Review: 強結合 EWSB 模型
Technicolor & Higgsless

基研研究会 電弱対称性の破れ

March 11–17, 2011

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- その模型は生きているのか？
- 模型はわかるが一体何がしたいのか？
- まだやるのか？惰性でやってるだけではないのか？
- その模型は何かの問題を解いているのか？
- 模型になってるのか？なにか予言できるのか？
- LHC で見つかるのか？検証できるのか？

- テクニカラー模型
  - 素粒子としてのヒッグス粒子は導入しない
  - TeV スケールに強い相互作用
  - くりこみ可能なゲージ相互作用に基づく
  - QCD-like 模型は強い予言能力を持つ
    * S パラメータ
    * クォーク・レプトンの質量の小ささ
    * テクニハドロンのスペクトルと性質
    * 実験でルールアウト（S パラメータ、大きすぎる FCNC)
  - ウォーキングテクニカラー模型の現象論には非摂動ダイナミクスが必要（格子シミュレーションの世界）
    * S パラメータ？
    * クォーク・レプトンの質量？ $\gamma_m \simeq 1$?
    * テクニハドロンのスペクトルと性質？
    * 実験での検証が難しい。

- ヒッグスレス模型
- 素粒子としてのヒッグス粒子は導入しない
- TeV スケールでは強い相互作用は現れない（TeV レゾナンスの相互作用が決定される）
- くりこみ可能ではない低エネルギー有効理論（予言能力は制限される）
  - 余剰次元ヒッグスレス模型
    - ワープした余剰次元バージョン：階層性

      $$M_W \sim 100 GeV, \quad M_{W'} \sim 1 TeV$$

      を出すためには中途半端なワープファクターが必要
    - フラットな余剰次元バージョン：階層性を出すために brane localized kinetic term を導入
    - walking technicolor のデュアル？

- デコンストラクション ヒッグスレス模型
  * N サイト模型にすれば余剰次元模型を良く近似できる
  * カイラル摂動論の方法で、有効理論としての系統的な展開が可能
  * 最小の脱構築模型（3 サイトヒッグスレス模型）
    $\Leftrightarrow$ BESS 模型（HLS 模型）

Technicolor

QCD-like Technicolor

- QCD でのカイラル対称性の力学的破れ

$$\langle \bar{q}q \rangle \neq 0 : \qquad SU(2)_L \times SU(2)_R \to SU(2)_V$$

と同様のメカニズムを電弱対称性の破れに使おうと言うアイデア

- TeV スケールで強結合となるあらたなゲージ相互作用（テクニカラーゲージ相互作用と、テクニフェルミオンを導入。たとえば $SU(N_{\mathrm{TC}})$ 1 ダブレットテクニカラー模型だと、

	$SU(2)_W$	$U(1)_Y$	$SU(N_{\mathrm{TC}})$
Q_L	$\underline{2}$	0	$\underline{N_{\mathrm{TC}}}$
U_R	$\underline{1}$	$1/2$	$\underline{N_{\mathrm{TC}}}$
D_R	$\underline{1}$	$-1/2$	$\underline{N_{\mathrm{TC}}}$

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

$SU(N_{\mathrm{TC}})$ による強い相互作用で生じるテクニクォーク凝縮

$$\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \sim 1TeV^3$$

で電弱ゲージ対称性が力学的に壊れる。

- 1 世代テクニカラー模型だと

	$SU(2)_W$	$SU(3)_c$	$U(1)_Y$	$SU(N_{\rm TC})$
Q_L	$\underline{2}$	$\underline{3}$	$1/6$	$\underline{N_{\rm TC}}$
U_R	$\underline{1}$	$\underline{3}$	$2/3$	$\underline{N_{\rm TC}}$
D_R	$\underline{1}$	$\underline{3}$	$-1/3$	$\underline{N_{\rm TC}}$
L_L	$\underline{2}$	$\underline{1}$	$-1/2$	$\underline{N_{\rm TC}}$
N_R	$\underline{1}$	$\underline{1}$	0	$\underline{N_{\rm TC}}$
E_R	$\underline{1}$	$\underline{1}$	-1	$\underline{N_{\rm TC}}$

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \qquad L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}$$

- クォーク・レプトンの質量を説明するには、ETC 相互作用

$$\bar{Q}\gamma^{\mu}qE_{\mu}$$

を導入する。ETC ゲージ粒子 E_{μ} が非常に重いと仮定すれば

$$\frac{1}{M_{\rm ETC}^2}\bar{Q}_L\gamma^{\mu}q_L\bar{q}_R\gamma^{\mu}Q_R \sim \frac{1}{M_{\rm ETC}^2}\bar{Q}_LQ_R\bar{q}_Rq_L$$

テクニフェルミオンの対凝縮によって、クォーク・レプトンに

$$\sim \frac{1TeV^3}{M_{\rm ETC}^2}$$

程度の質量が生じる。

$$\underbrace{\begin{matrix}\nu_{eR}\\ e_R\\ \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix}\\ u_R\\ d_R\\ \begin{pmatrix}u_L\\ d_L\end{pmatrix}\end{matrix}\;,\;\underbrace{\begin{matrix}\nu_{\mu R}\\ \mu_R\\ \begin{pmatrix}\nu_{\mu L}\\ \mu_L\end{pmatrix}\\ c_R\\ s_R\\ \begin{pmatrix}c_L\\ s_L\end{pmatrix}\end{matrix}\;,\;\underbrace{\begin{matrix}\nu_{\tau R}\\ \tau_R\\ \begin{pmatrix}\nu_{\tau L}\\ \tau_L\end{pmatrix}\\ t_R\\ b_R\\ \begin{pmatrix}t_L\\ b_L\end{pmatrix}\end{matrix}\;,\;\left\{\begin{matrix}N_R\\ E_R\\ \begin{pmatrix}N_L\\ E_L\end{pmatrix}\\ U_R\\ D_R\\ \begin{pmatrix}U_L\\ D_L\end{pmatrix}\end{matrix}\right\}_{i=1,\cdots N_{TC}}}_{\text{E''TC}}}_{\text{E'TC}}}_{\text{ETC}}$$

クォーク質量に対応する ETC スケール

q	u	d	s	c	b
m_q (GeV)	6MeV	10MeV	200MeV	1.5GeV	5GeV
$\Lambda^2_{E_q}$	2200TeV^2	1400TeV^2	68TeV^2	9TeV^2	2.7TeV^2

- すべてをゲージ相互作用で説明するとっても魅力的な可能性なんだけど、模型の構築という点では、、、
- クォーク・レプトンの実際の質量や、CKM 行列、KM 位相をすべて説明しうる ETC 模型を作るのにはまだ誰も成功していない。
- トップクォーク質量 170GeV を ETC の枠組みで説明するのには無理がある
- ETC ゲージ粒子が質量を獲得するためには、あらたに ETC ゲージ対称性を破る「テクニカラー」を導入する必要がある。

- 現象論的には、、、
  - 大きすぎる FCNC の問題
  - 軽すぎる pseudo Nambu-Goldstone boson の問題
  - 電弱精密測定との整合性の問題

大きすぎる FCNC の問題

s クォーク質量を説明する ETC ゲージ粒子の媒介によって、FCNC ４体フェルミオン相互作用

$$\frac{\lambda^2}{\Lambda_{E_s}^2}(\bar{s}_L \gamma^\mu d_L)^2, \qquad \Lambda_{\Delta S=2}^2 = \frac{\Lambda_{E_s}^2}{\lambda^2} \simeq 1000\text{TeV}^2$$

の存在が予言されてしまう。これは K-$\bar{K}$ 混合からの制限

$$\Lambda_{\Delta S=2}^2 > 10^6\text{TeV}^2$$

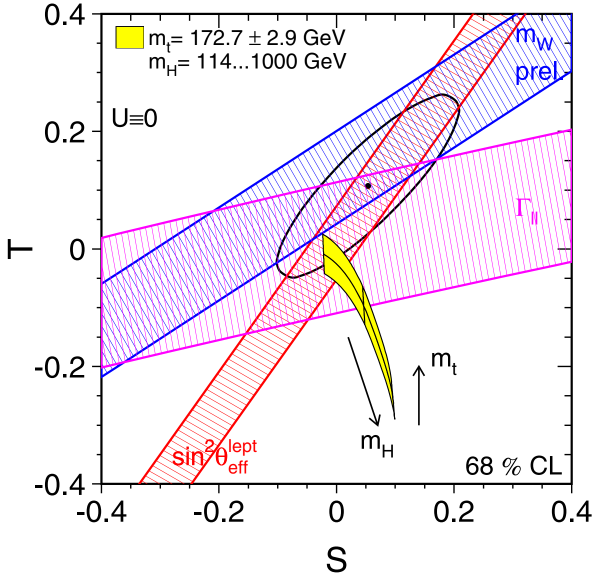
とあきらかに矛盾する。

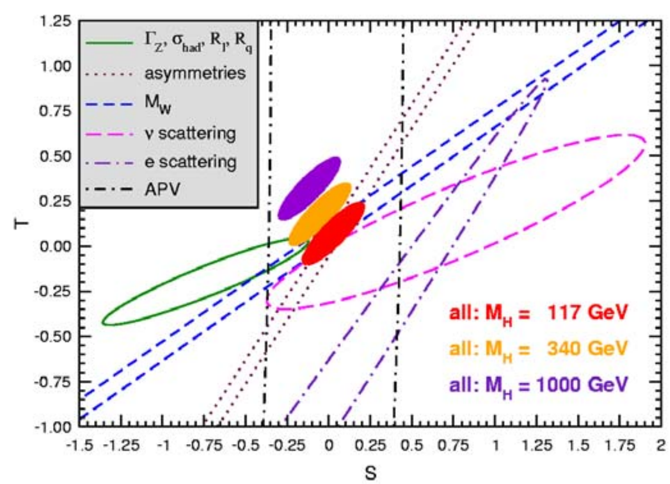
電弱精密測定との整合性

重いフェルミオンの１ループレベルでの S パラメータへの寄与

$$S = \frac{N_D}{6\pi}$$

S-T plot

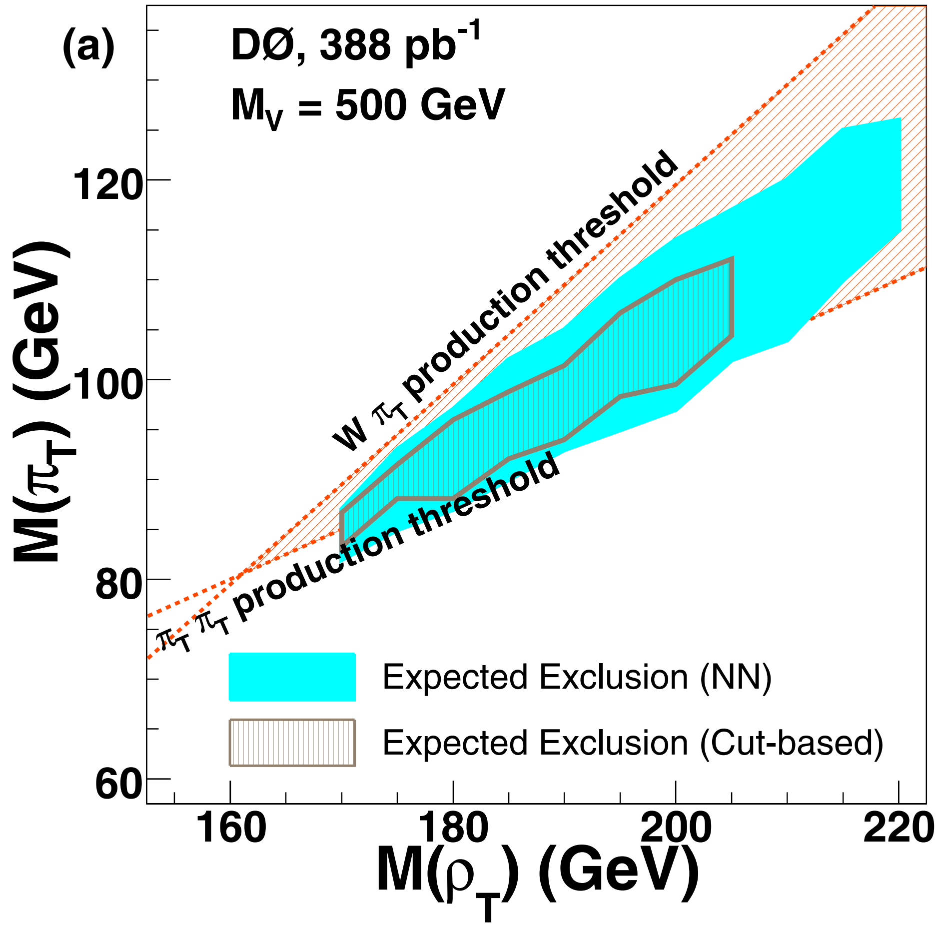




http://lepewwg.web.cern.ch/LEPEWWG/

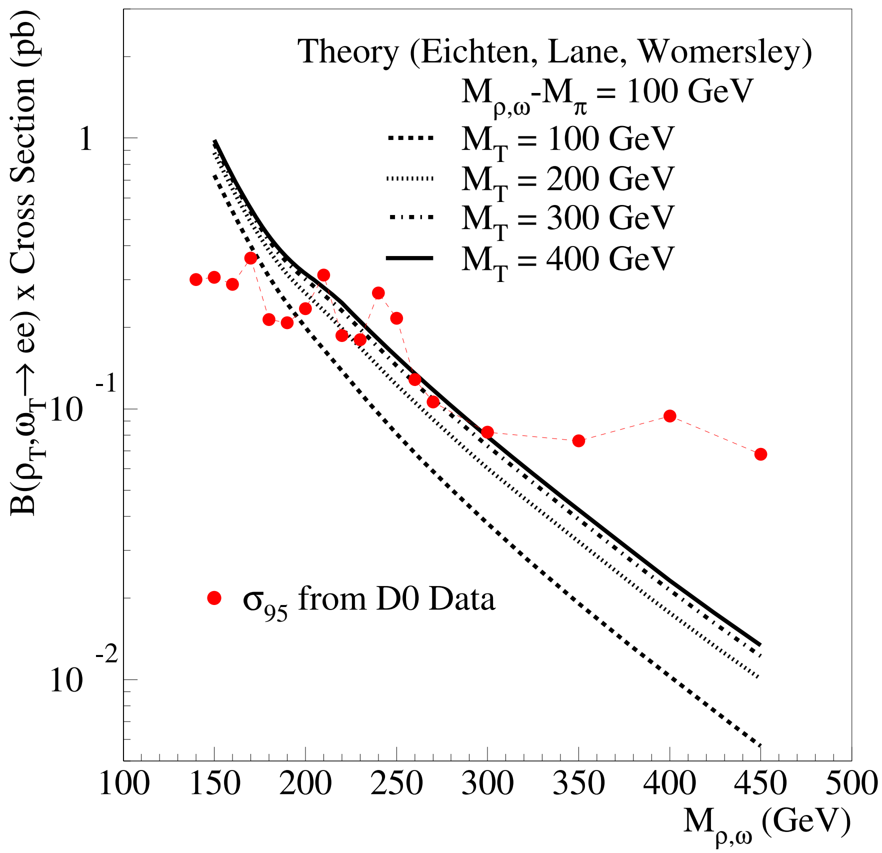
Erler and Langacker, in RPP2010

Tevatron での bound の例



Chivukula, Narain, Womersley, in RPP2010

Tevatron での bound の例



Chivukula, Narain, Womersley, in RPP2010

Walking Technicolor

- Λ_E を大きくしても、クォーク質量がそんなに小さくならないためのメカニズム
- テクニカラー模型でのクォーク質量

$$m_q = \frac{1}{\Lambda_{E_q}^2} \langle \bar{Q}Q \rangle$$

- 大きな異常次元を仮定（Λ_{TC} から Λ_{E_s} までのあいだのスケールで TC 相互作用が強いままであることを仮定）

$$m_s = \frac{1}{\Lambda_{E_s}^2} \left(\frac{\Lambda_{E_s}}{\Lambda_{TC}} \right)^{\gamma_m} \langle (\bar{Q}Q)_{\Lambda_{TC}} \rangle$$

- FCNC からの制限 $\Lambda_{E_s} > 10^6 \text{TeV}^2$ を満たすためには

$$\gamma_m \gtrsim 1$$

- 具体的に実現するには（Walking/Conformal Technicolor)
- 質量によらないくりこみ群で IR-FP をもつテクニカラーゲージ相互作用を考える。（質量が無視できる高エネルギー領域では、コンフォーマル対称性を持ち、強結合を保つ。）
- 低エネルギーでテクニクォークが対凝縮を起こし、質量を持つことで理論からデカップルし、テクニカラーゲージ相互作用は閉じ込めを起こす。

Walking/Conformal Technicolor

- くりこみ群
$$\mu^2 \frac{d}{d\mu^2}\alpha_{TC} = -b_0\alpha_{TC}^2 - b_1\alpha_{TC}^3 + \cdots$$

- IR-FP をもつには $\qquad b_0 > 0, \qquad b_1 < 0$

  が必要 (摂動論を仮定)

$$\begin{aligned} b_0 &= \frac{1}{12\pi}(11C_A - 4N_f T_R), \\ b_1 &= \frac{1}{24\pi^2}(17C_A^2 - N_f T_R(10C_A + 6C_F)) \end{aligned}$$

- N_f は十分小さくて S パラメータと矛盾しないことが必要
- IR-FP でのゲージ相互作用の値 α_{TC}^* テクニクォーク対凝縮を引き起こすのに十分なほどが大きいと仮定する。

$$\alpha_{TC}^* > \alpha_{\rm crit}$$

具体例: Minimal Walking Technicolor (Sannino Model)

- 模型

	$SU(2)_{TC}$	$SU(2)_W$	$U(1)_Y$
Q_L	$\underline{3}$	$\underline{2}$	0
U_R	$\underline{3}$	$\underline{1}$	1/2
D_R	$\underline{3}$	$\underline{1}$	$-1/2$

$$C_A = C_F = T_R = 2, \qquad N_f = 2.$$

- IR-FP を実現

$$b_0 = \frac{1}{2\pi} > 0, \qquad b_1 = \frac{17}{4\pi^2} < 0$$

- Not too bad S

$$S \simeq \frac{1}{2\pi}$$

コメント

- $SU(2)_W$ の Witten anomaly をキャンセルするためには $SU(2)_{TC}$ シングレットが必要

- $SU(2)_W$, $U(1)_Y$, ETC をスイッチオフするとグローバル対称性として、

$$SU(4)$$

が存在。テクニフェルミオン対凝縮により、低エネルギーでは

$$SU(4)/SO(4)$$

のテクニパイオン粒子 (24-16=8) で記述される。(W に食べられる 3 個の NG 粒子以外に、5 個のテクニパイオンが存在し、コライダー現象に影響する)

テクニカラーのまとめ

- QCD-like technicolor はすでにルールアウトされている。
- Walking technicolor がうまくいくためには、
  - 仮定しているダイナミクスが本当に存在するかの検証が必要。(格子ゲージ理論)
  - ダイナミクスが存在していたとしても、ETC の枠組みにうまく入れるのは、チャレンジしがいのある課題。
  - コライダー現象の解析には、テクニベクトル粒子の質量と結合定数の他に、テクニパイオンの性質が大きく影響する。テクニカラー模型全体に網をかけるクリーンなコライダーシグナルは (いまのところ) 見当たらない。そのようなコライダーシグナルを検討すべき時期にきている。

Higgsless

The role of the Higgs boson in the SM:

- Renormalizability :

  W and Z are gauge bosons (universality of weak interaction).

  Explicit breaking of electroweak gauge symmetry makes the theory *non-renormalizable*. We need, at least, one Higgs boson so as to feed W and Z masses in a renormalizable manner.

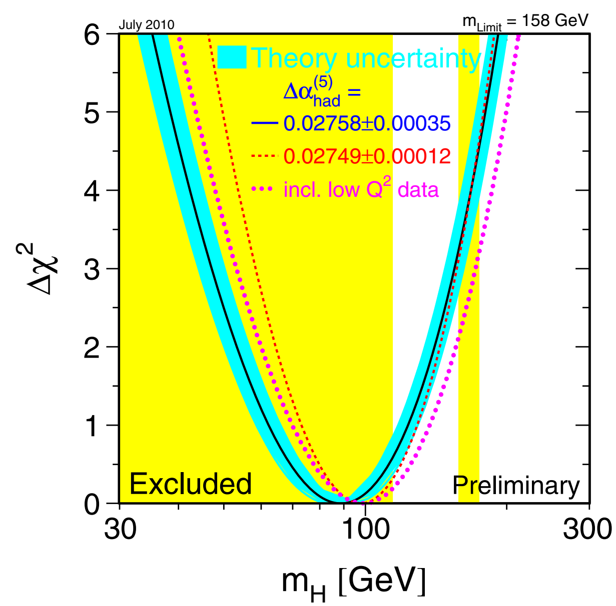
- Unitarity :

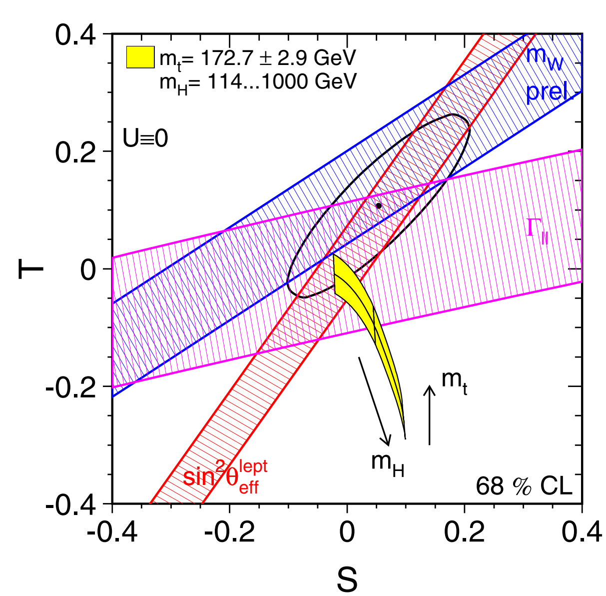
  The longitudinal W boson (W_L) scattering amplitude grows as the CM energy increases. If there is no Higgs boson, it eventually violates the unitarity.

Life without a Higgs

Renormalizability :

New physics (cutoff scale of SM) is believed to exist at TeV. In principle, renormalizability is not a primary issue in this sense. However, the lack of renormalizability usually implies a loss of robust predictability. How can we ensure the consistency with the existing precision electroweak measurements without introducing a Higgs boson then?





Unitarity

$W_L W_L$ scattering amplitude grows as the CM energy increases.

$$\mathcal{M} \propto \frac{s}{v^2}$$

The probability of the $W_L W_L$ scattering exceeds unity at the energy scale $s = 8\pi v^2$.

$$\Downarrow$$

unitarity violation

Two possibilities

Unitarity bound : $\sqrt{8\pi}v \simeq 1.2$TeV

- non-perturbative casse

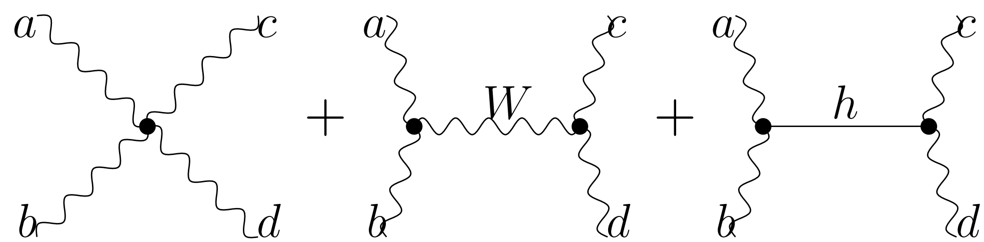
  The theory becomes non-perturbative above the unitarity bound. The unitarity should be recovered in a non-perturbative manner. (technicolor models, predictability may be lost.)

- perturbative case

  The $W_L W_L$ scattering behavior is modified thanks to the existence of particles lighter than the unitarity bound (predictable model.)

In the standard model, perturbative unitarity is guaranteed by the spin-0 Higgs exchange diagram.

$i\mathcal{M}(ab \to cd) =$



$+$ crossed.

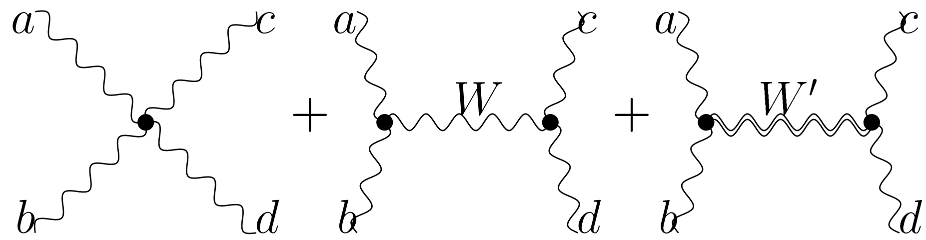
we notice that the $s \sim E^2$ term cancels

$$\mathcal{M}(ab \to cd) = \mathcal{M}_{\text{gauge}} + \mathcal{M}_{\text{Higgs}} = \frac{s}{v^2} \frac{M_h^2}{M_h^2 - s} \delta^{ab} \delta^{cd} + \cdots .$$

- The amplitude agrees with the low energy theorem at $s \ll M_h^2 = \lambda v^2$.
- The amplitude approaches to a constant λ at the region $s \gg M_h^2 = \lambda v^2$. The theory is perturbative if the constant λ is sufficiently small.

Can a spin-1 resonance unitarize the $W_L W_L$ scattering amplitude?

$$i\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) =$$



$+$crossed.

Answer: ***Yes!*** *if we suitably adjust WWW' coupling.*

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \frac{1}{3v^2}\left((s-u)\frac{M_{W'}^2}{M_{W'}^2 - t} + (s-t)\frac{M_{W'}^2}{M_{W'}^2 - u}\right)\delta^{ab}\delta^{cd} + \cdots$$

Cancellation of bad high-energy behavior is achieved through *exchange of massive spin-1 particle W'*.

Note, however,

we need to introduce yet another massive vector particle W'' so as to unitarize the $W'_L W'_L \to W'_L W'_L$ amplitude ....

$$\Downarrow$$

A tower of massive vector particles:

$$W, \quad W', \quad W'', \quad W''', \cdots$$

This situation is naturally realized in gauge theory with an *extra dimension*

A tower of massive Kaluza-Klein modes

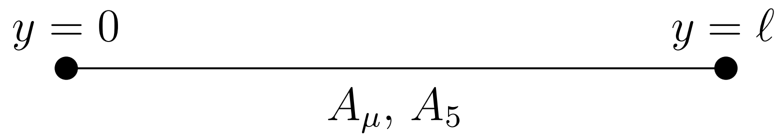
Chivukula, Dicus and He ; Csaki, Grojean, Murayama, Pilo and Terning

Gauge symmetry breaking through boundary conditions

Higgsless models in 5D

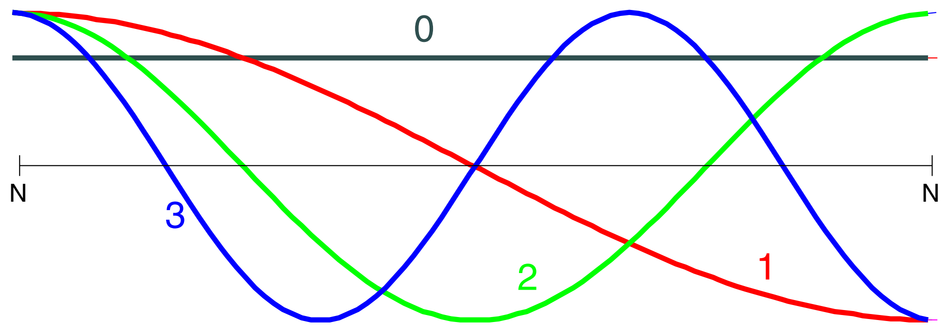
Gauge symmetry breaking through boundary conditions

5D gauge theory with an interval extra dimension



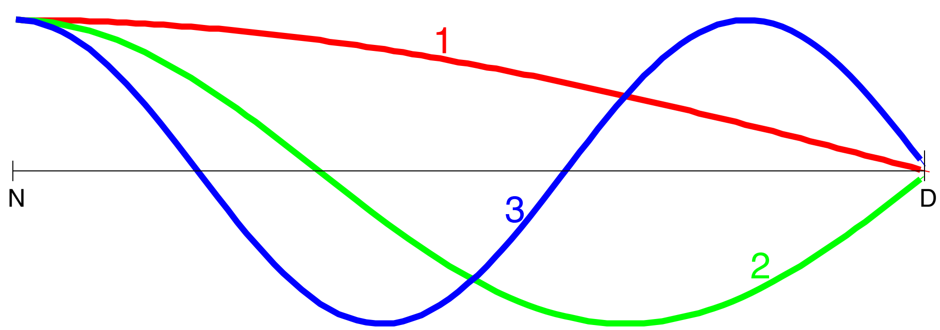
BC: Neumann(N)? Dirichlet(D)?

1. $\partial_y A_\mu(x,y)|_{y=0} = 0$ (N), $\quad \partial_y A_\mu(x,y)|_{y=\ell} = 0$ (N) [NN]



massless spin-1 field: unbroken 4D gauge symmetry

2. $\partial_y A_\mu(x,y)|_{y=0} = 0$ (N), $\quad A_\mu(x,y)|_{y=\ell} = 0$ (D) [ND]



absence of massless spin-1 field: 4D gauge sym is broken

4D gauge sym and spectrum

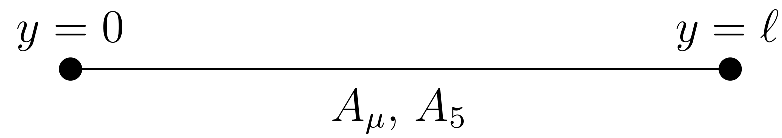
In addition to the massive spin-1 KK particles, we have

1. [NN]: massless spin-1 (unbroken 4D gauge sym)
   photon
2. [ND]: absence of massless particle (4D gauge syms are all broken)
3. [DN]: absence of massless particle (4D gauge syms are all broken)
   $W^{\pm}$, Z
4. [DD]: massless spin-0 (gauge and global syms are broken)

Applying this mechanism to EWSB, we can push up the unitarity vilation scale around 10TeV.

- R. Sekhar Chivukula, D. A. Dicus and H. J. He, "Unitarity of compactified five dimensional Yang-Mills theory," Phys. Lett. B **525**, 175 (2002) [arXiv:hep-ph/0111016].
- C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, "Gauge theories on an interval: Unitarity without a Higgs," Phys. Rev. D **69**, 055006 (2004) [arXiv:hep-ph/0305237].

Brane localized Higgs field (aka RS model)



BC: Neumann BC at both brane

$\partial_y A_\mu(x,y)|_{y=0} = 0$ (N), $\quad \partial_y A_\mu(x,y)|_{y=\ell} = 0$ (N) $\qquad$ [NN]

Brane localized Higgs ϕ:

$$S_{\text{Higgs}} = \int dy\delta(y-\ell+\epsilon)\left[(D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi)\right]$$

$\Downarrow$ (VEV v_b)

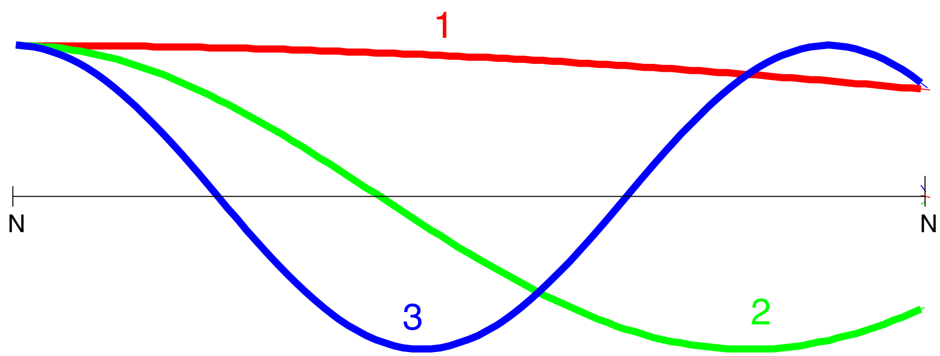
$$\int dy\delta(y-\ell+\epsilon)v_b^2 A_\mu A^\mu$$

KK mode equation for the gauge field

$$\left[-\partial_y^2 + \delta(y-\ell+\epsilon)g^2 v_b^2\right]\chi^{(n)}(y) = M_n^2\chi^{(n)}(y)$$

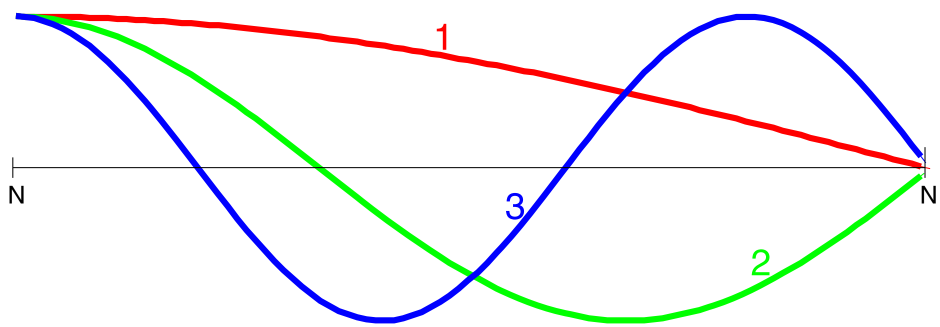
$\Leftrightarrow$ 1-dim Schroedinger eq. with δ-function repulsive force

1. finite v_b case



δ-function repulsive force at $y = \ell$ brane affects the wave-function form.

2. $v_b \to \infty$ case



δ-function repulsive force at $y = \ell$ brane affects the effective boundary condition at the brane.

Remarks

- Brane localized Higgs with an infinite VEV.

$$\Updownarrow \text{ (equivalent)}$$

Dirichlet BC (Higgsless)

- The KK gauge boson spectrum remains finite around the compactification scale even in the infinite Higgs VEV limit.
- Higgsless models can be regarded as a variant of usual RS model.

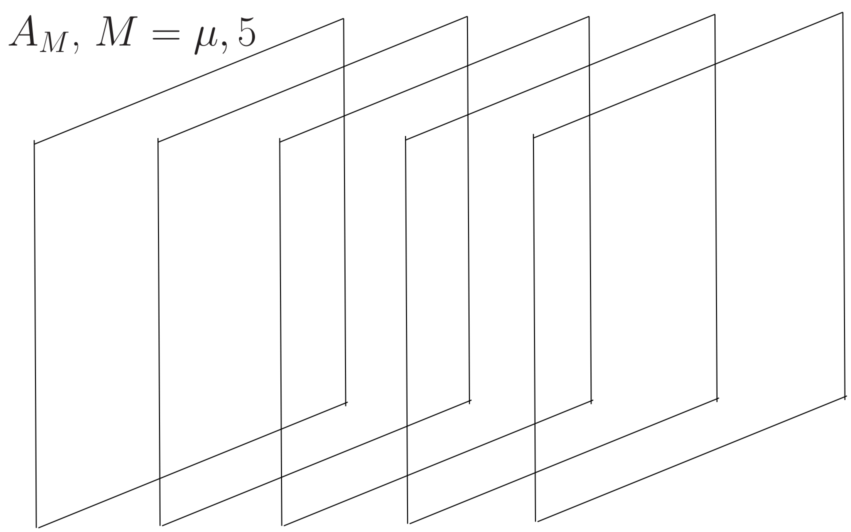
Effective theory viewpoint

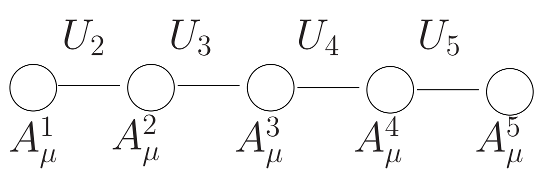
— Deconstruction —

Deconstruction of boundary conditions

Deconstruction (latticization) of extra dimension

Arkani-Hamed, Cohen and Georgi ; Hill, Pokorski and Wang





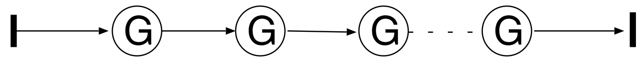
moose diagram

a : lattice spacing

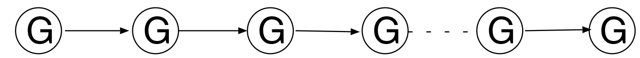
- $A^j_\mu = A_\mu(x, y = ja)$: gauge field at site j
- $U_j = \exp(i \int_{(j-1)a}^{ja} dy A_5(x, y))$: link field. non-linear σ model field.

Note: Moose model can be viewed as a generalization of Bando-Kugo-Yamawaki's Hidden Local Symmetry (HLS) model (Phys.Rep.164,217(1988)) + Georgi's vector symmetry model (NPB331,311(1990)).

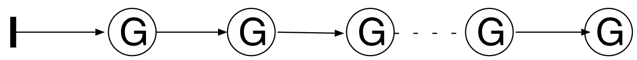
Deconstructions of an interval in "moose" notation:

[DD] 

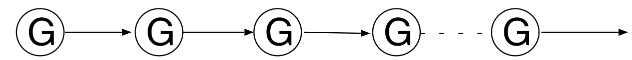
$$\#(U_j) = \#(A^j_\mu) + 1.$$

[NN] 

$$\#(U_j) = \#(A^j_\mu) - 1.$$

[DN] 

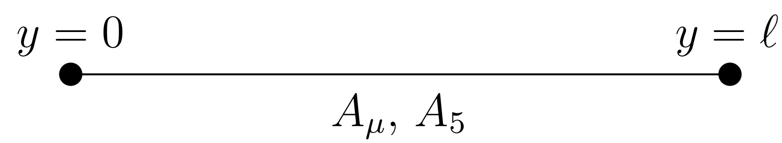
$$\#(U_j) = \#(A^j_\mu).$$

[ND] 

$$\#(U_j) = \#(A^j_\mu).$$

which correspond to 5D gauge theories with an interval compactification:

H.-J. He, hep-ph/0412113



[DD] $A_\mu(x,y)|_{y=0} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=0} = 0$ (N),
$A_\mu(x,y)|_{y=\ell} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=\ell} = 0$ (N).

[NN] $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $\quad A_5(x,y)|_{y=0} = 0$ (D),
$\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $\quad A_5(x,y)|_{y=\ell} = 0$ (D).

[DN] $A_\mu(x,y)|_{y=0} = 0$ (D), $\quad \partial_5 A_5(x,y)|_{y=0} = 0$ (N),
$\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $\quad A_5(x,y)|_{y=\ell} = 0$ (D).

Advantages for deconstruction in 5D Higgsless models

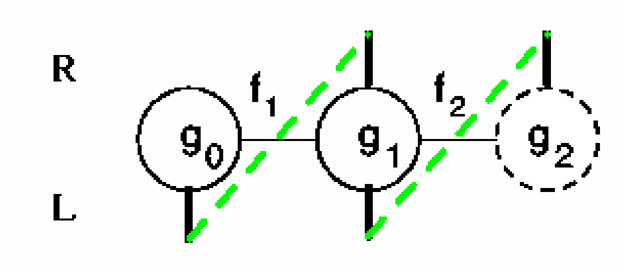
- Familiar language of spontaneous gauge symmetry breaking (gauged nonlinear σ model).
- Easier to understand the physics behind the delay of unitarity violation.
- Easier to calculate corrections to electroweak interactions.
- Allowing for arbitrary background 5D geometry, spatially dependent gauge couplings, and brane kinetic terms.
- Easier to perform loop analysis using well-known chiral perturbation method.

Very Low Energy Effective Theory

How can we contruct a model consistent with the existing precision electroweak and flavor measurements?

Three-site Higgsless model (Prof. Simmons' lecture)

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi and M.T., PRD72 075012 (2006);
See also Bando, Kugo, Yamawaki's HLS model Phys.Rep.164,217(1988).



$SU(2) \times SU(2) \times U(1)$ gauge theory

- The gauge sector is precisely that of the BESS model. Casalbuoni et al., PLB 155 95 (1985))
- Fermion mass terms:

$$\mathcal{L}_f = -m_1 \bar{\psi}_{L0} U_1 \psi_{R1} - M \bar{\psi}_{R1} \psi_{L1} - \bar{\psi}_{L1} U_2 \begin{pmatrix} m_{2u} & \\ & m_{2d} \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} + \text{h.c.}.$$

- For simplicity, we examine the case $f_1 = f_2 = \sqrt{2} v$ and work in the limit

$$\frac{g_0}{g_1} \ll 1, \quad \frac{g_2}{g_1} \ll 1, \qquad \text{and thus,} \quad g_W \simeq g_0, \quad g_Y \simeq g_2.$$

Fermion mass matrix: (seesaw like)

$$\begin{pmatrix} m_1 & 0 \\ M & m_{2f} \end{pmatrix} \equiv M \begin{pmatrix} \varepsilon_L & 0 \\ 1 & \varepsilon_{fR} \end{pmatrix}, \qquad \varepsilon_L \equiv \frac{m_1}{M}, \quad \varepsilon_{fR} \equiv \frac{m_{2f}}{M}$$

Light fermion mass:

$$m_f \simeq \frac{m_1 m_{2f}}{\sqrt{M^2 + m_{2f}^2}} = \frac{\varepsilon_L}{\sqrt{1+\varepsilon_{fR}^2}} m_{2f}$$

and its eigenstate

$$\psi_L^{f,\mathrm{light}} \simeq -\left(1 - \frac{\varepsilon_L^2}{2}\right) \psi_{L0}^f + \varepsilon_L \psi_{L1}^f$$

where we assumed $\varepsilon_{fR} \ll 1$.

Heavy (KK) fermion mass:

$$M_{f,KK} \simeq \sqrt{M^2 + m_{2f}^2} \simeq M$$

For $M \gg v$, we can integrate out the heavy KK-fermion. The fermion delocalization effect can then be replaced by an operator

$$\mathcal{L}'_f = -x_1 \bar{\psi}_L (i \not{D} U_1 \cdot U_1^\dagger) \psi_L, \qquad x_1 \equiv \varepsilon_L^2, \quad \varepsilon_L = \frac{m_1}{M}$$

ψ_L is a left-hand fermion at site-0,

$$D_\mu \psi_L = \partial_\mu \psi_L - i g_0 W_{0\mu} \psi_L .$$

S-parameter at tree level

$$S = \frac{4\pi}{g_1^2} \left(1 - \frac{2 g_1^2}{g_0^2} x_1 \right)$$

vanishes in the ideal delocalization limit:

$$x_1 = \frac{g_0^2}{2 g_1^2}, \qquad g_{W' f f} = 0.$$

c.f. Anichini, Casalbuoni, and De Curtis, PLB348 521 (1995).

Higgsless confronts electroweak precision tests at one-loop

Matsuzaki, Chivukula, Simmons, and M.T., PRD75, 073002 (2007)
Chivukula, Simmons, Matsuzaki, and M.T., PRD75, 075012 (2007)
Abe, Matsuzaki, and M.T., PRD78, 055020 (2008)
See also, Abe, Chivukula, Christensen, Hsieh, Matsuzaki, Simmons, and M.T., PRD79, 075016 (2009)

- $S = 0$ can be achieved by assuming the ideal delocalization limit $g_{W'ff} = 0$ in the tree level.
- We have no symmetry reason which guarantees the smallness of S and T parameters at the loop level.
- There do exist loop induced higher derivative operators contributing to S and T parameters in the electroweak chiral perturbation theory (Appelquist and Bernard).

$$\alpha_{(1)1}\text{tr}\left[W_{(0)\mu\nu}U_1 W_{(1)}^{\mu\nu}U_1^\dagger\right] + \alpha_{(2)1}\text{tr}\left[W_{(1)\mu\nu}U_2\frac{\tau^3}{2}B^{\mu\nu}U_2^\dagger\right]$$

$$\beta_{(2)}\frac{f_2^2}{4}\text{tr}\left[U_2^\dagger D_\mu U_2\tau^3\right]\text{tr}\left[U_2^\dagger D^\mu U_2\tau^3\right]$$

- Even if we assume coefficients of these higher derivative operators vanish at the cutoff scale Λ ($\Lambda \gg M_W'$), these coefficients can be generated through the electroweak chiral perturbation renormalization group:

$$\mu \frac{d}{d\mu} \alpha_{(i)1} = \frac{1}{6(4\pi)^2}, \quad \mu \frac{d}{d\mu} (\beta_{(2)} f_2^2) = \frac{3}{4(4\pi)^2} g_Y^2 f_2^2.$$

We evaluate the size of these low energy induced coefficents as

$$\alpha_{(i)1}(\mu) \simeq -\frac{1}{6(4\pi)^2} \ln \frac{\Lambda}{\mu}, \quad \beta_{(2)}(\mu) \simeq -\frac{3}{4(4\pi)^2} g_Y^2 \ln \frac{\Lambda}{\mu}$$

- Matching with the usual electroweak chiral perturbation theory (aka 2-site model), which includes

$$\alpha_1 \mathrm{tr} \left[W_{\mu\nu} U B^{\mu\nu} U^\dagger \right]$$

$$\beta \frac{f_2^2}{4} \mathrm{tr} \left[U^\dagger D_\mu U \tau^3 \right] \mathrm{tr} \left[U^\dagger D^\mu U \tau^3 \right]$$

At $\mu = M_{W'}$,

$$\alpha_1 = -\frac{v^2}{4M_{W'}^2}\left(1 - \frac{x_1}{2}\frac{M_{W'}^2}{M_W^2}\right) + \frac{1}{2}\alpha_{(1)1} + \frac{1}{2}\alpha_{(2)1}$$

$$\beta = \frac{1}{2}\beta_{(2)}$$

- We evaluate α_1 and β at $\mu = M_Z$ by solving RGE from $\mu = M_{W'}$ down to $\mu = M_Z$

$$\alpha_1|_{\mu=M_Z} = \alpha_1|_{\mu=M_{W'}} - \frac{1}{6(4\pi)^2}\ln\frac{M_{W'}}{M_Z}$$

$$\beta|_{\mu=M_Z} = \beta|_{\mu=M_{W'}} - \frac{3}{4(4\pi)^2}g_Y^2\ln\frac{M_{W'}}{M_Z}$$

- These operators contribute S and T parameters at $\mu = M_Z$ scale

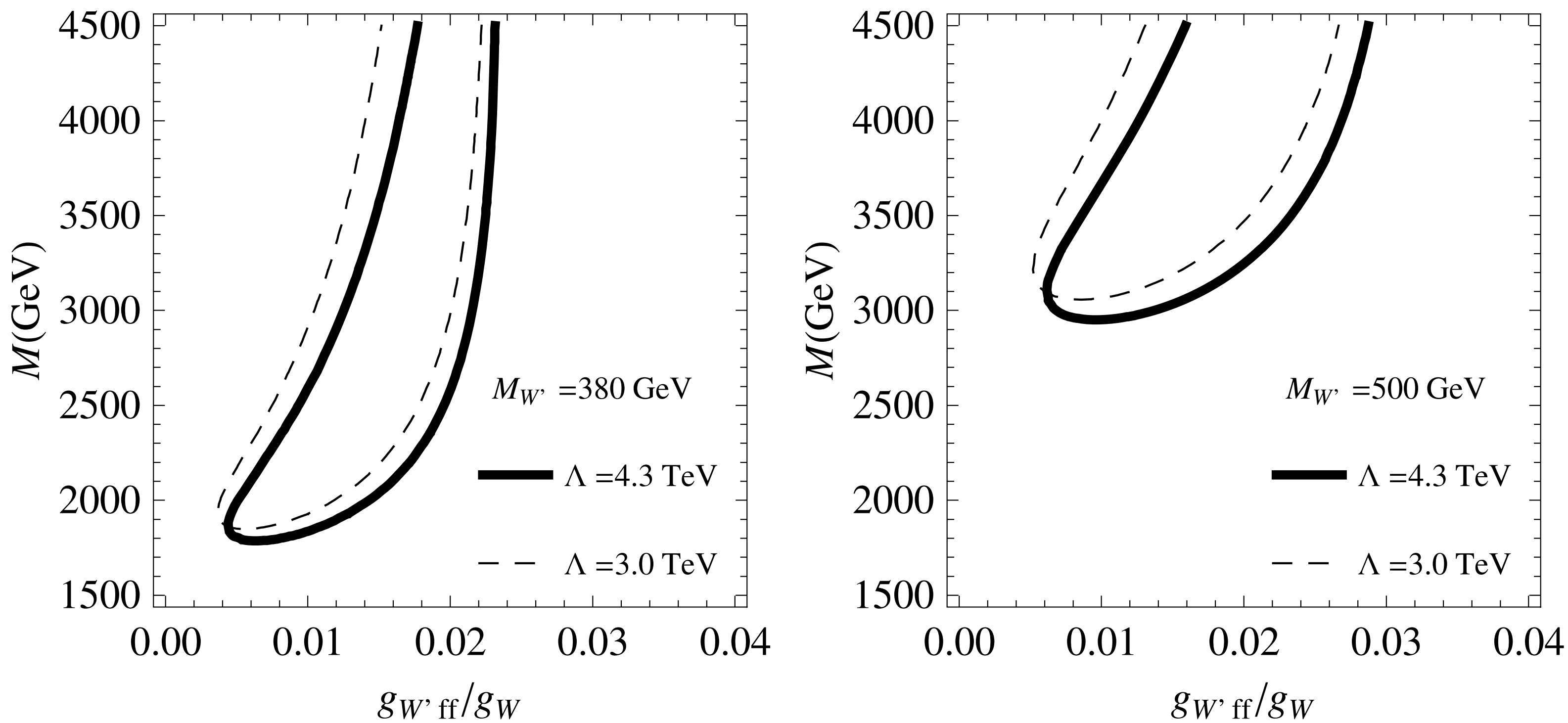
$$S \simeq \frac{4\pi v^2}{M_{W'}^2}\left(1 - \frac{x_1}{2}\frac{M_{W'}^2}{M_W^2}\right) + \frac{1}{6\pi}\ln\frac{\Lambda}{M_Z}$$

$$\alpha T \simeq -\frac{3g_Y^2}{4(4\pi)^2}\ln\frac{\Lambda}{M_{W'}} - \frac{3g_Y^2}{2(4\pi)^2}\ln\frac{M_{W'}}{M_Z} + \frac{1}{16\pi^2}\frac{{m_t}^4}{M^2 v^2 x_1^2},$$

where we have also added top and KK-top contribution to T parameter.

- Re-tuning of the delocalization parameter x_1 is required to make the theory consistent with the precision electroweak measurements. Corrections to the ideal delocalization: $g_{W'ff} \neq 0$.

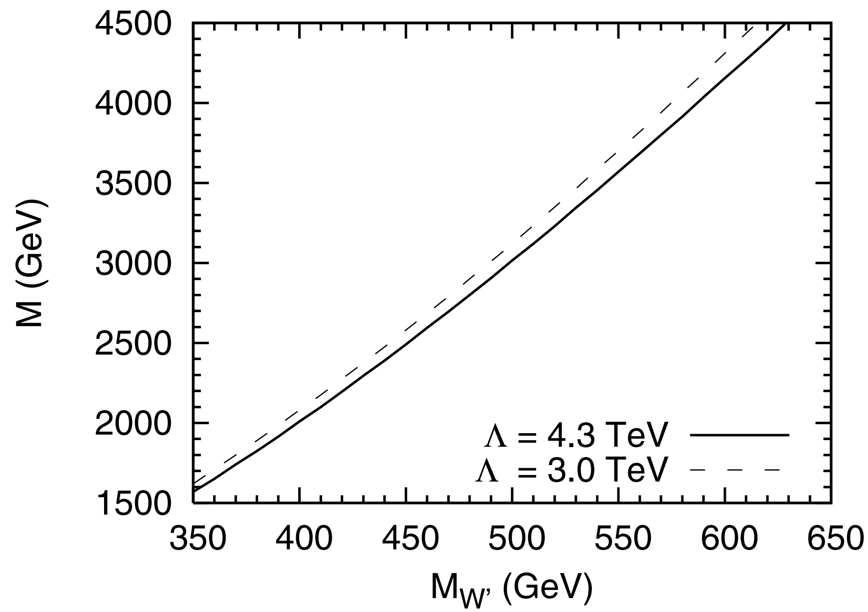
One loop constraint from precision electroweak measurements (95%CL):



T. Abe, S. Matsuzaki, and M.T., PRD78, 055020 (2008)

The cutoff dependence is small.

Tiny non-zero $W'ff$ coupling (correction to the ideal delocalization).



- The limit $M_{W'} \gtrsim 380$GeV is from the ZWW measurement at LEP2.

- The cutoff Λ should satisfy

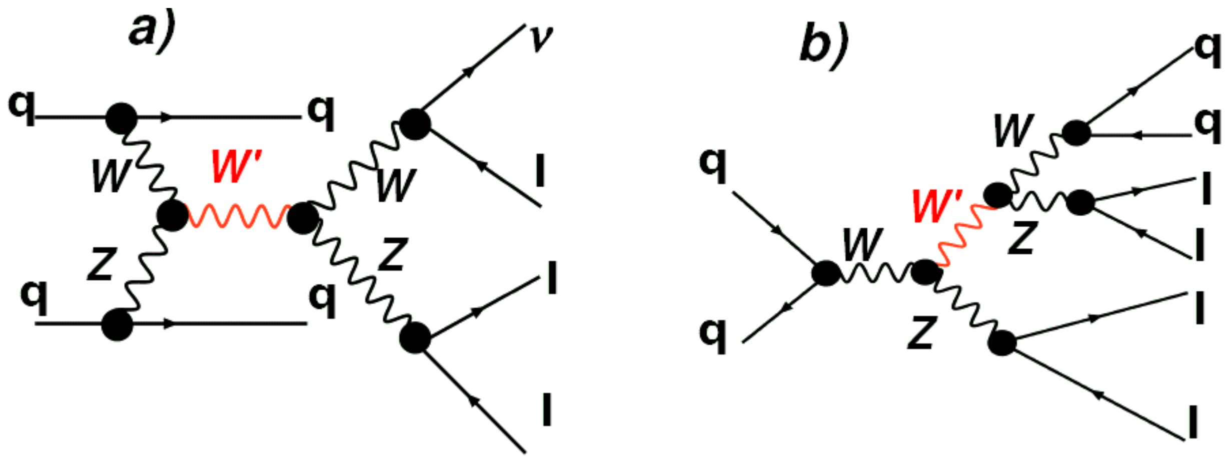
$$M < \Lambda \lesssim 4\pi f_1 = 4\pi f_2 = 4.3\text{TeV},$$

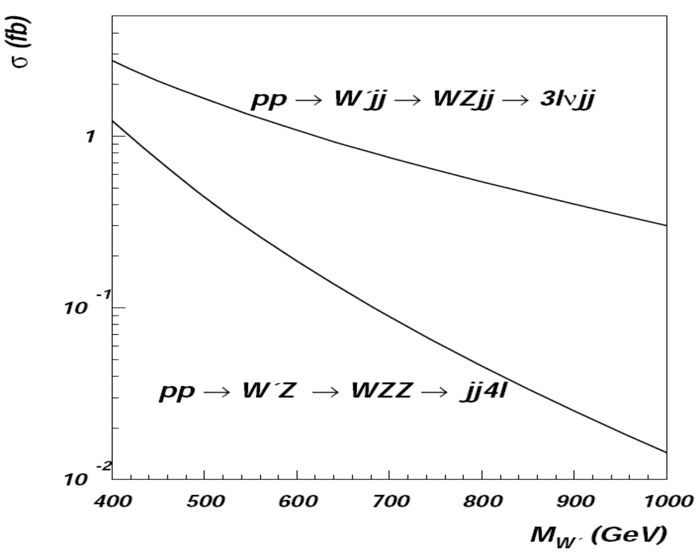
which implies

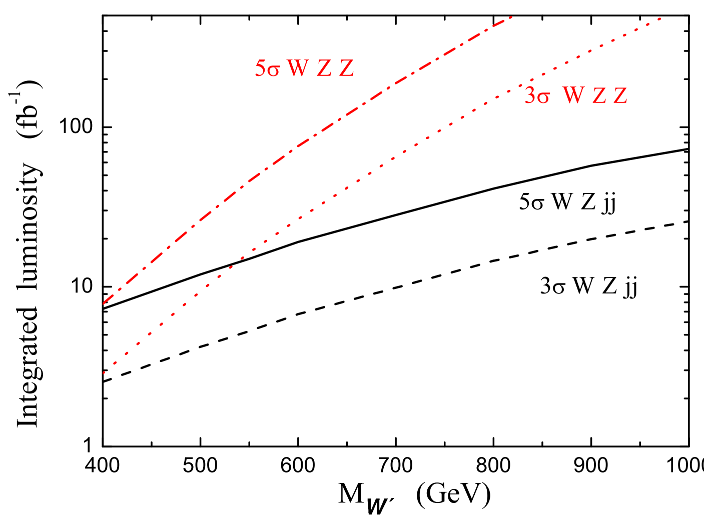
$$M_{W'} \lesssim 600\text{GeV}$$

W' production cross sections through $W'WZ$ vertex:

H.-J. He et al., arXiv:0708.2588

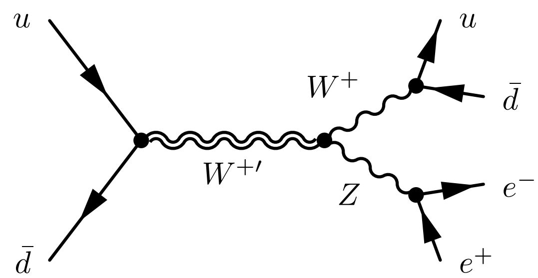


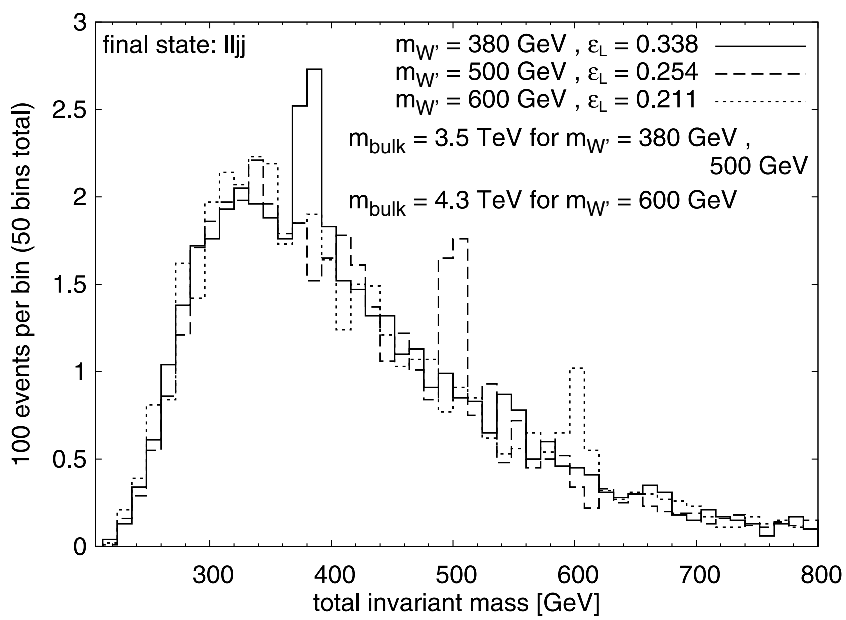




W' production cross sections at LHC through $W'ff$ vertex:

T. Ohl and C. Speckner, arXiv:0809.0023





100fb^{-1}

Higgsless confronts flavor precision tests at one loop

Abe, Chivukula, Simmons, and M.T., in preparation.
See also, Kurachi and Onogi, arXiv:1006.3414

- Flavor physics observables such as ϵ_K and $B(b \to s\gamma)$ are known to provide severe constraints on models with a warped extra dimension. See, e.g., Agashe, Azatov and Zhu, arXiv:0810.1016.
- Actually, in RS model with fully "anarchic" Yukawa couplings, an extremely severe KK gluon mass limit

$$M_{\rm KK} \gtrsim 33\text{TeV}$$

is obtained from the K-$\bar{K}$ mixing constraints.
(Cśaki-Falkowski-Weiler, arXiv:0804.1954)

- If this severe bound on $M_{\rm KK}$ equally applies to Higgsless models, it is almost impossible to solve the unitarity problem in the RS framework using the KK boson exchange.
- Here, we try to address flavor issues in the three site model by studying its flavor structures.

Flavor structure in the three site model

- Consider quark "Yukawa" sector of the three site model,

$$-\bar{q}_L^0 U_1 \mathbf{m}_1 q_R^1 - \bar{q}_L^1 \mathbf{M} q_R^1 - \bar{q}_L^1 U_2 \mathbf{m}_{2u} \left(\begin{array}{c} u_R^2 \\ 0 \end{array} \right) - \bar{q}_L^1 U_2 \mathbf{m}_{2d} \left(\begin{array}{c} 0 \\ d_R^2 \end{array} \right),$$

  where summation over flavor indices is implicit.

- We consider $SU(3)$ flavor rotations

$$q_L^0 \to L q_L^0, \quad q_L^1 \to L_D q_L^1, \quad q_R^1 \to R_D q_R^1,$$

$$u_R^2 \to R_u u_R^2, \quad d_R^2 \to R_d d_R^2$$

- If the mass-parameters were simultaneously changed as

$$\mathbf{m}_1 \to L \mathbf{m}_1 R_D^\dagger, \quad \mathbf{M} \to L_D \mathbf{M} R_D^\dagger, \quad \cdots$$

  the theory would be symmetric under these flavor rotations.

- Without any futher assumptions on these masses, one could go to a basis where $\mathbf{m}_1$ and $\mathbf{m}_{2d}$ are diagonal — but one would not have freedom to diagonalize the other $\mathbf{m}_{2u}$ and $\mathbf{M}$. Flavor is violated not only by $\mathbf{m}_{2u}$ but also by $\mathbf{M}$. (Non-minimal flavor violation). We expect the theory would be constrained severely from its precision flavor tests.

- In the three site model, we often assume both $\mathbf{m}_1$ and $\mathbf{M}$ are proportional to the identity matrix. (Minimal Flavor Violation, MFV). With MFV assumption, we can go to a basis where $\mathbf{m}_1$, $\mathbf{M}$ and $\mathbf{m}_{2d}$ are diagonal. The flavor vilation is governed solely by $\mathbf{m}_{2u}$ in this case.

- However, even in this case, flavor-violating contributions to $\mathbf{M}$ are induced at one-loop.

- We consider

$$\mathbf{M} = \begin{pmatrix} M & & \\ & M & \\ & & M \end{pmatrix} + \mathbf{\Delta M}, \qquad \delta \equiv \frac{\mathbf{\Delta M}}{M}$$

in the basis where $\mathbf{m}_1$ and $\mathbf{M}$ are diagonal. In this talk, we focus on the constraints of δ_{sd} derived from the K-$\bar{K}$ mixing. For more extensive study using varieties of quark and lepton flavor measurements, see Abe-Chivukula-Simmons-M.T.

We consider K-$\bar{K}$ mixing operator

$$C_1^K(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)$$

In the three site model, the coefficient C_1^K is calculated as

$$C_1^K = \frac{1}{v^2}\frac{m_1^4}{M^4}(\delta_{sd})^2$$

We assume the ideal delocalization

$$\frac{m_1^2}{M^2} = 2\frac{M_W^2}{M_{W'}^2},$$

which leads to

$$C_1^K \simeq 1.1\cdot 10^{-7}(\delta_{sd})^2\left(\frac{400\text{ GeV}}{M_{W'}}\right)^4 \text{ GeV}^{-2}.$$

95%CL allowed range obtained by UTfit group

$$-9.6 \cdot 10^{-13}\text{GeV} < \text{Re}(C_1^K) < 9.6 \cdot 10^{-13}\text{GeV},$$

and

$$-4.4 \cdot 10^{-15}\text{GeV} < \text{Im}(C_1^K) < 2.8 \cdot 10^{-15}\text{GeV}.$$

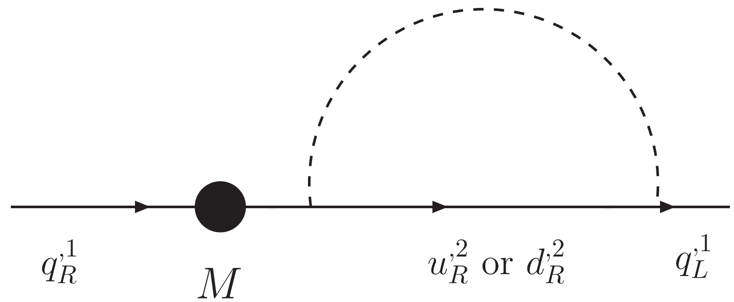
The bounds on δ_{sd} are therefore

$$-9.0 \cdot 10^{-6} < \text{Re}(\delta_{sd})^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 < 9.0 \cdot 10^{-6},$$

and

$$-4.1 \cdot 10^{-8} < \text{Im}(\delta_{sd})^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 < 2.6 \cdot 10^{-8}.$$

We next compare these limits with one-loop expected values



$$\delta_{sd}^{\mathrm{one-loop}} \sim \frac{1}{(4\pi)^2}\frac{m_t^2}{2v^2}\frac{M^2}{m_1^2}V_{ts}^* V_{td}$$

Assuming the ideal delocalization, we obtain

$$(\delta_{sd}^{\mathrm{one-loop}})^2 \sim (0.38 - 0.38i)\cdot 10^{-10}\left(\frac{M_{W'}}{400\mathrm{GeV}}\right)^4$$

which is consistent with the phenomenological bounds we obtained.

Unitarity in the WW Scattering

Unitarity in the $W_L W_L$ scatteing

Let us consider the longitudinally polarized W (W_L). It's polarization vector

$$\epsilon^{\mu}_{(L)} = \frac{E}{M_W}\begin{pmatrix} \frac{|\vec{p}|}{E} \\ \frac{\vec{p}}{|\vec{p}|} \end{pmatrix}, \qquad E^2 = |\vec{p}|^2 + M_W^2, \quad \epsilon_{(L)\mu}\epsilon^{\mu}_{(L)} = -1$$

grows for large $E \gg M_W$. Naive power counting in E suggests

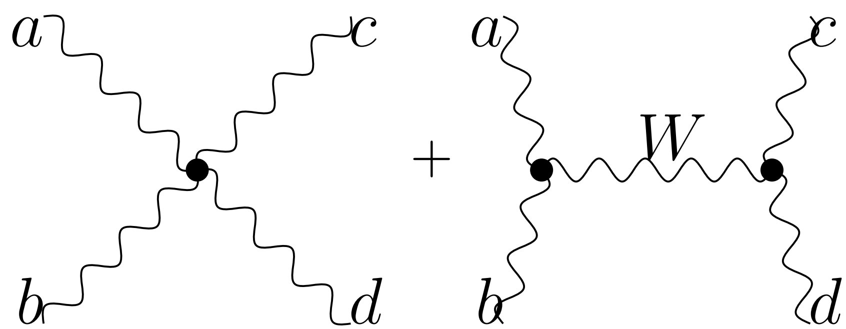
$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) \propto |\epsilon_{(L)\mu}|^4 \sim \frac{E^4}{M_W^4}$$

Unitarity seems to be violated in the high energy $W_L W_L$ scattering.

For simplicity, we consider the $g_Y = 0$ case. $(Z = W^3)$

Two Feynman diagrams contributing to the WW scattering

$i\mathcal{M}_{\text{gauge}}(ab \to cd) =$



$+$ crossed.

Contribution from the $4W$ vertex (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWWW}\frac{E^4}{M_W^4}\left\{-(1+\cos\theta)[3-\cos\theta-2\frac{M_W^2}{E^2}]\right.$$
$$\left.-(1-\cos\theta)[3+\cos\theta-2\frac{M_W^2}{E^2}]\right\}$$

t-channel W exchange (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWW}^2 \frac{E^4}{M_W^4} \left\{ (1-\cos\theta)[3+\cos\theta - 2\frac{M_W^2}{E^2}] + \frac{1}{2}(1+11\cos\theta)\frac{M_W^2}{E^2} \right\} + \cdots$$

u-channel W exchange (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWW}^2 \frac{E^4}{M_W^4} \left\{ (1+\cos\theta)[3-\cos\theta - 2\frac{M_W^2}{E^2}] + \frac{1}{2}(1-11\cos\theta)\frac{M_W^2}{E^2} \right\} + \cdots$$

Each diagram behaves $\sim E^4/M_W^4$ at high energy.
The leading E^4/M_W^4 term cancels in the amplitude thanks to the Ward-Takahashi identity

$$g_{WWWW} = g_{WWW}^2 = g_W^2$$

of the gauge symmetry.

$$\mathcal{M}_{\rm gauge}(ab \to cd) = \delta^{ab}\delta^{cd} g_W^2 \frac{E^4}{M_W^4}\frac{M_W^2}{E^2} + \cdots = \delta^{ab}\delta^{cd} g_W^2 \frac{E^2}{M_W^2} + \cdots$$

We use $E^2 = s/4$, $M_W^2 = g_W^2 v^2/4$

$$\mathcal{M}_{\rm gauge}(ab \to cd) = \frac{s}{v^2}\delta^{ab}\delta^{cd} + \frac{t}{v^2}\delta^{ac}\delta^{bd} + \frac{u}{v^2}\delta^{ad}\delta^{bc} + \cdots.$$

This form agrees with the low energy theorem (equivalence theorem)

B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. Lett. **38**, 883 (1977); Phys. Rev. D **16**, 1519 (1977).

This is clear because

NG boson $\Rightarrow W_L$

(Higgs mechanism)

Unitarity

If the $W_L W_L$ scattering amplitude is completely given by the low energy theorem

The probability of the $W_L W_L$ scattering exceeds unity at the $s = 8\pi v^2$ energy scale.

$$\Downarrow$$

unitarity violation

Two possibilities

Unitarity bound : $\sqrt{8\pi}v \simeq 1.2$TeV

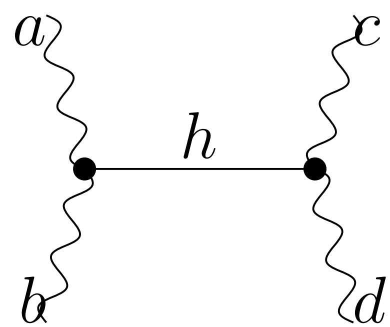
- perturbative case
  The $W_L W_L$ scattering behavior is modified thanks to the existence of particles lighter than the unitarity bound (predictable model.)

- non-perturbative casse
  The theory beoomes non-perturbative above the unitarity bound. The unitarity should be recovered in a non-perturbative manner. (predictability may be lost.)

Perturbative unitarity in the standard model Higgs sector

Higgs exchange diagram

$$i\mathcal{M}_{\rm Higgs}(ab \to cd) =$$



$+$ crossed.

$$\mathcal{M}_{\rm Higgs}(ab \to cd) = g_{hWW}^2 \frac{s^2}{M_W^4} \frac{1}{M_h^2 - s} \delta^{ab} \delta^{cd} + \cdots .$$

Using the standard model Higgs relation

$$g_{hWW} = \frac{M_W^2}{v}$$

we notice that the $s \sim E^2$ term cancels

$$\mathcal{M}(ab \to cd) = \mathcal{M}_{\text{gauge}} + \mathcal{M}_{\text{Higgs}} = \frac{s}{v^2} \frac{M_h^2}{M_h^2 - s} \delta^{ab} \delta^{cd} + \cdots .$$

- The amplitude agrees with the low energy theorem at $s \ll M_h^2 = \lambda v^2$.
- The amplitude approaches to a constant λ at the region $s \gg M_h^2 = \lambda v^2$. The theory is perturbative if the constant λ is sufficiently small.

It is easy to extend this to multi-Higgs models

From the perturbative unitarity requirement of the $W_L W_L$ scattering

$$4 \sum_n g_{h_{(n)}WW}^2 = (4 g_{WWWW} - 3 g_{WWW}^2) M_W^2$$

hWW coupling $\Leftrightarrow$ perturbative unitarity $\Leftrightarrow$ Essence of a "Higgs"

Physics ensuring the unitarity in the $W_L W_L$ scattering

- $\mathcal{O}(E^4)$ cancellation

$$g_{WWWW} = g_{WWW}^2$$

gauge symmetry

- $\mathcal{O}(E^2)$ cancellation

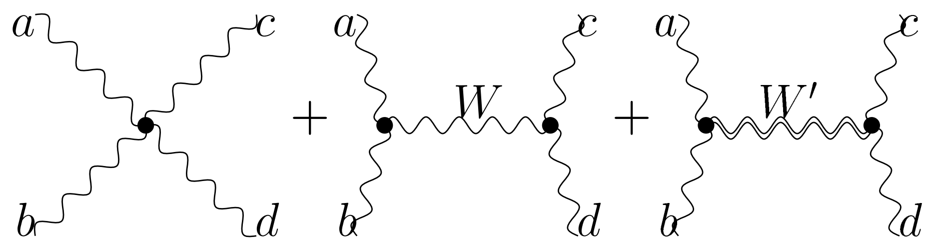
$$4\sum_n g_{h_{(n)}WW}^2 = (4g_{WWWW} - 3g_{WWW}^2)M_W^2$$

unitarity sum rules

The $h_{(n)}WW$ coupling determimes the Higgs property of $h_{(n)}$.

Can a spin-1 resonance unitarize the $W_L W_L$ scattering amplitude?

$$i\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) =$$



+crossed.

Answer: **Yes!** *if we suitably adjust WWW' coupling.*

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \frac{1}{3v^2}\left((s-u)\frac{M_{W'}^2}{M_{W'}^2 - t} + (s-t)\frac{M_{W'}^2}{M_{W'}^2 - u}\right)\delta^{ab}\delta^{cd} + \cdots$$

Cancellation of bad high-energy behavior is achieved through *exchange of massive spin-1 particle W'*.

Note, however,

we need to introduce yet another massive vector particle W'' so as to unitarize the $W'_L W'_L \to W'_L W'_L$ amplitude ....

$$\Downarrow$$

A tower of massive vector particles:

$$W, \quad W', \quad W'', \quad W''', \cdots$$

This situation is naturally realized in gauge theory with an *extra dimension*

A tower of massive Kaluza-Klein modes

Chivukula, Dicus and He ; Csaki, Grojean, Murayama, Pilo and Terning

Gauge symmetry breaking through boundary conditions

Unitarity sum rules

- Spin-0 exchange case (Higgs):

$$g_{WWWW} = g_{WWW}^2, \qquad g_{WWWW} M_W^2 = 4 \sum_n g_{h_{(n)}WW}^2$$

- Spin-1 exchange case (Higgsless):

$$g_{WWWW} = \sum_n g_{WWW_{(n)}}^2, \qquad 4 g_{WWWW} M_W^2 = 3 \sum_n g_{WWW_{(n)}}^2 M_{W_{(n)}}^2$$

- If there exist spin-0 and spin-1 simultaneously (gaugephobic Higgs): Cacciapaglia et al. `hep-ph/0611358`, `c.f. Hikasa and Igi`

$$g_{WWWW} = \sum_n g_{WWW_{(n)}}^2,$$

$$4 g_{WWWW} M_W^2 = 3 \sum_n g_{WWW_{(n)}}^2 M_{W_{(n)}}^2 + 4 \sum_n g_{h_{(n)}WW}^2$$

General Sum Rules

Chivukula, He, Kurachi, Simmons, M.T.

Phys.Rev.D78, 095003 (2008) [arXiv:0808.1682 [hep-ph]]

General Sum Rules

We try to genelarize the unitarity sum rule

$$g_{WWWW} = \sum_n g^2_{WWW_{(n)}}, \qquad 4g_{WWWW}M_W^2 = 3\sum_n g^2_{WWW_{(n)}} M^2_{W_{(n)}}$$

- Finite deconstruction
- Inelastic scattering

$$nn \to mm$$

- Generalize to the transverse gauge boson scattering such as $LL \to LT$, $LL \to TT$, $LT \to TT$.

$\Rightarrow$ important for the collider confirmation of the Higgsless scenario

We deduce the sum rules using the equivalence theorems

$$M(L_{(n)}L_{(n)} \to L_{(m)}L_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}\pi_{(m)}), \qquad E^4, E^2, (E^0)$$

$$M(L_{(n)}L_{(n)} \to L_{(m)}T_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}T_{(m)}), \qquad E^3, E^1$$

$$M(L_{(n)}L_{(n)} \to T_{(m)}T_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to T_{(m)}T_{(m)}), \qquad E^2, E^0$$

$$M(L_{(n)}T_{(n)} \to T_{(m)}T_{(m)}) \simeq M(\pi_{(n)}T_{(n)} \to T_{(m)}T_{(m)}), \qquad E^1$$

In the continuum limit ($N \to \infty$),

$$M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}\pi_{(m)}) = 0, \qquad M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}T_{(m)}) = 0$$

Results

$$G_4^{nnmm} = \sum_k G_3^{nnk} G_3^{mmk} = \sum_k G_3^{nmk} G_3^{nmk},$$

$$2(M_n^2 + M_m^2) G_4^{nnmm} + \sum_k (G_3^{nmk})^2 \left[\frac{(M_n^2 - M_m^2)^2}{M_k^2} - 3M_k^2 \right]$$

$$= \frac{4}{v^2} M_n^2 M_m^2 \tilde{G}_4^{nnmm},$$

$$\vdots$$

Here

$$G_4^{nm\ell k} \equiv g_{W_{(n)} W_{(m)} W_{(\ell)} W_{(k)}}, \quad G_3^{nm\ell} \equiv g_{W_{(n)} W_{(m)} W_{(\ell)}},$$

$$\tilde{G}_4^{nm\ell k} \equiv g_{\pi_{(n)} \pi_{(m)} \pi_{(\ell)} \pi_{(k)}},$$

In the continuum limit, we have $\tilde{G}_4^{nm\ell k} = 0$

These sum rules agree with the unitarity sum rule in the continuum limit

$$G_4^{nnnn} = \sum_k G_3^{nnk} G_3^{nnk},$$

$$4M_n^2 G_4^{nnnn} = 3 \sum_k M_k^2 (G_3^{nnk})^2,$$

Sum rules in the deconstructed Higgsless models

W mass matrix M_W^2, π mass matrix $\tilde{M}_W^2$ are given by ($g_Y = 0$),

$$M_W^2 = Q^T Q, \qquad \tilde{M}_W^2 = Q Q^T,$$

where

$$Q \equiv \frac{1}{2} \begin{pmatrix} g_0 f_1 & -g_1 f_1 & & & \\ & g_1 f_2 & -g_2 f_2 & & \\ & & \ddots & \ddots & \\ & & & g_{N-1} f_N & -g_N f_N \\ & & & & g_N f_{N+1} \end{pmatrix}.$$

Diagonalization

$$\begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & \ddots & \\ & & & M_{N+1} \end{pmatrix} = M_W^{\rm diag} = R^T Q^T \tilde{R} = \tilde{R}^T Q R$$

The matrices R, $\tilde{R}$ represent the KK gauge bson $W_{(n)}$ (NG boson $\pi_{(n)}$) "wave function" in the extra dimension.

G_3^{nmk}, $G_4^{nm\ell k}$ are given by the KK gauge boson wave function,

$$G_3^{nmk} = \sum_j g_j R_{j,n} R_{j,m} R_{j,k}, \quad G_4^{nm\ell k} = \sum_j g_j^2 R_{j,n} R_{j,m} R_{\ell,k} R_{j,k},$$

while $\tilde{G}_4^{nm\ell k}$ given by the NG boson wave function,

$$G_4^{nm\ell k} = \sum_j \frac{v^2}{f_j^2} \tilde{R}_{j,n} \tilde{R}_{j,m} \tilde{R}_{\ell,k} \tilde{R}_{j,k}.$$

A relation between $W_{(n)}$ and $\pi_{(n)}$ functions (WT identity)

$$\tilde{R} M_W^{\mathrm{diag}} = QR$$

"Completeness" of the wave function

$$\delta_{j,j'} = (RR^T)_{j,j'} = \sum_k R_{j,k} R_{j',k},$$

$$\delta_{j,j'} = (\tilde{R}\tilde{R}^T)_{j,j'} = \sum_k \tilde{R}_{j,k} \tilde{R}_{j',k}.$$

General sum rules

$\Updownarrow$

WT identities + completeness relations

Example:

Derivation of

$$\sum_i G_3^{nmi} G_3^{\ell ki} = G_4^{nm\ell k}$$

$$\begin{aligned}
\sum_i G_3^{nmi} G_3^{\ell ki} &= \sum_i \sum_{j,j'} g_j g_{j'} R_{j,n} R_{j,m} R_{j,i} R_{j',n} R_{j',m} R_{j',i} \\
&= \sum_{j,j'} g_j g_{j'} R_{j,n} R_{j,m} R_{j',n} R_{j',m} \delta_{j,j'} \\
&= \sum_j g_j^2 R_{j,n} R_{j,m} R_{j,n} R_{j,m} \\
&= G_4^{nm\ell k}
\end{aligned}$$

Summary of the WT identities and the completeness relations

$$G_4^{nm\ell k} = \sum_i G_3^{nmi} G_3^{\ell ki},$$

$$\begin{aligned}\frac{4}{v^2}\tilde{G}_4^{nm\ell k} &= G_4^{nm\ell k}(M_n^2 + M_m^2 + M_\ell^2 + M_k^2) \\ &\quad - \sum_i (G_3^{nmi}G_3^{\ell ki} + G_3^{n\ell i}G_3^{mki} + G_3^{nki}G_3^{m\ell i})M_i^2,\end{aligned}$$

$$(M_n^2 - M_m^2)(M_\ell^2 - M_k^2)\sum_i \frac{G_3^{nmi}G_3^{\ell ki}}{M_i^2} = \sum_i (G_3^{nki}G_3^{m\ell i} - G_3^{n\ell i}G_3^{mki})M_i^2,$$

$$\vdots$$

c.f., Sakai and Uekusa, Prog.Theo.Phys. 118 (2007) 315 [hep-th/0604121]

($\tilde{G}_4 = 0$ in the continuum limit)

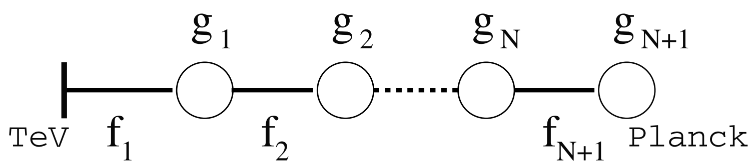
Warped Effective Theory

Chivukula, He, Kurachi, Simmons, M.T.

Phys.Rev.D75, 035005 (2007)

An important motivation of deconstruction: *low energy effective theory*

Warped deconstruction (g-flat case):



$$g_1^2 = g_2^2 = \cdots = g_{N+1}^2$$

$$f_1^2 < f_2^2 < \cdots < f_{N+1}^2$$

$$\frac{1}{r} f_1^2 = f_2^2, \quad \cdots \quad \frac{1}{r} f_N^2 = f_{N+1}^2$$

$$(0 < r < 1)$$

Cheng et al., PRD64:095003 (2001); Abe-Kobayashi-Maru-Yoshioka, PRD67:045019 (2003); Randall et al., JHEP0301:055 (2003)

Warp factor:

$$e^{b/2} = \frac{f_{N+1}}{f_1} = \frac{1}{r^{N/2}}, \qquad b \simeq 60$$

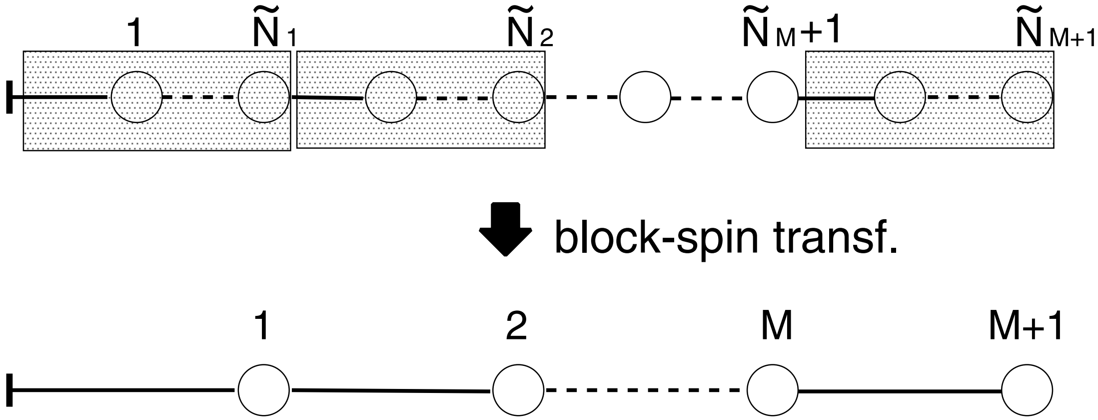
Note:

g-flat deconstruction cannot be regarded as *a low energy effective theory below μ.*

$\Leftarrow$ large hierarchy between scales $f_{N+1} \simeq M_{\rm planck} \gg f_1 \simeq \mu$

Use the *block-spin* analysis so as to obtain a *low energy effective theory*.

Block-spin transformation



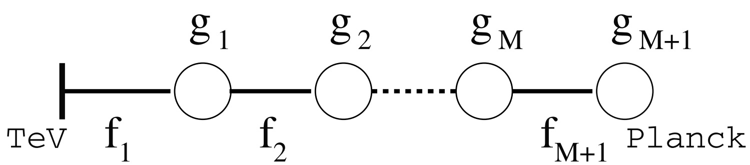
$$\frac{1}{\tilde{f}_1^2} = \sum_{i=1}^{\tilde{N}_1} \frac{1}{f_i^2}, \quad \frac{1}{\tilde{f}_2^2} = \sum_{i=\tilde{N}_1+1}^{\tilde{N}_2} \frac{1}{f_i^2}, \quad \cdots, \qquad \frac{1}{\tilde{g}_1^2} = \sum_{i=1}^{\tilde{N}_1} \frac{1}{g_i^2}, \quad \frac{1}{\tilde{g}_2^2} = \sum_{i=\tilde{N}_1+1}^{\tilde{N}_2} \frac{1}{g_i^2}, \quad \cdots$$

If we arrange $\tilde{N}_1, \tilde{N}_2, \cdots \tilde{N}_3$

$$\tilde{f}_1^2 = \tilde{f}_2^2 = \cdots = \tilde{f}_{M+1}^2,$$

we obtain an *f-flat deconstruction*.

Warped deconstruction (f-flat case):



Warp factor: $b \simeq 60$

$$f_1^2 = f_2^2 = \cdots = f_{M+1}^2$$

$$g_n^2 = bg^2 / \ln\left(\frac{M+2-n}{M+1-n}\right),$$

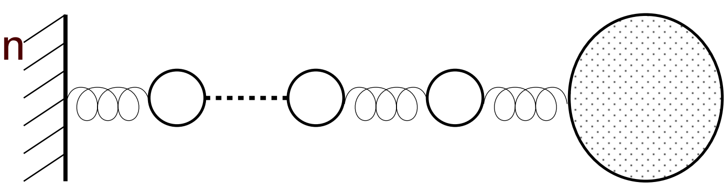
$$g_{M+1}^2 = g^2 / \left(1 - \frac{1}{b}\ln(M+1)\right)$$

Note:

The f-flat deconstruction of the warped metric naturally realizes

$$g_1^2 \simeq g_2^2 \simeq \cdots \simeq g_M^2 \gg g_{M+1}^2$$

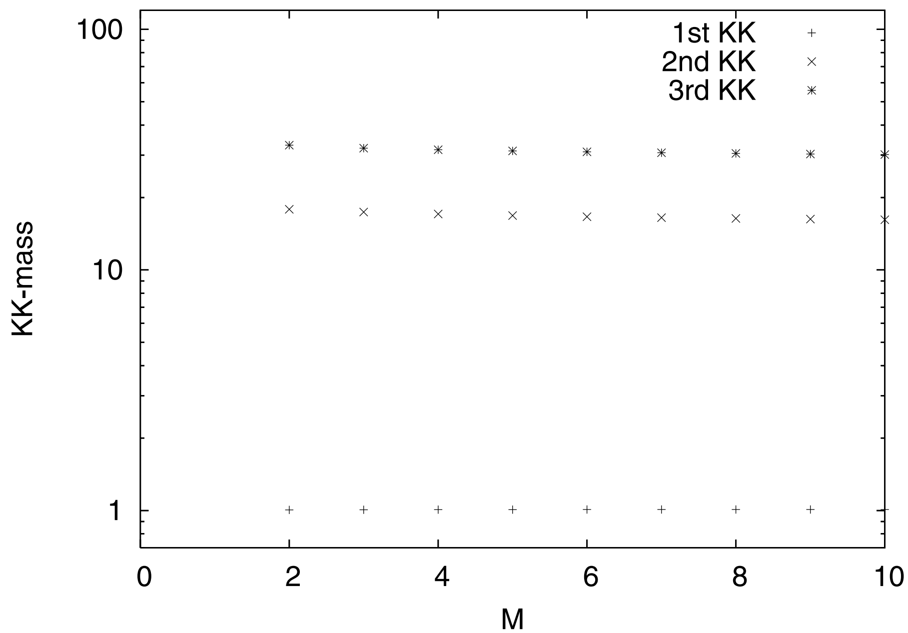
and thus the situation



which ensures $M_W \ll M_{W'}$.

f-flat deconstruction as a low energy effective theory

Small M (number of deconstruction sites) is enough to reproduce the low-energy KK spectrum:



$b = 60$ is assumed

cf. We need $N \gtrsim b = \mathcal{O}(100)$ sites in the f-flat deconstruction.

Summary

- Higgsless theory is an interesting alternative to the standard model Higgs, achieving tree level unitarity at 1TeV.
- We analyzed an effective theory (three site Higgsless model) at one-loop level and found the model is consistent with the available precicion electroweak measurements. The allowed ranges of the KK gauge boson coupling $g_{W'ff}$, the KK gauge boson mass $M_{W'}$, and the KK quark/lepton masses M are severely constrained, however.
- Assuming MFV at tree level, FCNC constraints can be satisfied easily even if we include one-loop effects. (with T. Abe, R.S.Chivukula, and E.H. Simmons)
- The KK gauge boson W' will be discovered at LHC in near future.