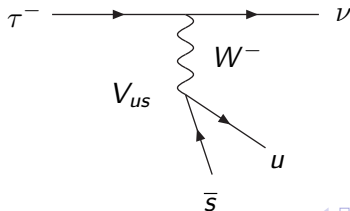


CP violation of $\tau \rightarrow \nu K \pi^0(\eta)$

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Feb. 21 /2008, Flavor Workshop (Sakunami)

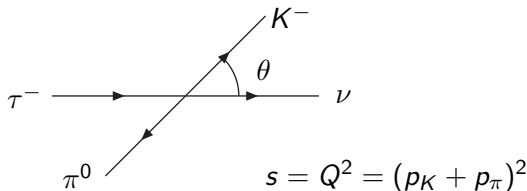
- What kind of new physics may be studied with CP violation of charged lepton decays
- What kind of measurement is needed ?
- Belle studied $\tau^\pm \rightarrow K_s \pi^\pm \nu$ (hep-ex 0706.2231, Physics Lett.B.). Also preliminary measurement of $\tau^\pm \rightarrow K^\pm \eta \nu$ is reported.
(By K. Hayasaka, YITP workshop at Kyoto)



CP violation measurement $\rightarrow$ unpolarized case.

Forward and Backward Asymmetry

$$\frac{dA_{\text{FB}}}{d\sqrt{s}} = \frac{d\Gamma}{d\sqrt{s}}|_{\cos\theta>0} - \frac{d\Gamma}{d\sqrt{s}}|_{\cos\theta<0} = -\frac{G_F^2 |V_{us}|^2}{2^5 \pi^3} \frac{(m_\tau^2 - s)^2}{m_\tau^3} \frac{m_\tau^2}{\sqrt{s}} I(s)^2 \text{Re} F_V F_S^*$$



Vector F_V and scalar F_S form factors

$$\langle K^-(p_K) \pi^0(p_\pi) | \bar{s} \gamma^\mu u | 0 \rangle = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) q_\nu F_V(Q^2) + Q^\mu F_S(Q^2)$$

$$\begin{aligned} \frac{d^2\Gamma}{d\sqrt{s}d\cos\theta} &= \frac{G_F^2|V_{us}|^2}{2^5\pi^3} \frac{(m_\tau^2 - s)^2}{m_\tau^3} l(s) \\ &\left(\left(\frac{m_\tau^2}{s} \cos^2\theta + \sin^2\theta \right) l(s)^2 |F_V(s)|^2 + \frac{m_\tau^2}{4} |F_s(s)|^2 \right. \\ &\quad \left. - \frac{m_\tau^2}{\sqrt{s}} l(s) \cos\theta \operatorname{Re}(F_V F_s^*) \right) \end{aligned}$$

- Forward and Backward Asymmetry $\rightarrow$ Interference bet. $L = 0$ and $L = 1$ states of $K\pi$ system.
- F_V and F_S have their own strong phases. (Non-perturbative input).
- By taking the direct CP asymmetry (CP conjugate process) $\bar{A} = \bar{A}(\tau^+ \rightarrow \bar{\nu} K^+ \pi^0)$, we may have

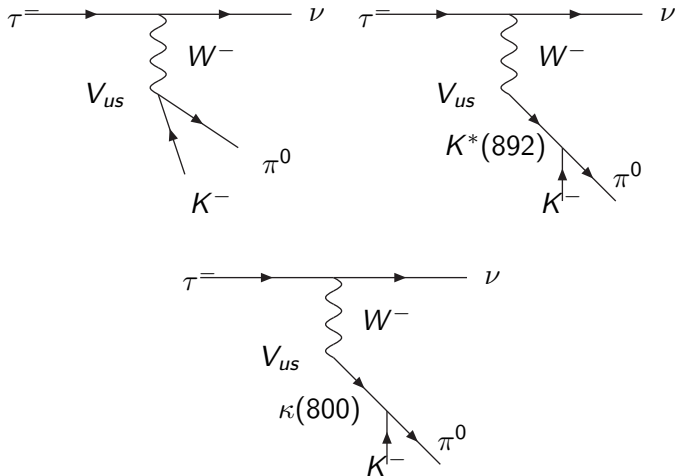
$$\frac{dA_{\text{CPV}}}{d\sqrt{s}} \sim \frac{d\bar{A}_{\text{FB}}}{d\sqrt{s}} - \frac{dA_{\text{FB}}}{d\sqrt{s}} \sim \text{Im}(F_V F_S^*) \sin \delta_{\text{New}} \sim \sin \delta_{\text{st}} \sin \delta_{\text{New}}$$

- Within the standard model, there is no CP violation because the vector part and scalar part has common weak phase V_{us} .

Calculation of the form factors F_V and F_S .

- Chiral perturbation is not valid when $\sqrt{s} \geq 800(\text{MeV})$
- Inclusion of effects of resonances $K^*(892) : 1^-$, $\kappa(800) : 0^+$ or higher resonances is important. Here we assume κ is a simple $\bar{s}q$ bound state.
- The chiral Lagrangian including vector and scalars:
The hadronic Model: Vector and scalar nonets: $(\text{SU}(3)_f)$

Feynman Diagrams



Effective Chiral Lagrangian including scalar and vectors

$$\begin{aligned}
 \mathcal{L} = & \frac{f^2}{4} \text{Tr} D U D U^\dagger + B \text{Tr} M (U + U^\dagger) \\
 & + \text{Tr} D_\mu S D^\mu S - M_\sigma^2 \text{Tr} S^2 \\
 & - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + M_\rho^2 \text{Tr} (V_\mu - \frac{\alpha_\mu}{g})^2 \\
 & + \frac{g_1}{4} \text{Tr} (D_\mu U D^\mu U^\dagger) (\xi S \xi^\dagger) \\
 & + g_2 \text{Tr} \left((\xi M \xi + \xi^\dagger M \xi^\dagger) S \right)
 \end{aligned}$$

$$U = \xi^2 \quad \xi = \exp(i \frac{\pi}{f})$$

$$\pi = \begin{pmatrix} \frac{\pi^0}{2} + \frac{\eta_8}{2\sqrt{3}} & \frac{\pi^+}{\sqrt{2}} & \frac{K^+}{\sqrt{2}} \\ \frac{\pi^-}{\sqrt{2}} & -\frac{\pi^0}{2} + \frac{\eta_8}{2\sqrt{3}} & \frac{K^0}{\sqrt{2}} \\ \frac{K^-}{\sqrt{2}} & \frac{K^0}{\sqrt{2}} & -\frac{\eta_8}{\sqrt{3}} \end{pmatrix} M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Description of vector and scalar nonets

$$S = \begin{pmatrix} \frac{\delta^0}{2} + \frac{\sigma}{2} & \frac{\delta^+}{\sqrt{2}} & \frac{\kappa^+}{\sqrt{2}} \\ \frac{\delta^-}{\sqrt{2}} & -\frac{\pi^0}{2} + \frac{\sigma}{2} & \frac{\kappa^0}{\sqrt{2}} \\ \frac{\kappa^-}{\sqrt{2}} & \frac{\kappa^0}{\sqrt{2}} & \sigma_{ss} \end{pmatrix} + S_0 \quad V = \begin{pmatrix} \frac{\rho}{2} + \frac{\omega}{2} & \frac{\rho^+}{\sqrt{2}} & \frac{K^{*+}}{\sqrt{2}} \\ \frac{\rho^-}{\sqrt{2}} & -\frac{\rho^0}{2} + \frac{\omega}{2} & \frac{K^{*0}}{\sqrt{2}} \\ \frac{K^{*-}}{\sqrt{2}} & \frac{K^{*0}}{\sqrt{2}} & \phi \end{pmatrix}$$

$$S_0 = \frac{g_2 M}{M_\sigma^2} \quad M = \text{Diag.}(m_u, m_d, m_s) \quad (\text{quark mass: } m_u = m_d)$$

$$\text{Tr} D_\mu S D^\mu S = \text{Tr} (\partial_\mu S + ig[V_\mu, S])(\partial^\mu S + ig[V^\mu, S])$$

generates $\kappa(0^+)$ and $K^*(1^-)$ mixing.

$$K_\mu^{*-} \rightarrow K_\mu^{*-} - ig \frac{(S_{03} - S_{01}) \partial_\mu \kappa^-}{M_{K^*} M_\rho}$$

SU(3)_f breaking for vector and scalar mesons.

$$M_{K^*}^2 = M_\rho^2 + g^2 (S_{03} - S_{01})^2 \quad M_\omega^2 = M_\rho^2 = M_\phi^2 \rightarrow \text{not good}$$

$$\frac{M_\sigma}{M_{K^*}} = \frac{M_\rho}{M_{K^*}} \quad M_\sigma \sim 690(\text{MeV})$$

Weak current with hadrons

$$\begin{aligned}
 \overline{u}_L \gamma_\mu s_L &= -\frac{1}{\sqrt{2}} F_K \partial_\mu K^- + \frac{M_\rho^2}{\sqrt{2}g} (K_\mu^{*-} - ig \frac{(S_{03} - S_{01})}{M_\rho M_{K^*}} \partial_\mu \kappa^-) \\
 &- i \frac{1}{2\sqrt{2}} \left(K^- \partial_\mu \pi^0 \left(\frac{F_\pi}{F_K} - \frac{M_\rho^2}{2g^2 F_K F_\pi} \right) - \partial_\mu K^- \pi^0 \left(\frac{F_K}{F_\pi} - \frac{M_\rho^2}{2g^2 F_K F_\pi} \right) \right) \\
 &- i \frac{\sqrt{3}}{2\sqrt{2}} \left(K^- \partial_\mu \eta_8 \left(\frac{F_8}{F_K} - \frac{M_\rho^2}{2g^2 F_K F_8} \right) - \partial_\mu K^- \eta_8 \left(\frac{F_K}{F_8} - \frac{M_\rho^2}{2g^2 F_K F_8} \right) \right) \\
 &- i \frac{1}{2} \left(\overline{K}^0 \partial_\mu \pi^- \left(\frac{F_\pi}{F_K} - \frac{M_\rho^2}{2g^2 F_K F_\pi} \right) - \partial_\mu \overline{K}^0 \pi^- \left(\frac{F_K}{F_\pi} - \frac{M_\rho^2}{2g^2 F_K F_\pi} \right) \right)
 \end{aligned}$$

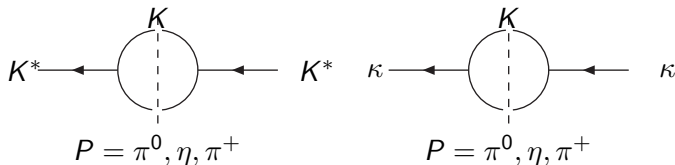
Form factors: Tree level results

$$\langle K^+(p_k)\pi^0(p_\pi)|\bar{s}\gamma^\mu u|0\rangle = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right) q_\nu F_V(Q^2) + Q^\mu F_S(Q^2)$$

$$\begin{aligned}F_V^{K^+\pi^0}(Q^2) &= \frac{1}{\sqrt{2}} \left(-\frac{R + R^{-1}}{2} + \frac{M_\rho^2}{2g^2 F_K F_\pi} \left(1 - \frac{M_\rho^2}{M_{K^*}^2 - Q^2}\right) \right) \\Q^2 F_S^{K^+\pi^0} &= -(m_s - m_u) \langle K^+\pi^0|\bar{u}s|0\rangle \\&= \frac{1}{2\sqrt{2}} \left(-\Delta_{K\pi}(R + R^{-1}) - \frac{Q^2}{M_\kappa^2 - Q^2} M_\kappa^2 (R - R^{-1}) \right. \\&\quad + \frac{Q^2}{M_\kappa^2 - Q^2} \left(\frac{m_\pi^2}{2\Delta} - \frac{m_K^2}{1 + \Delta} \right) (2\Delta R + (1 + \Delta)R^{-1}) \\&\quad \left. + \frac{\Delta_{K\pi}}{M_\kappa^2 - Q^2} (1 - \Delta) \left(-\frac{m_\pi^2}{2\Delta} R^{-1} + \frac{m_K^2}{1 + \Delta} R \right) \right)\end{aligned}$$

$$R = \frac{F_K}{F_\pi}, \Delta_{K\pi} = m_K^2 - m_\pi^2, \Delta = \frac{m_u + m_d}{2m_s} \sim \frac{1}{25}$$

The width of K^* and κ in propagators



The inverse propagator for K^* is modified as,

$$A = M_{K^*}^2 - Q^2 - iM_{K^*}\Gamma_{K^*}^*(Q^2)$$

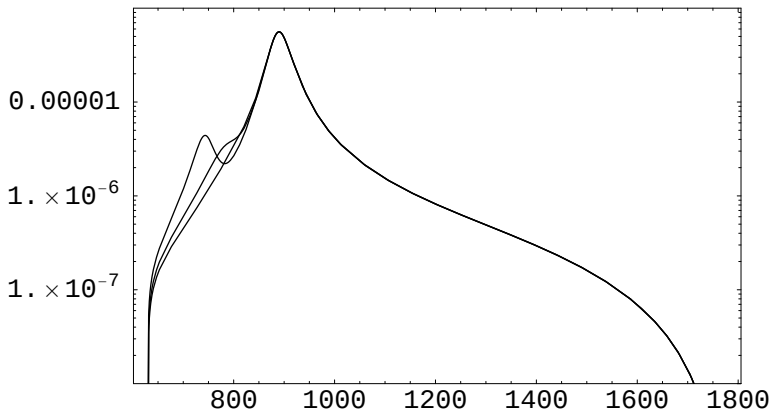
$$\Gamma_{K^*}(Q^2) = 3\frac{1}{48\pi M_{K^*}}\left(\frac{\nu_{K\pi}^3}{Q^4} + \frac{\nu_{K\eta}^3}{Q^4}\left(\frac{F_\pi}{F_8}\right)^2\right)g_{K^*K\pi}^2$$

$$g_{K^*K\pi} = \frac{M_\rho^2}{4gF_KF_\pi} \sim 3.246 \rightarrow \Gamma[M_{K^*}] = 50.8(\text{MeV})$$

$$\frac{d\text{Br}}{d\sqrt{s}} (M_\kappa = 750, 800, 850)$$

$$\text{Br}(K^+ \rightarrow \pi^0 K^+ \nu)_{\text{th}} = 0.448^{+0.004\%}_{-0.003\%}$$

$$1/2\text{Br}[\tau^\pm \rightarrow K_0 \pi^\pm \nu]_{\text{Belle}} = 0.404 \pm 0.002 \pm 0.013\%$$



Numerical Results of the vector and scalar form factors

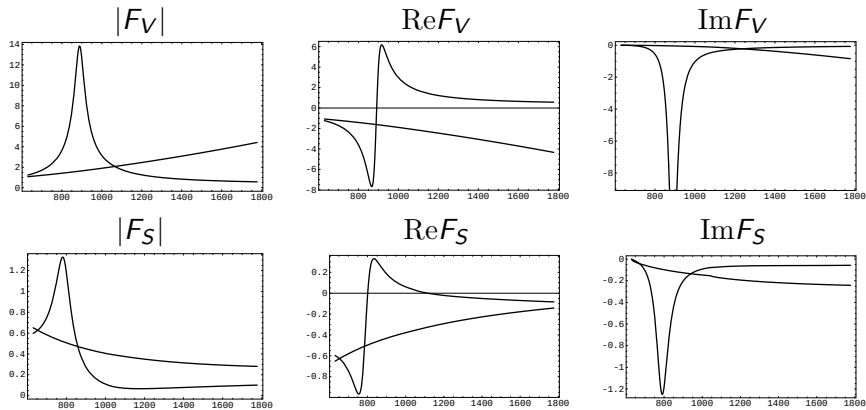


Figure: Form factors (comparison with CHPT)

Double differential rate and Forward Backward Asymmetry

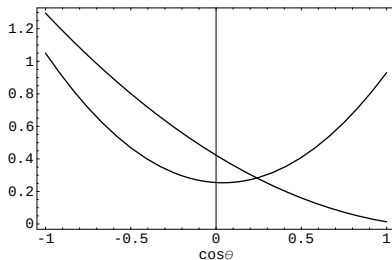
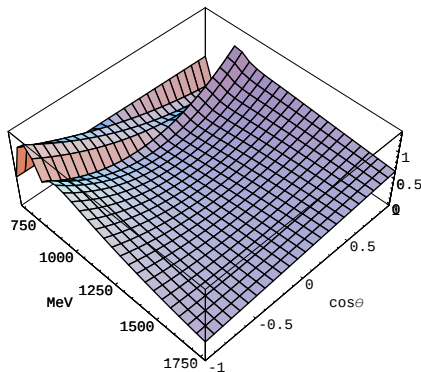


Figure: left. $\frac{d^2\Gamma}{d\sqrt{s}d\cos\theta} / \frac{d\Gamma}{d\sqrt{s}}$ double differential rate

Figure: right. $\cos\theta$ distribution for fixed $\sqrt{s} = 700, 900(\text{MeV})$

Large Forward Backward Asymmetry near the threshold region $\sqrt{s} \sim 700$ (MeV)

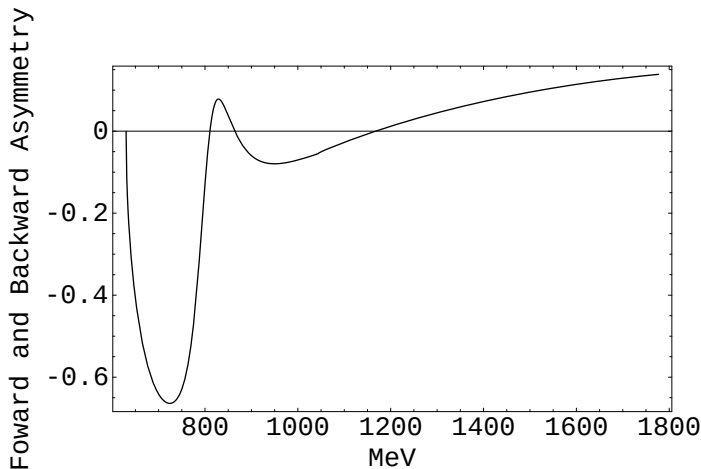


Figure: Forward and Backward Asymmetry

New Physics Effect (Flavor Diagonal New Physics):

$$\tau_R \rightarrow \nu_{\tau L}$$

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}} V_{us} \bar{\tau} \gamma_{\mu} (1 - \gamma_5) \nu_{\tau} \bar{u} \gamma^{\mu} (1 - \gamma_5) s + G_{\tau i} \bar{\tau} (1 - \gamma_5) \nu_i \bar{u} (1 + \gamma_5) s + \text{h.c.}$$

- The 3rd generation flavor diagonal interaction $G_{\tau\tau}$ can interfere with the standard model interaction.
- $G_{\tau\tau}$ is a complex number with a CPV phase differs from the phase of V_{us} .
- How large $G_{\tau\tau}$ contribution ? $a = \frac{\sqrt{2} G_{\tau\tau}}{G_F V_{us}}$

$$\begin{aligned} & \mathcal{M}(\tau^- \rightarrow K^- \pi^0 \nu) \\ &= -\frac{G_F}{\sqrt{2}} V_{us}^* \left[\left(g^{\mu\nu} - \frac{Q^{\mu} Q^{\nu}}{Q^2} \right) q_{\nu} F(Q^2) \right. \\ & \quad \left. + Q^{\mu} F_s(Q^2) \left\{ 1 - \frac{Q^2}{m_{\tau}(m_s - m_u)} a^* \right\} \right] \bar{u}_{\nu} \gamma_{\mu} (1 - \gamma_5) u_{\tau} \end{aligned}$$

New Physics interaction and source of CP violation

$$-\mathcal{L} = y_{1ij} \overline{e_{Ri}} \tilde{H}_1^\dagger l_{Lj} + y_{2ij} \overline{e_{Ri}} H_2^\dagger l_{Lj} + y_{2i}^\nu \overline{\nu_{Ri}} \tilde{H}_2^\dagger l_{Li} + \text{h.c.}$$

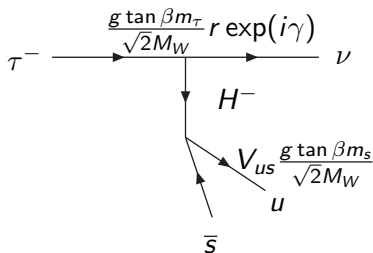
The charged lepton acquires mass from two Higgs y_1, y_2

$$H_1 = e^{i\frac{\theta}{2}} \begin{pmatrix} \frac{v_1 + h_1 - i \sin \beta A}{\sqrt{2}} \\ -\sin \beta H^- \end{pmatrix} \quad H_2 = e^{i\frac{\theta}{2}} \begin{pmatrix} -\cos \beta H^+ \\ \frac{v_2 + h_2 - i \cos \beta A}{\sqrt{2}} \end{pmatrix}$$

The relative phase of two Higgs θ (CP violation of Higgs potential.)

$$\begin{aligned} 1 - \frac{Q^2}{m_\tau m_s} a &= 1 - \frac{Q^2}{m_\tau m_s} \tan^2 \beta \frac{m_\tau m_s}{M_H^2} \left(1 - \frac{\sqrt{2} M_W}{m_\tau} \frac{y_{2\tau\tau}'^* e^{i\frac{\theta}{2}}}{g \sin \beta} \right) \\ &= 1 - \tan^2 \beta \frac{Q^2}{M_H^2} \exp(i\gamma) r \end{aligned}$$

Charged Higgs contribution to $G_{\tau\tau} = \frac{G_F}{\sqrt{2}} V_{us} \tan \beta^2 \frac{m_\tau m_s}{M_H^2} re^{i\gamma}$



Forward and backward CP asymmetry

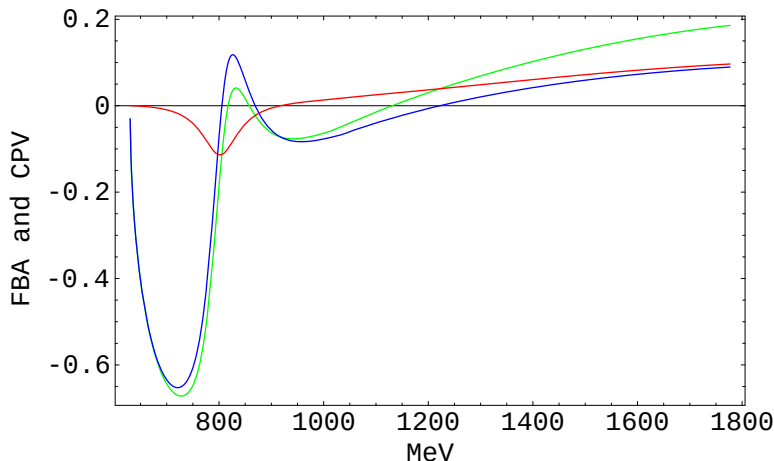
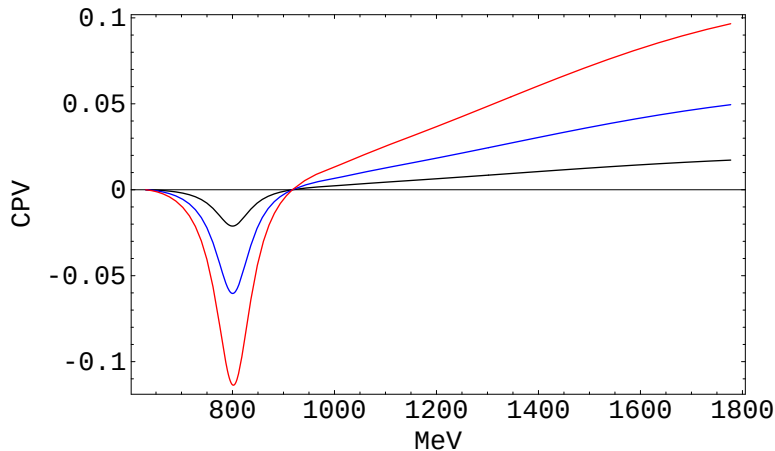


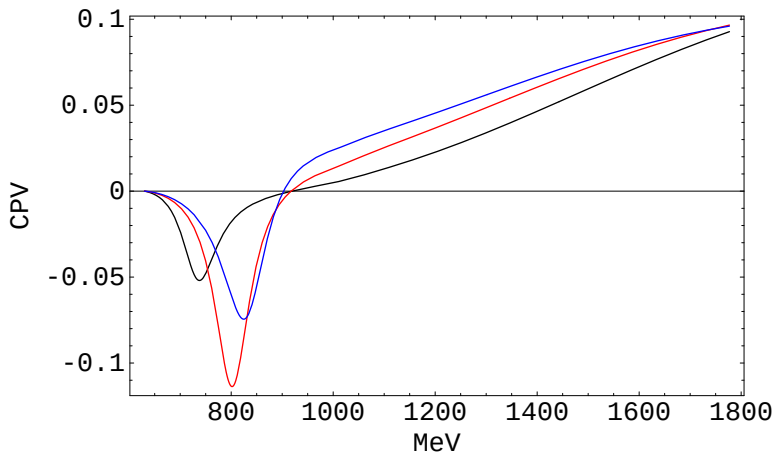
Figure: Forward and Backward CP Asymmetry (Red)

$M_H = 200, \tan \beta = 50, r = 2, \gamma = \pm \frac{\pi}{2}$

CP phase dependence $\gamma = \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{18}$



Dependence on the mass of κ $M_\kappa = 700, 800, 850(\text{MeV})$



$$\tau^\pm \rightarrow K^\pm \eta \nu$$

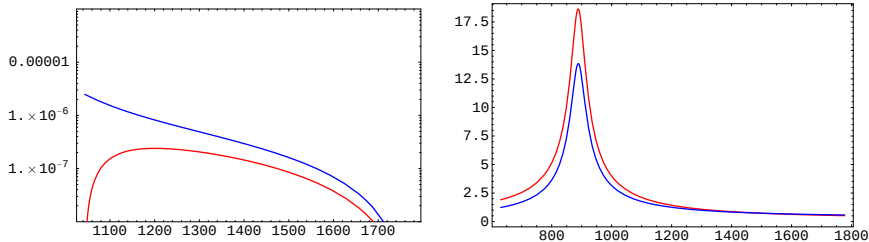
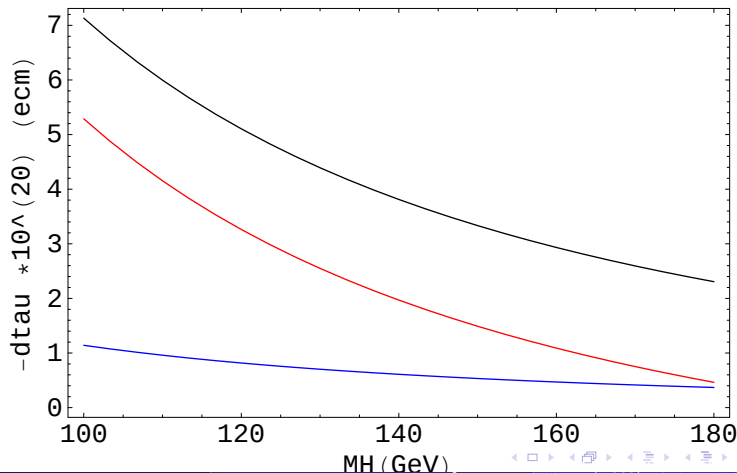


Figure: Left: The hadronic invariant mass distributions for $\tau \rightarrow K\eta\nu$ (RED) and $\tau \rightarrow K\pi\nu$ (BLUE). Right: The vector form factors $|F_{K\eta}|$ (RED) vs $|F_{K\pi}|$ (BLUE).

$\text{Br} = 8.58 \times 10^{-5} \leq \text{Br}|_{\text{exp}} = (1.62 \pm 0.10) \times 10^{-4}$. (Belle preliminary)
 Our result of ηK is a half of the experimental value.

$$d_\tau \simeq \sin(2\gamma) \frac{m_\tau}{16\pi^2 M_H^2} G_F m_\tau^2 \tan \beta^2 r^2 I_4\left(\frac{m_\tau^2}{M_H^2}\right)$$



We study CP violation of $\tau^\pm \rightarrow K^\pm \pi \nu_\tau$

- Realistic calculation of the form factors; i.e., We study the vector and scalar form factors including $\kappa(800)$ and K^* . The hadron invariant mass spectrum is obtained.
- We study Forward and backward asymmetry and find the large asymmetry (not CP) $\sim 60\%$ near the threshold region.
- Including the new physics contribution from charged Higgs exchange, we have seen that CPV can be as large as 10%.
($M_H = 200, r = 2, \tan \beta = 50, \gamma = \frac{\pi}{2}$)
- CP violating source is identified as from non-minimal ($r \neq 1, \gamma \neq 0$) type of two Higgs doublet model structure and CPV of the vacuum expectation value of Higgs.