CP Violation and FCNC Processes in Gauge-Higgs Unification Scenario

@ cpv from b factories to tevatron and lhcb
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I. Introduction

The standard model has unsettled problems in its Higgs sector:
(1) The hierarchy problem (how to guarantee $M_W \ll \Lambda$ naturally?) (SUSY, ADD, R-S, ..)
(3) In spite of the great success of the Kobayashi-Maskawa model, the origin of CP violation still seems to be not conclusive yet.
(4) The origin of hierarchical fermion masses and flavor mixings? there is no guiding principle (symmetry) to restrict the interactions of Higgs

We discuss “Gauge-Higgs Unification (GHU)” scenario as an attractive candidate of New Physics, which is expected to shed some lights on these problems relying on higher dimensional gauge symmetry.
“Gauge-Higgs unification” scenario

unified theory of gauge (s=1) & Higgs (s=0) interactions

: realized in higher dimensional gauge theory

\[ A_M = (A_\mu, A_y) \quad (5D) \quad A_y^{(0)}(x) = H(x) : \text{ Higgs} \]

the idea of gauge-Higgs unification itself is not new:

• N.S. Manton, Nucl. Phys. 58(’79)141.


The scenario was revived:

• H. Hatanaka, T. Inami and C.S.L., Mod. Phys. Lett. A13(’98)2601

The quantum correction to \( m_H \) is finite because of the higher dimensional gauge symmetry \( \rightarrow \) A new avenue to solve the hierarchy problem without invoking SUSY
Related scenarios

- **dimensional deconstruction**: latticized 5D gauge theory. @ N → ∞ limit, the effective potential for H just coincides with what we obtained.

- **little Higgs model**: 4D theory, where G/H of global symmetry provides Higgs as a N-G, may be “dual” to 5D GHU, where A_y associated with G/H of higher dimensional local gauge symmetry provides Higgs (“holographic principle”).

- The low energy limit of superstring theory, e.g. 10 dim. SUSY Y.-M. theory, is a sort of GHU.
Application to the cosmology

- (ultra) natural inflation (N. Arkani-Hamed, H.-C. Cheng, P. Creminelli and L. Randall, Phys.Rev.Lett. 90(’03)221302; T. Inami, Y. Koyama, S. Minakami & C.S.L., Progr. Theor. Phys. (09), to appear) : $A_y^{(0)}$ may be a natural candidate for the inflaton, as the local gauge symmetry stabilizes the potential under the quantum correction

**Minimal GHU standard model:** $SU(3)$ on $M^4 \times (S^1/Z_2)$  
(Kubo, C.S.L. and H. Yamashita, Mod. Phys. Lett. A17(’02)2249)

(N.B.) In the GHU, gauge group should be enlarged, as the Higgs belongs to adjoint repr., while SM Higgs is $SU(2)$ doublet. Recall that in the heterotic string theory, Higgs belonging to the fundamental repr. of $E_6$ comes from adjoint repr. of $E_8$.  


SU(3) → SU(2) x U(1) breaking due to non-trivial $Z_2$-parity assignment (Kawamura):

$$\Psi(-y) = \mathcal{P} \gamma^5 \Psi(y) \quad (\mathcal{P} = \text{diag}(+,+,+,-))$$

Zero-modes of Gauge-Higgs sector:

$$A^{(0)}_{\mu} = \frac{1}{2} \left( \begin{array}{ccc}
W^3_{\mu} + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W^+_{\mu} & 0 \\
\sqrt{2}W^-_{\mu} & -W^3_{\mu} + \frac{B_\mu}{\sqrt{3}} & 0 \\
0 & 0 & -\frac{2}{\sqrt{3}}B_\mu
\end{array} \right)$$

$$A^{(0)}_5 = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc}
0 & 0 & \phi^+ \\
0 & 0 & \phi^0 \\
\phi^- & \phi^0 & 0
\end{array} \right)$$

Higgs doublet

Exactly what we need for the SU(2) x U(1) SM!

$$8 \rightarrow 3 + 1 + 2 \times 2$$
Let us note that in GHU the following issues are non-trivial:

• to break CP
• to realize fermion mass hierarchy
• to accommodate flavor mixing

, as the Yukawa coupling, being gauge coupling, is real and universal among generations, to start with.

On the other hand, once these are realized, the scenario provides new types of mechanisms for CP and flavor violation, which we hope are predictive.
II. CP violation in GHU

Note that as long as higher dimensional gauge theory itself is CP invariant, without phase, CP violation should be a sort of “spontaneous” breaking.

We have discussed two possibilities:

1. CP violation due to compactification

One of a few possibilities to break CP symmetry is to invoke the manner of compactification, which determines the vacuum state of the theory.
Higher dimensional C, P transformations defined by, e.g. C matrix satisfying $C^\dagger \Gamma_M C = - (\Gamma_M)^t$, do not reduce to 4-dim. ones, in general, since they also act on the internal space of fermions.

→ modification is necessary

Interestingly, the modified CP transformation acts on the extra space coordinates non-trivially: it acts as a complex conjugation of the complex homogeneous coordinates for the extra space (in even dimensions).

Take D=6 case for an illustrative purpose.

In the base, where 6D spinor decomposes into two 4-D spinors,

\[ \psi_6 = \begin{pmatrix} \psi \\ \tilde{\psi} \end{pmatrix} \]

gamma matrices are given as

\[ \Gamma^\mu = \gamma^\mu \otimes I_2 = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & \gamma^\mu \end{pmatrix}, \Gamma^y = \gamma^5 \otimes i \sigma_1 = \begin{pmatrix} 0 & i \gamma^5 \\ i \gamma^5 & 0 \end{pmatrix}, \Gamma^z = \gamma^5 \otimes i \sigma_2 = \begin{pmatrix} 0 & \gamma^5 \\ -\gamma^5 & 0 \end{pmatrix} \]

The C matrix satisfying \( C^\dagger \Gamma^M C = -(\Gamma^M)^t \) is easily known to be

\[ C = C_4 \otimes \sigma_2 \quad (C_4 = i \gamma_0 \gamma_2) \quad (\sigma_2^\dagger \sigma_i \sigma_2 = -(\sigma_i)^t) \]

Thus, we modify C and P as

\[ P: \psi_6 \rightarrow (\gamma^0 \otimes \sigma_3) \psi_6, \quad C: \psi_6 \rightarrow (c_4 \otimes \sigma_3) \bar{\psi}_6^t \quad (c_4 = i \gamma_0 \gamma_2). \]
Accordingly the transformation properties of a vector

\[ V^M = \bar{\Psi}_6 \Gamma^M \Psi_6 \]

is uniquely determined and we find (y,z: extra-space coordinates):

\[ P : (y, z) \rightarrow (y, z) \quad C, \quad CP : (y, z) \rightarrow (y, -z) \]

Thus, introducing a complex coordinate

\[ \omega = y + iz \]

CP transf. is nothing but a complex conjugation:

\[ CP : \quad \omega \rightarrow \omega^* \]

(e.g.) Consider Type-I superstring theory with 6-dimensional Calabi-Yau manifold defined by

\[ \sum_{a=1}^{5} (\omega^a)^5 - C(\omega^1 \omega^2 \cdots \omega^5) = 0 \]

CP is broken if the coefficient C is complex (by “complex structure”).
In fact, resultant Yukawa couplings is known to have a CP violating phase for complex C (M. Matsuda, T. Matsuoka, H. Mino, D. Suematsu and Y. Yamada, Prog. Theor. Phys. 79(’88)174).

If we impose phenomenological requirement, of no FCNC at the tree level, the CP phase disappears, unfortunately.

The purpose of our work:

to realize CP violation in the framework of GHU (higher dimensional gauge theories), not string theory, with much simpler compact spaces, such as orbifold.

We discussed the CP violation in the 6-dimensional U(1) GHU model due to the compactification on the orbifold $T^2/Z_4$. 
Easy to know that CP transformation is not compatible with the condition of orbifolding.

In terms of a complex coordinate

\[ \omega = y + iz \quad (y, z \text{ : extra space coordinates}) \]

the \( Z_4 \) orbifold condition is written as

\[ i\omega \sim \omega \]

After the CP transf., the condition reads as

\[ (-i)\omega^* \sim \omega^* \quad : \text{"orientation-changing operator"} \]
Thus, CP tranf. is not compatible with orbifolding condition, and CP symmetry is expected to be broken.

(The model) 6D QED

\[ \mathcal{L}_{QED} = \bar{\psi}_6 \left\{ i \partial^M (i \partial_M + g A_M) - m_B \right\} \psi_6 - \frac{1}{4} \langle \partial_M A_N - \partial_N A_M \rangle^2 \ (M, N = 0 - 3, y, z) \]

\[ \psi_6 = \begin{pmatrix} \psi \\ \psi \end{pmatrix} \]
$Z_4$ eigenvalue $t$ ($t = \pm 1, \pm i$ ($t^4 = 1$)):

$$A_{\mu}(x, i\omega) = A_{\mu}(x, \omega) \quad (t = 1, \ \omega \equiv \frac{y + iz}{\sqrt{2}}).$$

$$A_\omega(x, i\omega) = (-i)A_\omega(x, \omega) \quad (t = -i).$$

$$A_\omega = \frac{A_y - iA_z}{\sqrt{2}}$$

$$\psi(x, i\omega) = (-i)\psi(x, \omega) \quad (t = -i)$$

$$\Psi(x, i\omega) = \Psi(x, \omega) \quad (t = 1)$$

The presence of eigenvalues $t = \pm i$ signal the possibility of CP violation.
The interaction vertices for non-zero KK photons generally have CP violating phases:

\[ A_{\mu}^{(m,n)} \]

\[ \Psi'(0) \]

\[ i g_4 \gamma^\mu \left( \frac{1}{m_f^{(m,n)}} \frac{m + i n}{R} L \right) \]

\[ \psi'(m,n) \]

- Such obtained CP violating phases are confirmed to remain even after the re-phasing of the fields.
- We also have identified “Jarlskog-type” parameters in our model.
The EDM of electron, as a typical CP violating observable, however, is found to vanish at 1-loop level. Unfortunately, we anticipate that we cannot get EDM even at higher loops.

Let us recall that

$$P : (y, z) \rightarrow (y, z) \quad C, \quad CP : (y, z) \rightarrow (y, -z)$$

Thus, P symmetry is not violated by the compactification, as is naively expected in QED.

Since, EDM necessitates both of P and CP violations, we anticipate EDM vanishes in our model, although we expect EDM will get contributions in a realistic theory including the SM, since P should be violated anyway in such a realistic theory.

(NB)

Anyway, CP violation is realized in “QED” with only 1 generation -> new type of mechanism of CP violation, different from that of KM.
2. CP violation due to the VEV of the Higgs
(w./ Y. Adachi and N. Maru, Phys. Rev. D 80(‘09)055025)

Another possibility to break CP is due to the VEV of some field which has odd CP eigenvalue. We argue that the VEV $\langle A_y \rangle$ of the Higgs, or the VEV of Wilson-loop plays the role.

We show that neutron EDM gets contribution already at 1-loop level in the model, though we assume the presence of only 1 generation.

(The model)
5-D SU(3) GHU model compactified on an orbifold $S^1/Z_2$ with a massive bulk fermion in a fundamental representation.

A $Z_2$-odd bulk mass term is introduced for the fermion:

$$M \varepsilon(y) \bar{\psi} \psi \quad (\varepsilon(y) : \text{the sign function})$$

In this case, the orbifold is too simple to break CP, thus only possibility seems to be due to $\langle A_y \rangle$. 
(N.B.)
To get EDM, both P and CP have to be broken. P symmetry, however, is broken anyway by the orbifolding ($\{\gamma^0, \gamma^5\} = 0$).

In 5D CP transf. can be defined just as in the 4D case:
\[
CP : \quad \psi(x^\mu, y) \rightarrow i\gamma^0 \gamma^2 \psi(x^\mu, y)^* 
\]

Correspondingly, the transformations of space-time and fields are fixed as.
\[
CP : \quad x^\mu \rightarrow x_\mu, \quad y \rightarrow y, \quad A_\mu(x^\mu, y) \rightarrow -A^\mu(x_\mu, y)^t, \quad A_y(x^\mu, y) \rightarrow -A_y(x_\mu, y)^t.
\]

Thus we realize that the VEV of $A_y$ has an odd CP-eigenvalue and leads to CP violation.
Actually, when the $Z_2$–odd bulk mass is switched off, we can perform a chiral rotation for $\Psi$, so that the coupling of $A_y$ becomes scalar type and therefore $A_y$ has even CP eigenvalue. Hence, to get physical CP violating effects, the interplay between the VEV $\langle A_y \rangle$ and the bulk mass is crucial.

(The neutron EDM)
EDM appears already at 1-loop level:

![Diagram](image)

Figure 1: The diagrams contributing to EDM at one-loop by the neutral current
The experimental upper bound on the EDM imposes a meaningful lower bound on the compactification scale,

\[ M_c = \frac{1}{R} \geq 2.6 \text{(TeV)}. \]

(NB)

Again, CP violation is realized with only 1 generation \( \rho_{\text{new}} \) type of mechanism of CP violation, different from that of KM.
III. FCNC in GHU

(w./ Y. Adachi, N. Kurahashi and N. Maru, 1005.2455 [hep-ph])

To achieve flavor violation is non-trivial issue in GHU, since Yukawa couplings are originated from gauge coupling, which is universal for all flavors. As a new feature of higher dimensional model with $Z_2$-orbifolding, $Z_2$-odd bulk masses

$$\epsilon(y) M_i \bar{\psi}_i \psi_i \quad (\epsilon(y) : \text{sign function})$$

are allowed, with $M_i$ being different depending on each flavor.

→ new source of the flavor violation, specific to higher dimensional model.
The bulk masses lead to the localization of Weyl fermions and, therefore, exponentially suppressed Yukawa couplings,

\[ \sim g e^{-RM_i} \quad (R \text{ : the size of extra dimension}) \]

(N.B.) In this scenario, the observed hierarchical fermion masses are understood as the result of an originally universal mass of weak scale, with the hierarchy being naturally realized by the exponential factor without fine tuning (though the top mass is a problem).
The bulk masses can be off-diagonal concerning generations, $M_{ij}$, which seems to yield flavor mixing.

Unfortunately, it is not the case: for each representation $R$ of gauge group, a general form of the bulk mass terms

$$M(R)_{ij} \varepsilon(y) \bar{\psi}(R)_i \psi(R)_j \ (i, j : \text{generation index})$$

can be diagonalized by a suitable unitary transformation, leaving the kinetic term invariant.

We are led to introduce brane localized mass terms, which are needed anyway to make exotic states heavy and decouple from the low energy effective theory, as is explained below.
The Model

We consider a five dimensional $SU(3) \otimes SU(3)_{\text{color}}$ model compactified on $S^1/Z_2$. As matter fields, we introduce n-generations of bulk fermion:

$$\psi^i(3) = Q_3^i \oplus d^i,$$

$$\psi^i(\bar{6}) = \Sigma^i \oplus Q_6^i \oplus u^i \ (i = 1, 2, \ldots, n)$$

To eliminate redundant quark doublets (and $\Sigma^i$), brane-localized mass term is introduced (G. Burdman and Y. Nomura, ’03),

$$\int dy \sqrt{2\pi R} \delta(y) \tilde{Q}^i_R(x) \left[ \eta_{ij} Q^j_{3L}(x, y) + \lambda_{ij} Q^j_{6L}(x, y) \right] + ...$$
\( \eta_{ij}, \lambda_{ij} \) may have off-diagonal elements and lead to flavor mixings.

(N.B.)

Still, the flavor violation due to the bulk masses plays an important role for the flavor mixings: in the limit of degenerate bulk masses, \( M_1 = M_2 = \ldots \), flavor mixings are known to disappear. → the interplay between bulk and blane-localized masses is crucial.

This type of flavor mixing is a genuine feature of GHU.

**Natural flavor conservation in GHU**

FCNC has played crucial roles in the discussion of the viability of New Physics. We ask if “natural flavor conservation” is met, i.e. if FCNC processes at tree level are forbidden in GHU.
The condition by Glashow-Weinberg to guarantee natural flavor conservation

“fermions with the same electric charge and the same chirality should possess the same quantum numbers”

is satisfied in our model.

But, the new sauce of flavor violation, i.e. non-degenerate bulk masses, is known to lead to FCNC due to the exchanges of non-zero KK modes of gauge bosons already at the tree level.

(N.B.)

FCNC processes in pure zero-mode sector are still forbidden, as the mode functions for zero-mode gauge bosons are “flat” in extra-dimension. Thus the FCNC processes are suppressed by the inverse powers of the compactification scale, which are under control (the decoupling of KK particles).
FCNC process: $K^0 \leftrightarrow \bar{K}^0$ mixing

We considered the dominant contribution due to the exchange of gluons with non-zero KK modes:

We find
(1) There is a “chiral enhancement factor”,

\[
\left( \frac{m_K}{m_s + m_d} \right)^2
\]

(3) A lower bound of the compactification scale is obtained by comparing the SM prediction (T. Inami & C.S.L., Buras et al., ..) and the data:

\[
\frac{1}{R} > \mathcal{O}(10) \text{TeV}
\]
Summary

1. The gauge-Higgs unification scenario guarantees a finite Higgs mass (in the viewpoint of hierarchy problem) and is an attractive scenario for the physics beyond the standard model.

2. In the scenario the Higgs interactions are governed by gauge principle and it may shed some light on the arbitrariness problem of the interactions.

3. As the Yukawa couplings are real to start with, CP violation is a challenging issue. Two new mechanism of “spontaneous” CP violations were proposed, one by the compactification and one by the VEV of the Higgs → CP violation beyond KM and unitarity triangle → may be (?) responsible for possible deviations from KM scheme discussed in this workshop.

4. As the Yukawa couplings are universal, fermion mass hierarchy and flavor mixing are also challenging issues.
5. In this scenario, hierarchical fermion masses are the result of an (originally) universal mass of weak scale, with the hierarchy being naturally realized by the exponential factor $e^{-RM_i}$ without fine tuning.

7. The non-degenerated bulk mass $M_i$ is a new source of flavor violation beyond the argument of Glashow-Weinberg, and leads to FCNC at the tree level.

7. In the case of $K^0 \leftrightarrow \bar{K}^0$ mixing, the exchange of non-zero KK modes of gluon at the tree level yields the amplitude suppressed by the compactification scale (decoupling), and the data put a lower bound $\frac{1}{R} > O(10) \text{TeV}$. 