$B_{d(s)} - \bar{B}_{d(s)}$ mixing and $b \rightarrow d(s)$ transitions in general SUSY models

Talk at CPV from B factories to Tevatron to LHCb

Tohoku University (Sep. 1-2, 2010)

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with Jae-Hyun Park (was at Tohoku)
Can we understand such a large deviation (in SUSY models)?
Contents

- Status of the SM and CKM matrix
- SUSY FCNC/CP Problems
- Earlier Literatures
- $s \rightarrow d$ transition: $\epsilon_K$ and $\epsilon'/\epsilon_K$
- $b \rightarrow d$ transition: $B_d - \overline{B_d}$ mixing and $B \rightarrow X_d \gamma$
- $b \rightarrow s$ transition: $B_s - \overline{B_s}$ mixing, $B \rightarrow X_s \gamma$ and $B_d \rightarrow \phi K_s$ CP asymmetry
  - $B_s - \overline{B_s}$ mixing in SUSY models
  - Implications on SUSY (flavor) models
- Concluding Remarks
My talk is based on the following papers

- “$B^0 - \bar{B}^0$ mixing, $B \to J/\psi K_s$ and $B \to X_d \gamma$ in general MSSM.” P. Ko, Jae-hyeon Park, G. Kramer, Eur.Phys.J.C25:615-622,2002.


“Addendum to: Implications of the measurements of $B_s - \overline{B_s}$ mixing on SUSY models.” P. Ko, Jae-hyeon Park.
**CKM matrix**

- Mixing matrix connecting weak interaction eigenstates and mass eigenstates of quarks.

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}
\]

- CKM matrix is **hierarchical** and has one **CP** phase.

\[
V = \begin{pmatrix}
  1 - \lambda^2 / 2 & \lambda & A\lambda^3(\rho - i\eta) \\
  -\lambda & 1 - \lambda^2 / 2 & A\lambda^2 \\
  A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}
\]

- Unitarity condition, \( V^\dagger V = VV^\dagger = 1 \), yields unitarity triangles (UT).
Unitarity triangle on the \((\rho, \eta)\) plane

- SM fit of \((\rho, \eta)\)

- In the presence of new physics, constraints on \((\rho, \eta)\) coming from one loop processes such as \(\epsilon_K, \Delta m_d, \) and \(\Delta m_s\), may be weaker

- Even if the shape of the UT is the same as this SM fit, there are processes with large deviations within SUSY models
SUSY FCNC/CP problem

Supersymmetry is symmetry between a fermion and a boson, which has many nice motivations such as resolution of gauge hierarchy problem, gauge coupling unification, and dark matter. But SUSY must be broken if it exists.

Supersymmetrizing SM doubles the particle spectrum, introducing more than 100 new parameters in the soft SUSY breaking sector.

Soft SUSY breaking parameters are complex and flavor violating, and a generic supersymmetric standard model results in huge FCNC and CP violation.

There must be some mechanism which controls FCNC and CP. This may be due to the SUSY breaking mediation mechanism and/or some flavor symmetry.
In particular, quark and squark mass matrices are not diagonalized simultaneously in general → Gluino mediated FCNC, which could easily dominate the SM amplitudes (∼ EW strength) → SUSY flavor problem

Possible Solutions
  - Universality (at some scale)
  - Alignment using some flavor symmetries
  - Decoupling (Effective SUSY models): Cohen, Kaplan, Nelson
    ← Disfavored by muon \((g - 2)\)
  - \(A_{SL}\) can tell something [ Randall and Su, NPB (1999) ]
  - All the related observables should be considered altogether [ Ko, Kramer, Park (2002) ]
Mass insertion approximation is a useful tool to present flavor violation in the sfermion sector. 
\((\delta^d_{12})_{LL}\) : dimensionless transition strength from \(\tilde{s}_L\) to \(\tilde{d}_L\).

We can do the same for \(b_A \to d_B\) and \(b_A \to s_B\) 
\((A, B = L, R : \text{chiralities of superpartners of squarks})\)

If \(\delta \sim O(1)\), large FCNC and CPV with strong couplings

SUSY FCNC/CP problem \(\delta\)’s should be small 
\(\lesssim 10^{-1} - 10^{-3}\) depending on \(AB = LL, RR, LR, RL\)
Current CKMology says New Physics should be flavor/CP blind to a very good approximation → Better to have $\delta = 0$

Even if we set $\delta$’s to zero by hand at one energy scale, it is regerated by RG evolution. → Cannot make it vanish at all scales

Either we consider $\delta$’s as parameters at EW scale, or assume $\delta$’s vanish at some scale (messenger scale) where Soft SUSY breaking terms are generated

mSUGRA makes an ad hoc assumption of universal scalar masses at $M_{\text{Planck}}$ or $M_{\text{GUT}}$ scale ($\delta$’s are zero), and the $\delta$’s are generated by RG evolution

Can we do better than simply assuming it ?

Yes (Gauge mediation, Anoamaly mediation, Dilaton dominated SUSY breaking ,....)
Basic Strategies

- Once again, “Flavor physics and CP violation” such as $B \rightarrow X_s \gamma$, $B_s \rightarrow \mu^+ \mu^-$, $\epsilon_K$,... in SUSY models depend strongly on Soft SUSY Breaking sector, which is not well understood yet.

- Without complete understanding of SUSY breaking, we have to rely on
  - Mass Insertion Approximation (MIA) to include gluino-squark loop contribution, OR
  - Work in some well motivated specific scenarios mSUGRA, GMSB, Dilaton Dominated SB (string theory), AMSB, ... where gluino-squark loop contributions ($\delta$’s) are under control, and study the implications on flavour physics.
## Implications for SUSY flavor models

| Model                     | $|\delta_{d,LL}^{23}|$ | $|\delta_{d,RR}^{23}|$ | $\tan \beta = 3$ | $\tan \beta = 10$ |
|---------------------------|---------------------|---------------------|------------------|------------------|
| LNS (A)                   | $\lambda^2$        | $\lambda^4$        | ✓                | ✓                |
| NS ; CHM (A)              | $\lambda^2$        | 1                   | ×                | ×                |
| NR (A)                    | $\lambda^2$        | $\lambda^8$        | ✓                | ✓                |
| CHM (NA)                  | $\lambda^2$        | $\lambda^{1/2}$    | ×                | ×                |
| BHRR, PT (NA)             | $\lambda^2$        | $\lambda^2$        | $\phi_s$         | ✓                |
| HM (NA)                   | $\lambda^3$        | $\lambda^5$        | ✓                |                  |
| PS (NA)                   | $\lambda^2$        | $\lambda^4$        | ✓                |                  |
| CKN (D)                   | $\lambda^2$        | $LL \gg RR$        |                  | ✓                |

Status of some models analyzed Randall and Su, for the two different values of $\tan \beta$. ($A=$Abelian, $NA=$Nonabelian, $D=$Decoupling) ($\cdot$) incompatible with $\phi_s$ but safe otherwise; ($\phi_s$) compatible with $\phi_s$ and safe; ($\sqrt{\text{✓}}$) currently okay but dangerous; ($\times$) disfavored.
$s \rightarrow d$ transition (12 Mixing)

$\epsilon_K$ and $\text{Re}(\epsilon'/\epsilon_K)$
SUSY contributions to $\epsilon_K$

Diagrams:
SUSY contributions to $\epsilon' / \epsilon$

Diagram:

\[
(\delta_{12}^{d})_{LL}
\]

\[
\tilde{s}_{L} \quad \tilde{d}_{L}
\]

\[
\tilde{g}
\]
Fully Supersymmetric CPV in the kaon system

- CP violating parameters in the kaon system
  - \( \epsilon_K = e^{i\pi/4} \left(2.280 \pm 0.013\right) \times 10^{-3} \) : CP violation in the \( K^0 - \overline{K^0} \) mixing (\( \Delta S = 1 \))
  - \( \text{Re}(\epsilon'/\epsilon_K) = (18 \pm 4) \times 10^{-4} \) : CP violation in the decay amplitude (\( \Delta S = 1 \))

- These two can be accommodated by the KM phase in the Glashow-Salam-Weinberg’s standard model (SM)

- SM prediction for \( \text{Re}(\epsilon'/\epsilon_K) \) :
  - Buras et al. (before 1999) : \( 5 \times 10^{-4} \)
  - Bertolini et al. : \( 5 - 30 \times 10^{-4} \)
  - Large Hadronic Uncertainties \( \rightarrow \) Need Lattice QCD Calculations after all
Fully SUSY CPV in K

- Can SUSY explain such a large $\text{Re}(\epsilon'/\epsilon_K)$? Answer: The folklore was “No” again before 1999, until Masiero and Murayama showed that it is possible.

- P. Ko et al.: Both $\epsilon_K$ and $\text{Re}(\epsilon'/\epsilon_K)$ can be explained in terms of a single SUSY parameter $(\delta^d_{12})_{LL}$, even if the KM phase is zero, without conflict with the $e/n$ EDM’s.

$\rightarrow$ Fully SUSY CP violation is possible in the MSSM with a single CPV parameter $(\delta_{LL})_{12}$

- Key Point: Double mass insertion can be important at large $|\mu \tan \beta| \sim O(5 - 10) \text{ TeV}$

- Completely different from Masiero and Murayama’s mechanism, and no problem with neutron EDM in our model.
Double Mass insertion

- Double mass insertion can be important in the large $\tan \beta$ region
- Diagrams:

(Baek, Jang, Ko, Park, PRD(2000))
Fully SUSY CPV in K-Cont’d

- $|\left(\delta_{12}^d\right)_{LL}| \sim O(10^{-3} - 10^{-2})$ with the phase $\sim O(1)$ saturates $\epsilon_K$

- This parameter can lead to a sizable $\text{Re}(\epsilon'/\epsilon_K)$ through the $(\delta_{12}^d)_{LL}$ insertion followed by the Flavor Preserving (FP) $(LR)$ mass insertion

\[
\propto (\delta_{22}^d)_{LR} \equiv m_s (A_s^* - \mu \tan \beta) / \tilde{m}^2 \sim O(10^{-2}),
\]

- This FP $LR$ insertion is generically present in any SUSY models

- $(\delta_{12}^d)_{LR}^{\text{ind}} = (\delta_{12}^d)_{LL} (\delta_{22}^d)_{LR} \sim 10^{-5}$ with $O(1)$ phase

- The same mechanism can happen in $b \to s$ transitions
Different predictions for $K \rightarrow \pi \nu \nu$ from the SM
$b \rightarrow d$ Transition (13 Mixing)

$B_d - \bar{B}_d$ mixing, and $B_d \rightarrow X_d \gamma$
1-3 Mixing: $B_d - \bar{B}_d$ mixing, and $B_d \rightarrow X_d \gamma$

[ Ko, Kramer, Park, EJPC (2003) ]

- Amp (tot) = Amp (SM) + Amp (SUSY: $\tilde{g}$-down squark) for $B^0 - \bar{B}^0$ mixing and $B_d \rightarrow X_d \gamma$
- Mass insertion approximation with $m_{\tilde{g}} = \tilde{m} = 500$ GeV
- Scan over one of $\delta_{13}^d$'s as well as $\gamma(\phi_3)$ (KM angle)
- Constraints

$$\Delta m_d = (0.472 \pm 0.017) \text{ ps}^{-1}$$
$$\sin 2\beta_{J/\psi} = 0.79 \pm 0.10$$
$$B(B \rightarrow X_d \gamma) < 1 \times 10^{-5}$$
1-3 Mixing : Cont’d

Predictions

\[ A_{ll} \equiv \frac{N(BB) - N(\bar{B}\bar{B})}{N(BB) + N(\bar{B}\bar{B})} \approx \text{Im} \left( \frac{\Gamma_{12} \approx \Gamma_{SM}^{12}}{M_{12}^{SM} + M_{12}^{SUSY}} \right) \]

\[ A_{CP}^{b\rightarrow d\gamma} \equiv \frac{\Gamma(B \rightarrow X_d \gamma) - \Gamma(\bar{B} \rightarrow \bar{X}_d \gamma)}{\Gamma(B \rightarrow X_d \gamma) + \Gamma(\bar{B} \rightarrow \bar{X}_d \gamma)} \]

Data : \( A_{ll}^{\exp} = (-0.13 \pm 0.60 \pm 0.56)\% \) (BELLE)

Consider two cases:

- Single \( (\delta_{13}^d)_{LL} \) insertion
- Single \( (\delta_{13}^d)_{LR} \) insertion
LL insertion - I
Hatched region for $B(B \rightarrow X_d \gamma) > 1 \times 10^{-5}$

- $A_{ll}$ can have sign opposite to that of SM value

- $B \rightarrow X_d \gamma$ strongly constrains $|\langle \delta^d_{13} \rangle_{LL}| \lesssim 0.2$
  \[\simeq -60^\circ \lesssim \gamma \lesssim 60^\circ\]

- $-15\% \lesssim A_{CP}^{b\rightarrow d\gamma} \lesssim +20\%$
LR insertion - I
Hatched region for $B(B \to X_d \gamma) > 1 \times 10^{-5}$

$B \to X_d \gamma$ even more strongly constrains 

$|(\delta_{13}^d)_{LR}| \lesssim 10^{-2}$

$\sim 30^\circ \lesssim \gamma \lesssim 80^\circ$

Nevertheless, $-25\% \lesssim A_{CP}^{b \to d \gamma} \lesssim +30\%$

Not much effect on $A_{ll}$.

Can expect large deviations in $B \to X_d \gamma$ even if 

$\gamma = \gamma_{SM}$
Implications of the recent measurements of $B_s - \bar{B}_s$ mixing on SUSY models
$B_s - \bar{B}_s$ mixing in SM

- Dominated by the box diagram with $W - t$ in the loop
- The mixing is almost real within the SM, and depend on $V_{ts}$
- Any phase in the mixing is a clear signal of physics beyond the SM
- $\Delta M_d / \Delta M_s$ depends on $|V_{td}|^2 / |V_{ts}|^2$ with less hadronic uncertainties than individuals
  → Important for CKM Phenomenology
First observations of $B_s - \overline{B_s}$ mixing

- The WA before 2006: $\Delta M_s > 14.4 \text{ ps}^{-1}$
- D0: $17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1}$
- CDF: $\Delta M_s = (17.33^{+0.42}_{-0.21} \text{(stat)} \pm 0.07 \text{(sys)}) \text{ ps}^{-1}$
- Constraint on $V_{ts}$ from $\Delta M_d / \Delta M_s$
  $$|V_{td}| / |V_{ts}| = 0.208^{+0.008}_{-0.007} \text{(stat + sys)}$$
- The Belle result from $b \rightarrow d \gamma$:
  $$|V_{td}| / |V_{ts}| = 0.199^{+0.026}_{-0.025} \text{(exp)}^{+0.018}_{-0.015} \text{(theor)}$$
- Excellent agreement of two measurements
  $\rightarrow$ Another test of the CKM paradigm and strong constraint on new physics scenarios
Model independent approach –I

$B_q^0 - \overline{B_q^0}$ Mixing ($q = d$ or $s$) and Observables

- $M_{12}^q = (1 + h_q e^{2i\sigma_q}) M_{12}^{q\text{SM}}$
- $\Delta M_q = |1 + h_q e^{2i\sigma_q}| M_{12}^{q\text{SM}}$
- $S_{\psi_K} = \sin[2\beta + \arg(1 + h_d e^{2i\sigma_d})]$
- $S_{\psi_\phi} = \sin[2\beta_s + \arg(1 - h_s e^{2i\sigma_s})]$
- $A_{SL}^q = \text{Im} \left[ \frac{\Gamma_{12}^q}{M_{12}^q(1 + h_q e^{2i\sigma_q})} \right]$
- $\beta_s = \arg \left[ -\frac{(V_{ts} V^*_{tb})}{(V_{cs} V^*_{cb})} \right] \approx 1^\circ$
- $\Gamma_{12}^q$: the absorptive part of the $B_q^0 - \overline{B_q^0}$ mixing
D0 result on semileptonic CP asymmetry:

\[
A_{SL} \equiv \frac{\Gamma(b\bar{b} \to \mu^+\mu^+X) - \Gamma(b\bar{b} \to \mu^-\mu^-X)}{\Gamma(b\bar{b} \to \mu^+\mu^+X) + \Gamma(b\bar{b} \to \mu^-\mu^-X)}
\]

\[\simeq 0.506A_{SL}^d + 0.494A_{SL}^s\]

\[= -0.957 \pm 0.251 \pm 0.146\%\]

BaBar, Belle and CLEO: \(A_{SL}^d = (-4.7 \pm 4.6) \times 10^{-4}\)

So one gets \(A_{SL}^s = -0.0146 \pm 0.0075\)

SM prediction: \(\sim -2 \times 10^{-5}\)
$B_s - \bar{B}_s$ mixing in SUSY models

- Additional contributions from $H^- - t$, $\chi^- - \tilde{U}_i$ and $\tilde{D}_i - g(\tilde{\chi}^0)$

- In generic SUSY models, the squark-gluino loop is parametrically stronger, since it is strong interaction

- Assume that the dominant new physics contribution comes from down squark-gluino loop diagrams

- (see also Ciuchini and Silvestrini; Khalil, Endo and Mshima; Baek ...)

- See Ko, Kramer, Park, Eur.J.Phys. (2002) for $B_d - \bar{B}_d$ mixing, $A_{SL}^d$ and CPV in $B \rightarrow X_d \gamma$

- See Kane, Ko, Kolda, Park, Wang$^2$, PRL (2003) and PRD (2004) for $B_d \rightarrow \phi K_s$ and $B_s - \bar{B}_s$ mixing and related issues
New Physics in $b \rightarrow s$
Before the CDF/D0 measurements
**LL or RR-I (Kane, Ko, Kolda, Park, Wang)**

### LL plots for $m_{\tilde{g}} = \tilde{m} = 400$ GeV

- $(\delta^d_{23})_{LL}$ can not significantly lower $S_{\phi K}$.

  \[ S_{\phi K} \gtrsim 0.05 \quad \text{for} \quad m_{\tilde{g}} = \tilde{m} = 250 \text{ GeV} \]

- Updated Value: $S_{\phi K} = 0.34 \pm 0.21$ (FPCP04)

- Now $S_{\phi K} = 0.47 \pm 0.19$ (Hazumi)
**LL or RR-II**

- But large effects possible in $B_s-\bar{B}_s$ mixing
  Both in the modulus and the phase

- Large $\Delta M_s$ & CP asymmetry in $B_s \rightarrow J/\psi\phi$
  Nice subjects at hadron machines

- $RR$ is similar to $LL$ except for $B \rightarrow X_s\gamma$. 
\textbf{LR for } m_{\tilde{g}} = \tilde{m} = 400 \text{ GeV}

-0.6 < S_{\phi K} < 1 \quad \text{for} \quad |(\delta_{23}^d)_{LR(RL)}| \sim 10^{-2}

- \quad A_{CP}^{b \to s\gamma} \text{ can be large compared w/ SM prediction}

- \quad \text{Not much effect on } B_s - \bar{B}_s \text{ mixing}
After the CDF/D0 measurements
**LL insertion** \( (\tan \beta = 3) \)

(a) \( \text{Re} (\delta_{31}^{UL}) \) vs. \( \text{Im} (\delta_{31}^{UL}) \)

(b) \( B(\bar{b}\to s\gamma) \times 10^{-4} \) vs. \( \phi_s \)

(c) \( S_{CP}^{\delta_{31}} \) vs. \( \phi_s \)

(d) \( A_{CP}^{\delta_{31}} \) vs. \( \phi_s \)
Captions

- Allowed regions on (a) \((\text{Re}(\delta_{23}^d)_{LL}, \text{Im}(\delta_{23}^d)_{LL}))\), and correlation between \(\phi_s\) and each of (b) \(B(B \rightarrow X_s \gamma)\), (c) \(S_{\phi K}\), and (d) \(A_{\text{CP}}^{b \rightarrow s \gamma}\).

- The hatched gray region leads to the lightest squark mass < 100 GeV.

- The hatched region is excluded by the \(B \rightarrow X_s \gamma\) constraint.

- The cyan region is allowed by \(\Delta M_s\).

- The blue region is allowed by the \(\Delta M_s\) and \(\phi_s\).

- The black square is the SM point.

- In Fig. (a), bands bounded by red dashed and solid curves correspond to 1\(\sigma\) and 2\(\sigma\) ranges of \(S_{\phi K}\), respectively.
LL insertion ($\tan \beta = 10$)
**RR insertion (tan β = 3)**

(e) \[ \text{Graph 1} \]

(f) \[ \text{Graph 2} \]

(g) \[ \text{Graph 3} \]

(h) \[ \text{Graph 4} \]
**RR insertion** \((\tan \beta = 10)\)
$LL = RR$ case ($\tan \beta = 3$)

(m) 

(n) 

(o) 

(p)
$LL = RR$ \textbf{case} ($\tan \beta = 10$)
$LL = -RR$ case ($\tan \beta = 3$)

(u) 

(v) 

(w) 

(x)
$LL = -RR$ case ($\tan \beta = 10$)
$a_{SL}$ for $\tan \beta = 3$

() $LL$

() $RR$

() $LL = RR$

() $LL = -RR$
Captions

- The hatched gray region leads to the lightest squark mass $< 100$ GeV.
- The hatched region is excluded by the $B \rightarrow X_s \gamma$ constraint.
- The cyan region is allowed by $\Delta M_a$.
- The blue region allowed both by $\Delta M_s$ and $\phi_s$.
- The black square is the SM point.
- The red dashed and solid lines mark the $1\sigma$ and $2\sigma$ ranges of $a_{SL}$, respectively.
$a_{SL}$ for $\tan \beta = 10$

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Implications for SUSY models

- mSUGRA (?) or GMSB: Universal soft masses at some scale $M_X$, ...
  $\rightarrow \delta (M_X) = 0$

- $\delta$’s are generated by RG evolutions

- For example, in mSUGRA,

  $$(m_{LL}^2)_{ij}(\mu = M_{weak}) \approx -\frac{1}{8\pi^2} Y_t^2 (V_{CKM})_{3i} (V^*_{CKM})_{3j} \left(3m_0^2 + a_0^2\right) \log\left(\frac{M_*}{M_{weak}}\right)$$

- $(\delta^d_{LL})_{23} \approx 9 \times 10^{-3}$ and $(\delta_{LL})_{13} \approx 8 \times 10^{-3} \times e^{-i2.7}$

- $(\delta^d_{LL})_{23}$ is real, no CPV phase $\rightarrow$ No effect on $S_{\phi K}$
\( \delta_{RR} \) from SUSY GUT with Seesaw

For example, in SU(5)+RHN’s, Moroi argues

\[
(m_d^2)_{ij} \simeq -\frac{1}{8\pi^2} [Y_N^\dagger Y_N]_{ij} (3m_0^2 + A^2) \log \frac{M_*}{M_{GUT}}
\]

\[
\simeq -e^{-i(\phi_i^{(L)} - \phi_j^{(L)})} \frac{y_{\nu k}^2}{8\pi^2} [V_L^*]_{ki} [V_L]_{kj} (3m_0^2 + A^2) \log \frac{M_*}{M_{GUT}}.
\]

\[
|{(\delta_{RR})_{23}}| \simeq 2 \times 10^{-2} \left( \frac{M_{N_3}}{10^{14} \text{ GeV}} \right) \text{ with } O(1) \text{ phase}
\]

And RG induced \( \delta \)'s can be small enough to evade the constraint from \( B_s - \bar{B}_s \) mixing, and the double mass insertion can induce effective RL insertion

\( \rightarrow \) Can affect \( S_{\phi K} \)
Induced $LR$ or $RL$ from Double Mass Insertion

\[
(\delta^d_{LR})_{23}^{\text{ind}} = (\delta^d_{LL})_{23} \times \frac{m_b (A_b - \mu \tan \beta)}{\tilde{m}^2}.
\]

- $(\delta^d_{LL,RR})_{23} \sim 10^{-2} \rightarrow (\delta^d_{LR,RL})_{23}^{\text{ind}} \sim 10^{-2}$, if $\mu \tan \beta \sim 30 \text{ TeV}$.
- Natural if $\tan \beta$ is large $\sim 40$.
- For larger $LL, RR$ mixing, even smaller $\mu \tan \beta$ would suffice to induce the needed $LR, RL$ mass insertions of a size $10^{-2} - 10^{-3}$.
- $\delta_{LL,RR}$'s in SUSY flavor models are generically complex, the induced $(\delta^d_{LR})_{23}^{\text{ind}}$ could carry a new CP violating phase leading to deviation in $S_{\phi K}$.
### Implications for SUSY flavor models

| Model                  | $|\delta_{d,LL}^{23}|$ | $|\delta_{d,RR}^{23}|$ | $\tan \beta = 3$ | $\tan \beta = 10$ |
|------------------------|------------------------|------------------------|------------------|------------------|
| LNS (A)                | $\lambda^2$            | $\lambda^4$            | $\cdot$          | $\checkmark$     |
| NS ; CHM (A)           | $\lambda^2$            | 1                       | $\times$         | $\times$         |
| NR (A)                 | $\lambda^2$            | $\lambda^8$            | $\cdot$          | $\checkmark$     |
| CHM (NA)               | $\lambda^2$            | $\lambda^{1/2}$        | $\times$         | $\times$         |
| BHRR, PT (NA)          | $\lambda^2$            | $\lambda^2$            | $\phi_s$         | $\checkmark$     |
| HM (NA)                | $\lambda^3$            | $\lambda^5$            | $\cdot$          | $\cdot$          |
| PS (NA)                | $\lambda^2$            | $\lambda^4$            | $\cdot$          | $\checkmark$     |
| CKN (D)                | $\lambda^2$            | $LL \gg RR$            | $\cdot$          | $\checkmark$     |

Status of some models analyzed Randall and Su, for the two different values of $\tan \beta$. ($A=$Abelian, $NA=$Nonabelian, $D=$Decoupling) ($\cdot$) incompatible with $\phi_s$ but safe otherwise; ($\phi_s$) compatible with $\phi_s$ and safe; ($\sqrt{\cdot}$) currently okay but dangerous; ($\times$) disfavored.
Digression on $(g - 2)_{\mu}$ in effective SUSY


\[ \Delta a_{\mu} = (31.6 \pm 7.9) \times 10^{-10} \]

- Baek, Ko, Park: EPJC 24, 613 (2002)
  - Strong correlation between $B(\tau \to \mu \gamma)$ and $(g - 2)_{\mu}$ in effective SUSY model
  - $B(\tau \to \mu \gamma) < 4.4 \times 10^{-8}$
  - $a_{\mu}^{\text{SUSY}} \lesssim 2(0.6) \times 10^{-10}$ for $\tan \beta = 3(30)$

- See plots
Plots for \((g - 2)_{\mu}\) in effective SUSY
Conclusion

- $B_s - \bar{B}_s$ mixing excludes some SUSY flavor models based on (non)abelian flavor symmetries.
- The LL or RR insertions for small $\tan \beta$ case cannot be large as in the past ($\lesssim 0.5$).
- Large $\tan \beta$ case is strongly constrained by $b \to s\gamma$ (independent of $m_A$) and by $B_s \to \mu^+\mu^-$ for light $m_A$; For moderately high $\tan \beta$, $O(1)$ value of $\phi_s$ tends to conflict with $B \to X_s\gamma$ in the four cases considered here.
- The $LL = \pm RR$ case is even more strongly constrained by $\Delta M_s$ measurement.
- The LR or RL insertions consistent with $b \to s\gamma$ is still OK with $\Delta M_s$, since it does not affect the $B_s - \bar{B}_s$ mixing; however for the same reason, it cannot make an $O(1)$ difference in $\phi_s$. 
Most important is to reach the experimental sensitivity to confirm/falsify the SM predictions for $A_{SL}$.