The Standard Model Prediction for the Muon $g-2$ and Related Topics

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I. Muon $g-2$
  - introduction
  - our evaluation of the LO hadronic contribution

II. EW precision study

III. Summary

Based on **KH, A.D. Martin, D. Nomura and T. Teubner (HMNT)**,
Phys. Lett. **B557** (2003) 69; *Phys. Rev.* **D69** (2004) 093003;
Phys. Lett. **B649** (2007) 173.
and **G.-C. Cho, KH, Y. Matsumoto and D. Nomura**, work in progress.

These slides have been prepared by D. Nomura

Muon $g-2$ — Introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad (\vec{s} = \frac{1}{2}\vec{\sigma} \ \text{(spin)}, \qquad g = 2 + 2F_2(0))$$

where

$$\overline{u}(p+q)\Gamma^\mu u(p) = \overline{u}(p+q)\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)

Today, still important, since...

- ★ One of the **most precisely measured** quantities

$$\boxed{a_\mu^{\text{exp}} = 11\ 659\ 208.0(6.3) \times 10^{-10}} \ [\mathbf{0.5ppm}]$$

- ★ **Extremely useful** in **probing/constraining physics beyond the SM**

Recent Ups and Downs of Muon $g-2$

		EXP − TH
Feb '01	new exp. result (BNL)	**2.6** σ
Nov '01	The 'famous' **l-by-l sign error found** (Knecht & Nyffeler)	2.6 $\sigma \to$ **1.6** σ
Dec '01	new $e^+e^- \longrightarrow \pi^+\pi^-$ data (CMD-2)	
July '02	new exp. result (BNL)	1.6 $\sigma \to$ **2.6** σ
Aug '02 —	new eval. of the LO had. contribution using the new CMD-2 data (DEHZ, HMNT, Jegerlehner)	2.6 $\sigma \to$ **3.0** σ (DEHZ, e^+e^-) **3.3** σ (HMNT, e^+e^-) **(0.9** σ**)** (DEHZ, τ)
Aug '03	**error found in the CMD-2 data analysis**	3.3 $\sigma \to$ **2.4** σ
Dec '03	new eval. of the **l-by-l** contribution (Melnikov & Vainshtein)	2.4 $\sigma \to$ **2.0** σ
Jan '04	**new exp result (BNL)**	2.0 $\sigma \to$ **2.9** σ
Feb '04	improved **QED** calculation (Kinoshita & Nio)	2.9 $\sigma \to$ **2.7** σ
July '04	new F_π data from KLOE	
June '05	new $e^+e^- \to \pi^+\pi^-$ data from SND	
May '06	error found in the SND analysis	
Oct '06	new $e^+e^- \to \pi^+\pi^-$ data from CMD-2	
Nov '06 —	updated analysis of the LO had contrib.	**3.4** σ (HMNT)

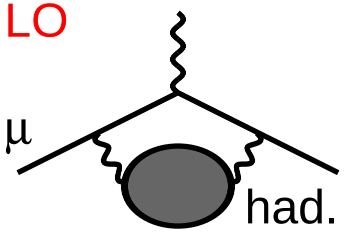
Standard Model Prediction for Muon $g-2$

QED contribution	11 658 471.809 (0.016) $\times 10^{-10}$	Kinoshita & Nio
EW contrib.	15.4 (0.2) $\times 10^{-10}$	Czarnecki et al
Hadronic contrib.		
LO hadronic	**689.4 (4.5)** $\times 10^{-10}$	HMNT
NLO hadronic	-9.8 (0.1) $\times 10^{-10}$	HMNT
light-by-light	13.6 (2.5) $\times 10^{-10}$	Melnikov & Vainshtein
Theory TOTAL	**11 659 180.4 (5.1)** $\times 10^{-10}$	
Experiment	**11 659 208.0 (6.3)** $\times 10^{-10}$	world avg. (2006)

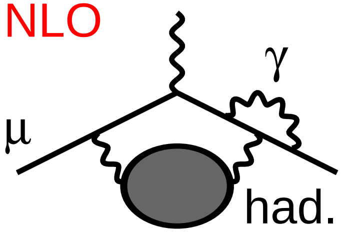
Good evaluation of the **LO hadronic** contribution **vital!**

n.b.: hadronic contributions:

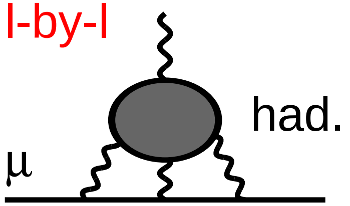
LO



NLO



l-by-l



The QED contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \; 0.765857410\,(27)\,(\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ \; 24.05050964\,(43)\,(\alpha/\pi)^3$

Barbieri, Laporta, Remiddi, … , Czarnecki, Skrzypek, MP '04

$+ \; 130.992\,(8)\,(\alpha/\pi)^4$ [See Nio's talk]

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04 & '05

$+ \; 663\,(20)\,(\alpha/\pi)^5$ [See Nio's talk]

Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.

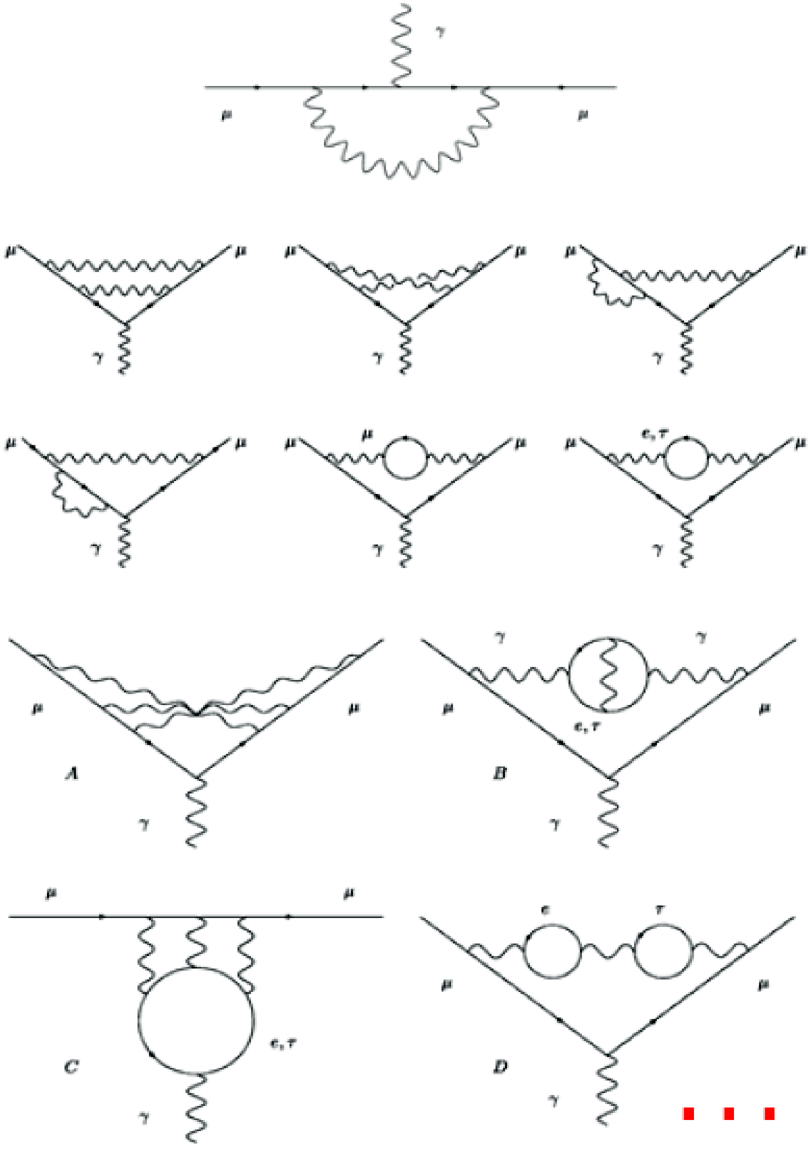
Aoyama-Hayakawa-Kinoshita-Nio, . . .

Adding up, I get:

$a_\mu^{QED} = 116584718.09\,(0.14)\,(0.08) \times 10^{-11}$

mainly from 5-loop unc ↲ ↳ from new $\delta\alpha$

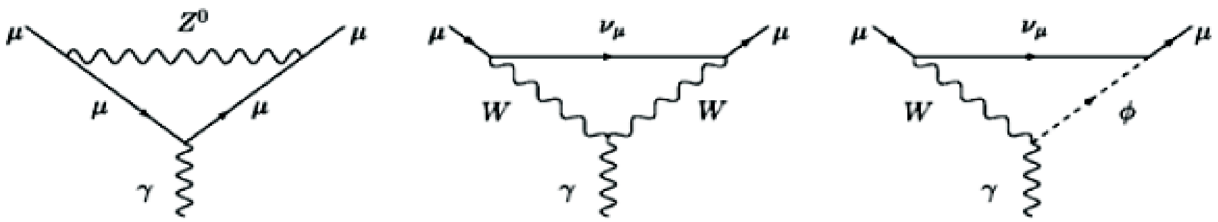
with $\alpha = 1/137.035999709\,(96)$ [0.7 ppb]



Passera, talk at Tau06

The Electroweak contribution to a_μ

- **One-Loop Term:**



$$a_\mu^{\rm EW}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

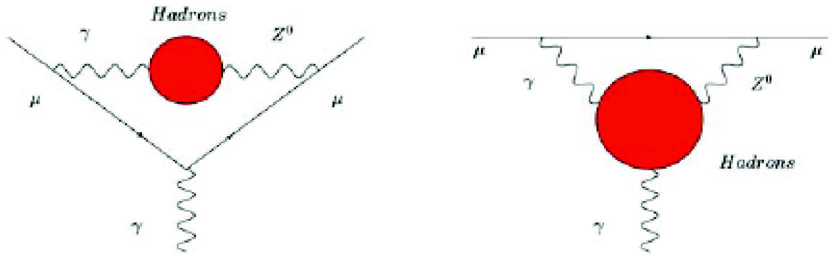
- **One-Loop plus Higher-Order Terms:**

$$\boxed{a_\mu^{\rm EW} = 154\ (2)\ (1) \times 10^{-11}}$$

Kukhto et al. '92; Czarnecki, Krause & Marciano '95; Knecht, Peris, Perrottet & de Rafael '02; Czarnecki, Marciano & Vainshtein '02; Degrassi & Giudice '98; Heinemeyer, Stockinger & Weiglein '04; Gribouk & Czarnecki '05; Vainshtein '03.

Higgs mass, M_top error, three-loop nonleading logs

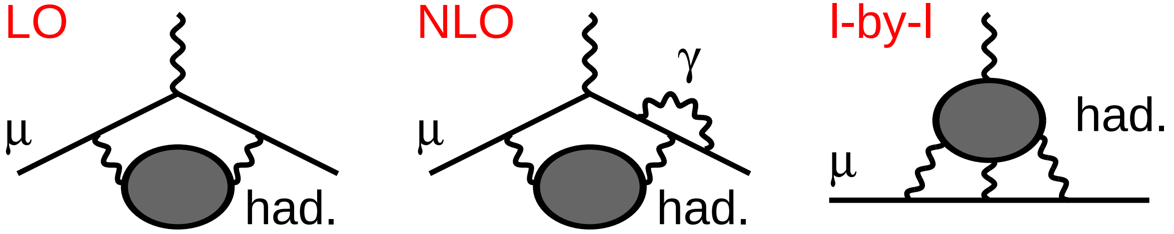
Hadronic loop uncertainties:



Passera, talk at Tau06

Hadronic contributions

$$\boxed{a_\mu^{\text{had}} = a_\mu^{\text{had, LO}} + a_\mu^{\text{had, NLO}} + a_\mu^{\text{l-by-l}}}$$

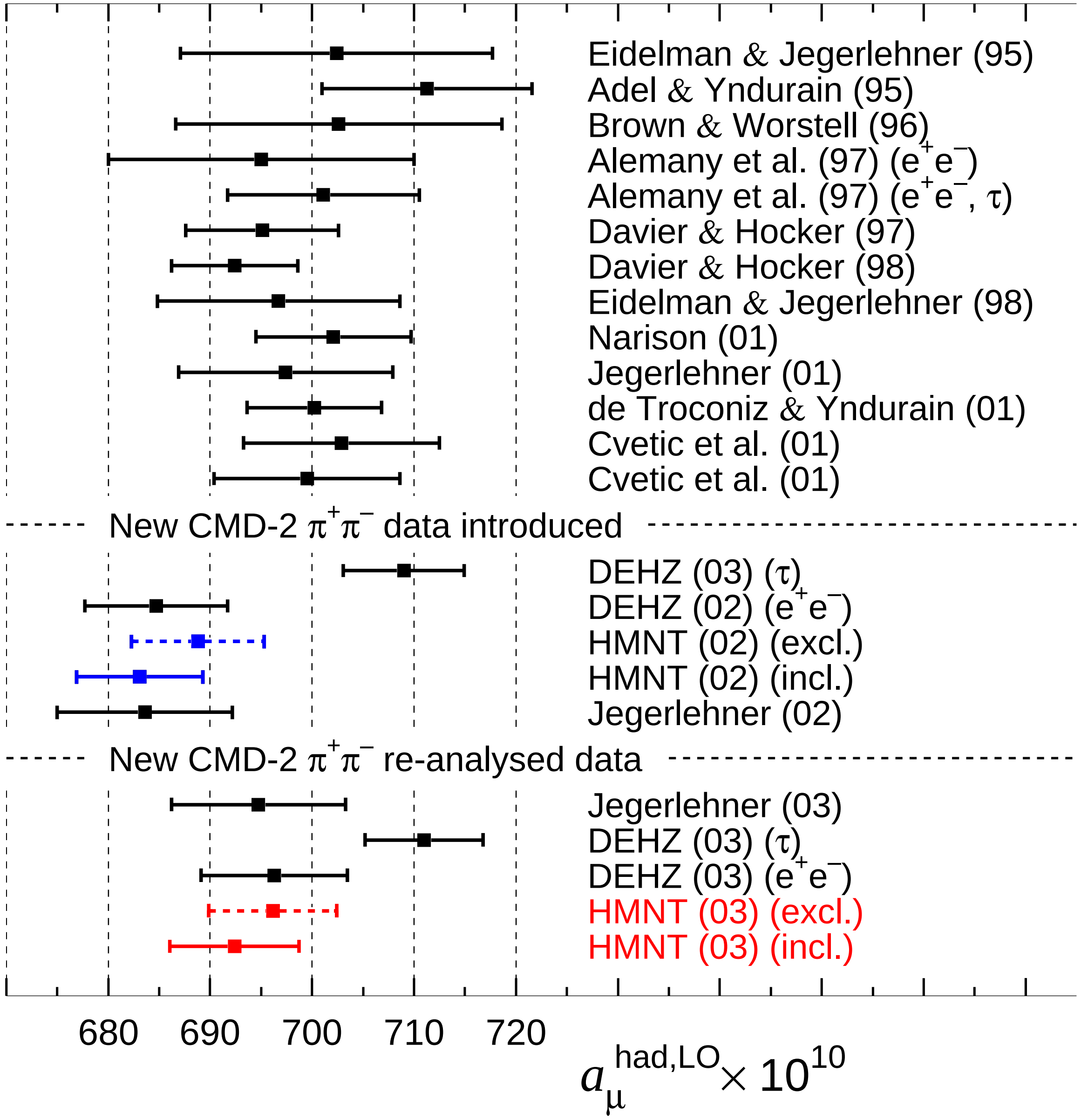


LO and NLO: calculable from exp. data

l-by-l: NOT calculable from exp. data, have to rely on model to some extent (model on pion form factor, large N_c expansion, ...)

There are some attempts to calculate them using lattice (Blum, Hayakawa-Blum-Izubuchi-Yamada, Aubin-Blum, . . .), but still suffering from large systematic uncertainties.

Recent Evaluations of $a_\mu^{\mathrm{had,LO}}$

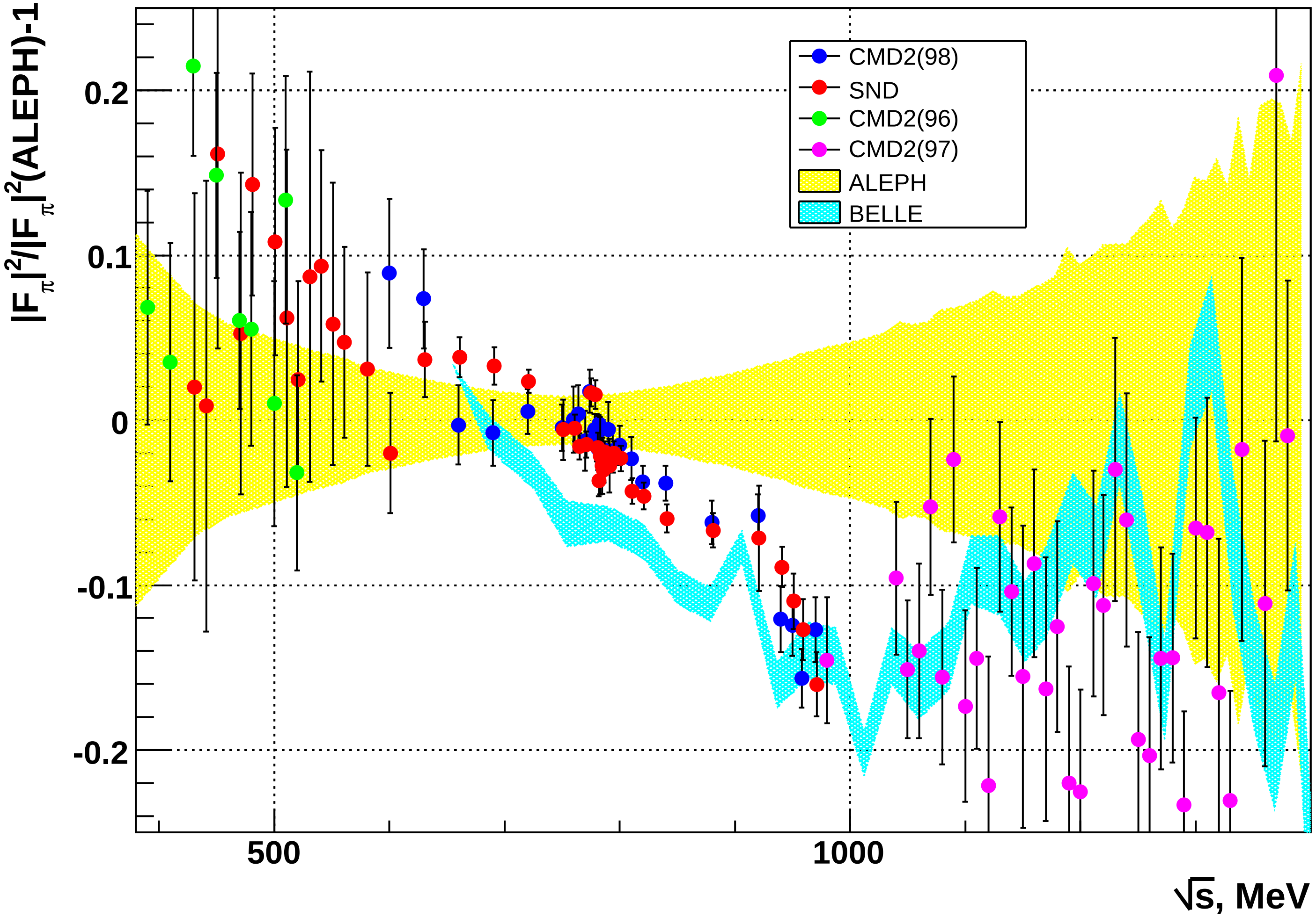


✓ e^+e^--based evaluations
— convergent

× Diff. between e^+e^--based and τ-based evaluations
— must be explained!

$F_\pi(q^2)$ from $e^+e^- \to \pi^+\pi^-$ vs that from τ decays

S. Eidelman, ICHEP06



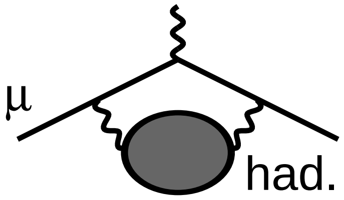
Light Blue: preliminary data of τ decays at Belle

Yellow: based on τ decays at ALEPH

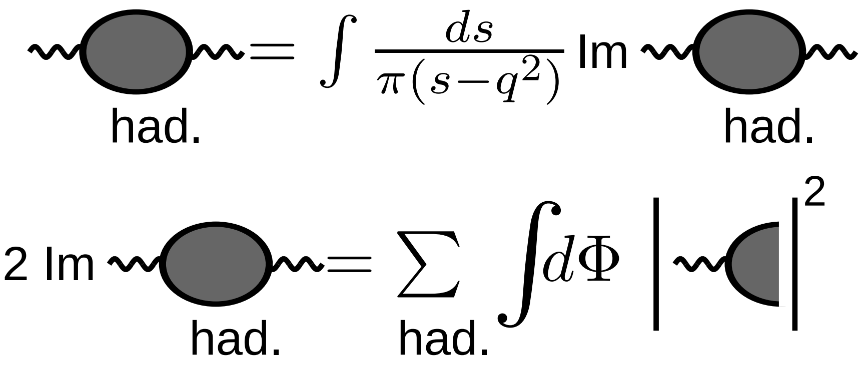
$\Longrightarrow$ Possible problem in τ-decay data?

Evaluating $a_\mu^{\text{had,LO}}$

The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.

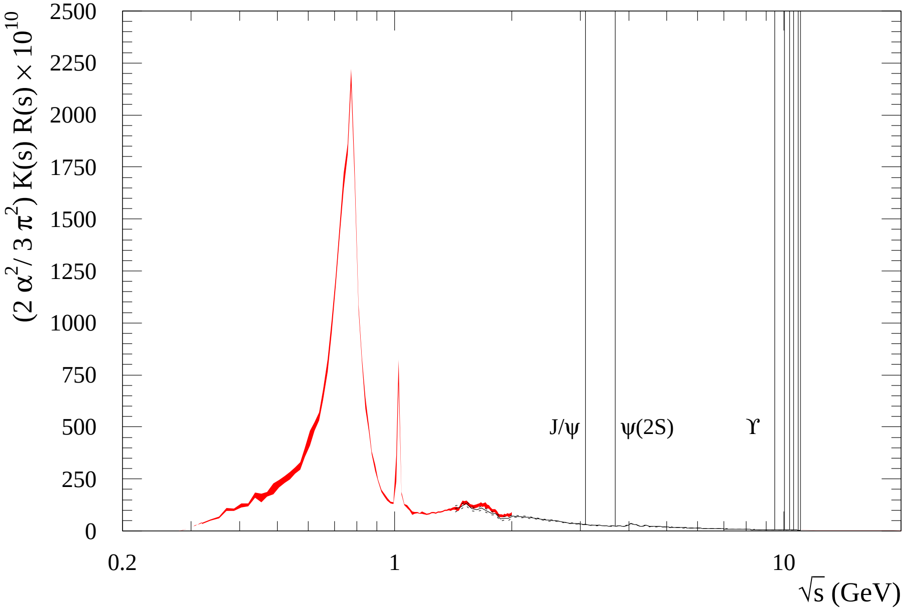


$$\boxed{a_\mu^{\text{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\text{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\text{had}}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

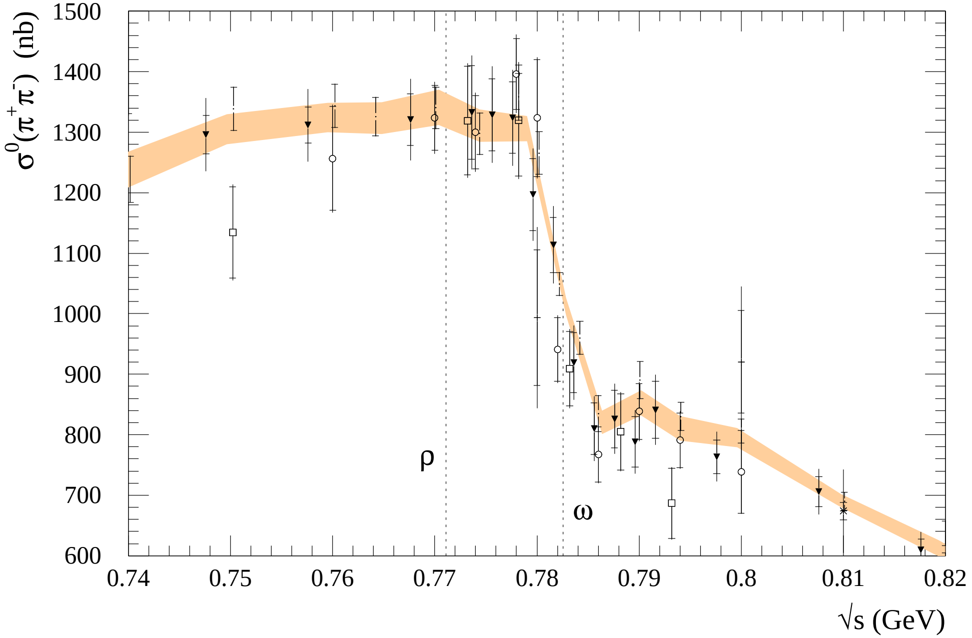
- We have to rely on exp. data for $\sigma_{\text{had}}(s) \Longrightarrow$ Good data crucial



- We have to use a large number (>80) of data sets $\Longrightarrow$ Statistically correct treatment/combination of data sets important

How to Combine data sets — “Clustering”

1. We model the true value of R by a piecewise-constant $\overline{R}_m$ within a Cluster of a given (min.) size.



2. Construct the χ^2 function as

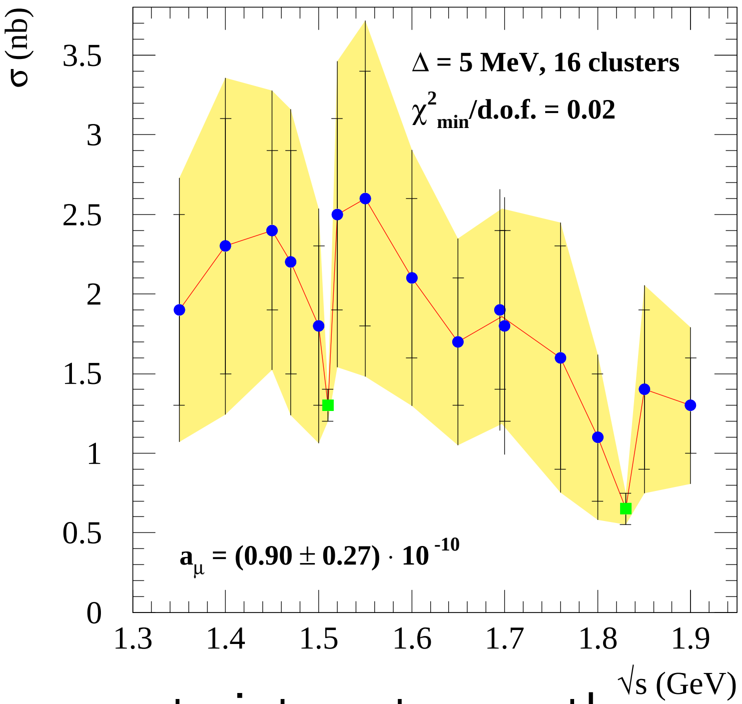
$$\chi^2(\overline{R}_m, f_k) = \sum_{k=1}^{\#\text{ofexp.}} \left(\frac{1-f_k}{df_k}\right)^2 + \sum_{m=1}^{\#\text{ofClus.}} \sum_{i=1}^{N_{\{k,m\}}} \left(\frac{R_i^{\{k,m\}} - f_k\overline{R}_m}{dR_i^{\{k,m\}}}\right)^2$$

from the raw data $R_i^{\{k,m\}} \pm dR_i^{\{k,m\}}$ and the **normalization uncertainty** of the k-th exp df_k.

3. Minimize it w. r. t. $\overline{R}_m$ and f_k.

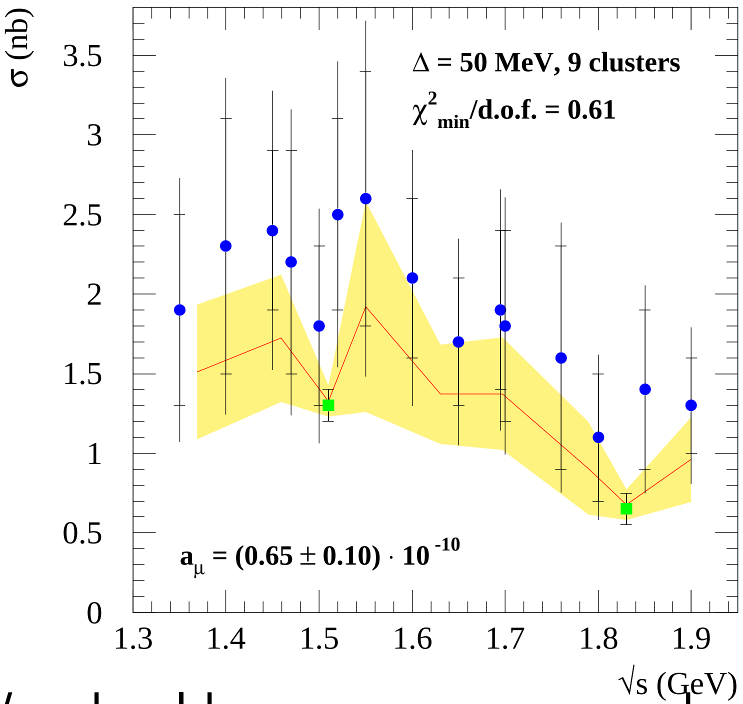
Combining data sets ("Clustering") — Toy Example

Suppose we have two data sets — one good (green), the other poor (blue)



If we are to integrate over the raw data, the result would be like this — we are:

- overestimating the error
- overestimating the mean (in this case)

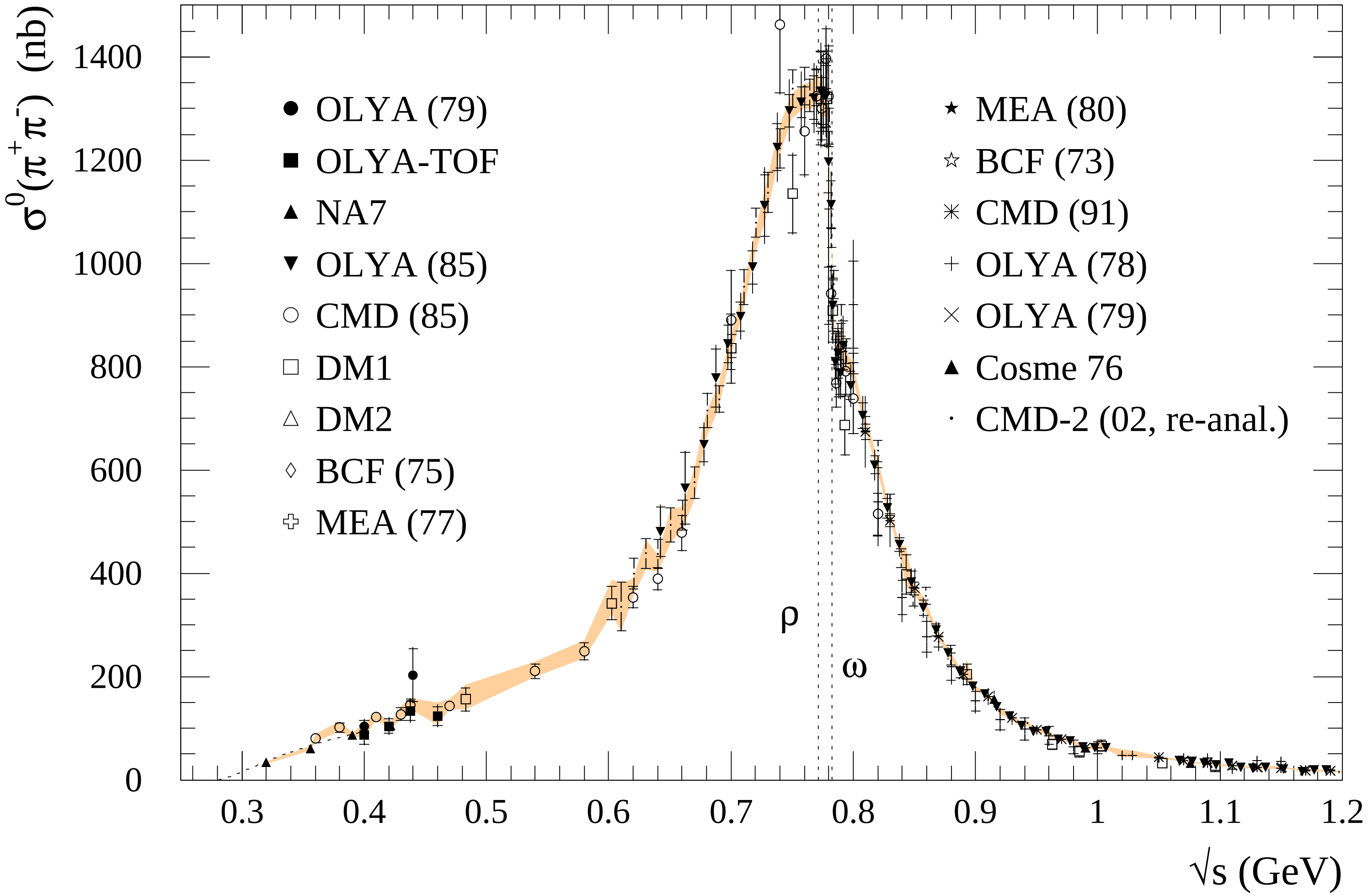


We should average over nearby data points ("clustering")

Advantages

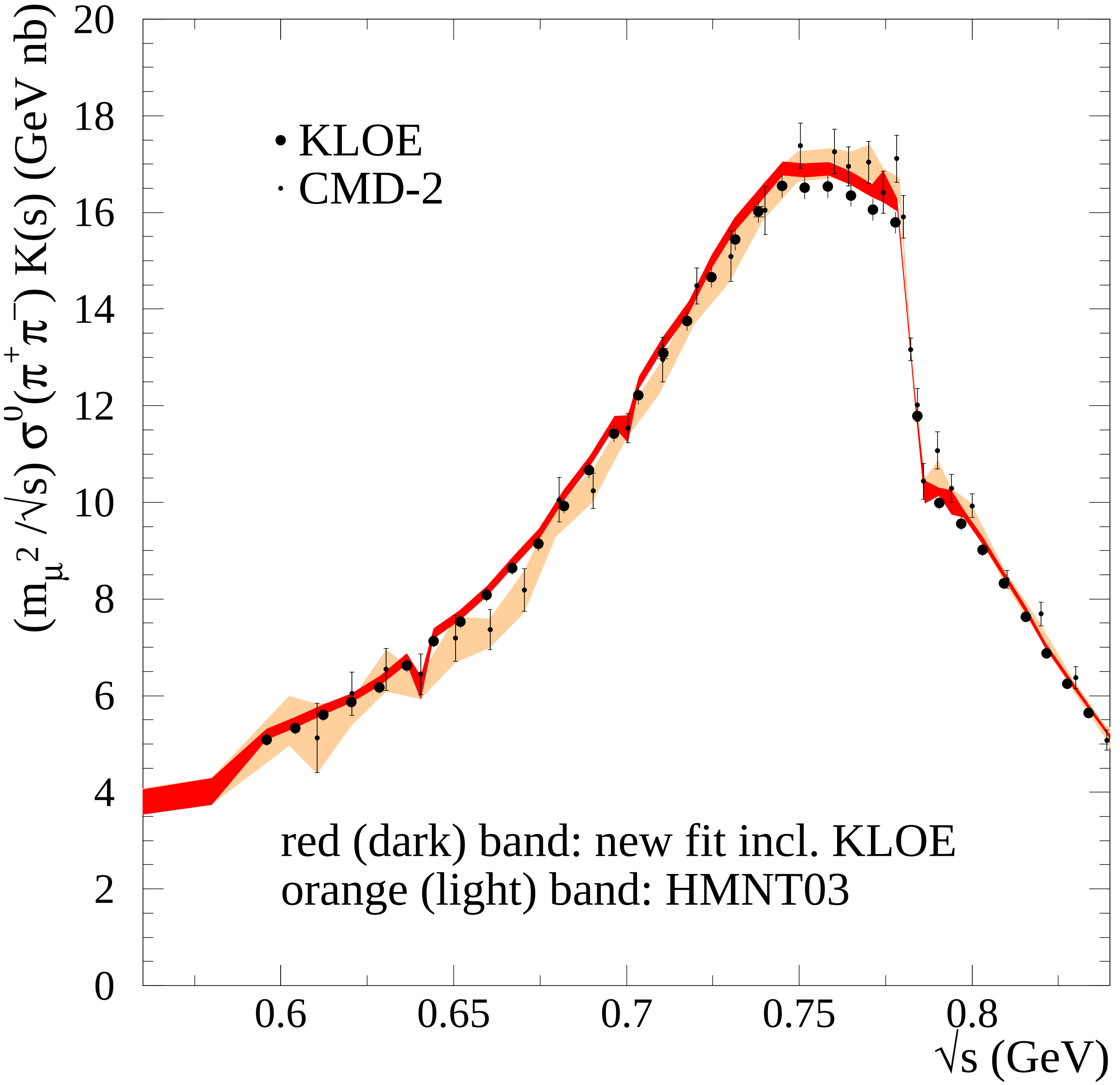
- overall normalization uncertainty of the poor data set fixed by the good one
- combining effect (N times data in one bin $\Longrightarrow$ error reduced by a factor of $1/\sqrt{N}$)

Clustering — Real Data ($e^+e^- \to \pi^+\pi^-$)



$\pi^+\pi^-$: by far the **most important** channel — **73 %** of total $a_\mu^{\mathrm{had,LO}}$

Comments on the KLOE data



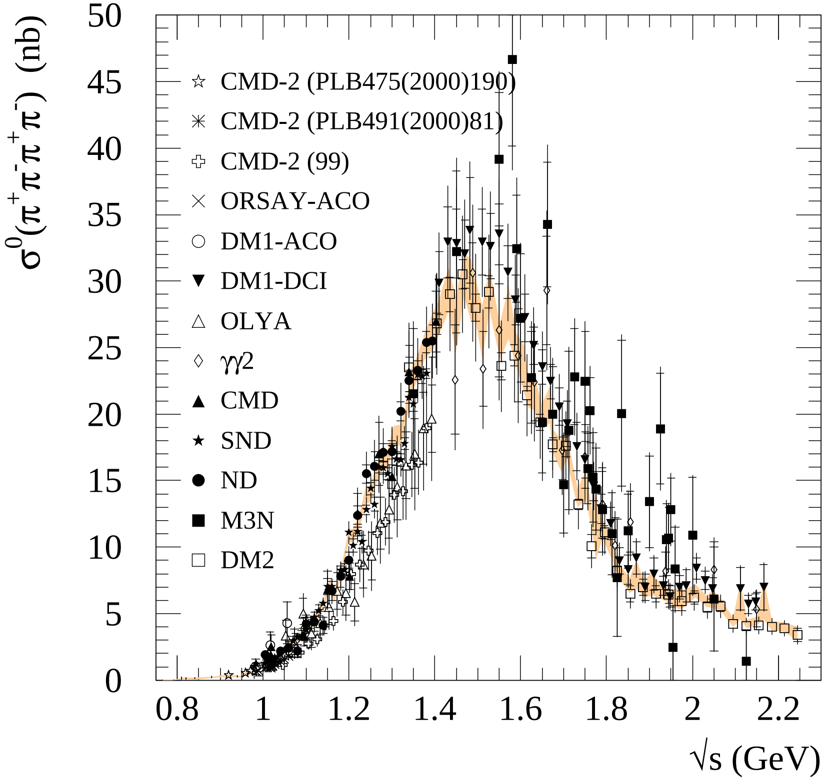
New data of the pion form factor appeared from **KLOE** (`hep-ex/0407048`) using $e^+e^- \to \pi^+\pi^-\gamma$

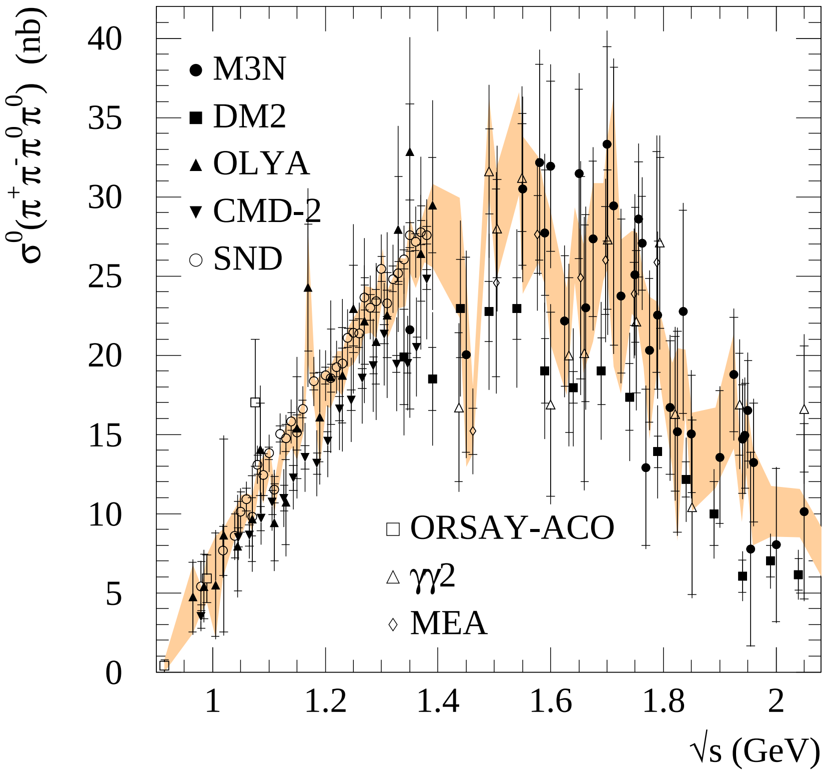
✓ Good quality data (small error)

× **Inconsistent shape** with CMD-2 ⟵ **not yet understood why**

We (HMNT) combined them only after integrating over the e^+e^- data and the KLOE data separately.

Clustering — More “Difficult” Channels (e.g. $e^+e^- \rightarrow 4\pi$)





$\mathbf{2\pi^+2\pi^-}$ and $\boldsymbol{\pi^+\pi^-2\pi^0}$: $\chi^2_{\rm min}/{\rm d.o.f}$ not good (2.00 and 1.28) — we have inflated the error by a factor of $\sqrt{\chi^2_{\rm min}/{\rm d.o.f}}$

Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K_S^0K_L^0$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma2$ [56], MEA [57]
$\omega(\rightarrow\pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma2$ [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma2$ [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\rightarrow\pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma2$ [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K_S^0\pi K$	DM1 [76], DM2 [74, 75]
K_S^0X	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (< 2 GeV)	$\gamma\gamma2$ [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (> 2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes their previously published data for $\pi^+\pi^-$ [11], $\pi^+\pi^-\pi^0$ [13] and $K_S^0K_L^0$ [14].

channel	inclusive (1.43,2 GeV)		exclusive (1.43,2 GeV)	
	$a_\mu^{\mathrm{had,LO}}$	$\Delta\alpha_{\mathrm{had}}(M_Z^2)$	$a_\mu^{\mathrm{had,LO}}$	$\Delta\alpha_{\mathrm{had}}(M_Z^2)$
$\pi^0\gamma$ (ChPT)	0.13 ± 0.01	0.00 ± 0.00	0.13 ± 0.01	0.00 ± 0.00
$\pi^0\gamma$ (data)	4.50 ± 0.15	0.36 ± 0.01	4.50 ± 0.15	0.36 ± 0.01
$\pi^+\pi^-$ (ChPT)	2.36 ± 0.05	0.04 ± 0.00	2.36 ± 0.05	0.04 ± 0.00
$\pi^+\pi^-$ (data)	502.78 ± 5.02	34.39 ± 0.29	503.38 ± 5.02	34.59 ± 0.29
$\pi^+\pi^-\pi^0$ (ChPT)	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00	0.00 ± 0.00
$\pi^+\pi^-\pi^0$ (data)	46.43 ± 0.90	4.33 ± 0.08	47.04 ± 0.90	4.52 ± 0.08
$\eta\gamma$ (ChPT)	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\gamma$ (data)	0.73 ± 0.03	0.09 ± 0.00	0.73 ± 0.03	0.09 ± 0.00
K^+K^-	21.62 ± 0.76	3.01 ± 0.11	22.35 ± 0.77	3.23 ± 0.11
$K_S^0K_L^0$	13.16 ± 0.31	1.76 ± 0.04	13.30 ± 0.32	1.80 ± 0.04
$2\pi^+2\pi^-$	6.16 ± 0.32	1.27 ± 0.07	14.77 ± 0.76	4.04 ± 0.21
$\pi^+\pi^-2\pi^0$	9.71 ± 0.63	1.86 ± 0.12	20.55 ± 1.22	5.51 ± 0.35
$2\pi^+2\pi^-\pi^0$	0.26 ± 0.04	0.06 ± 0.01	2.85 ± 0.25	0.99 ± 0.09
$\pi^+\pi^-3\pi^0$	0.09 ± 0.09	0.02 ± 0.02	1.19 ± 0.33	0.41 ± 0.10
$3\pi^+3\pi^-$	0.00 ± 0.00	0.00 ± 0.00	0.22 ± 0.02	0.09 ± 0.01
$2\pi^+2\pi^-2\pi^0$	0.12 ± 0.03	0.03 ± 0.01	3.32 ± 0.29	1.22 ± 0.11
$\pi^+\pi^-4\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.12 ± 0.12	0.05 ± 0.05
$K^+K^-\pi^0$	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0K_L^0\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K_S^0\pi^\mp K^\pm$	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K_L^0\pi^\mp K^\pm$ (isospin)	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K\bar{K}\pi\pi$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	3.63 ± 1.34	1.33 ± 0.48
$\omega(\rightarrow\pi^0\gamma)\pi^0$	0.64 ± 0.02	0.12 ± 0.00	0.83 ± 0.03	0.17 ± 0.01
$\omega(\rightarrow\pi^0\gamma)\pi^+\pi^-$	0.01 ± 0.00	0.00 ± 0.00	0.07 ± 0.01	0.02 ± 0.00
$\eta(\rightarrow\pi^0\gamma)\pi^+\pi^-$	0.07 ± 0.01	0.02 ± 0.00	0.49 ± 0.07	0.15 ± 0.02
$\phi(\rightarrow$ unaccounted)	0.06 ± 0.06	0.01 ± 0.01	0.06 ± 0.06	0.01 ± 0.01
$p\bar{p}$	0.00 ± 0.00	0.00 ± 0.00	0.04 ± 0.01	0.02 ± 0.00
$n\bar{n}$	0.00 ± 0.00	0.00 ± 0.00	0.07 ± 0.02	0.03 ± 0.01
$J/\psi, \psi'$	7.30 ± 0.43	8.90 ± 0.51	7.30 ± 0.43	8.90 ± 0.51
$\Upsilon(1S-6S)$	0.10 ± 0.00	1.16 ± 0.04	0.10 ± 0.00	1.16 ± 0.04
inclusive R	73.96 ± 2.68	92.75 ± 1.74	42.05 ± 1.14	81.97 ± 1.53
pQCD	2.11 ± 0.00	125.32 ± 0.15	2.11 ± 0.00	125.32 ± 0.15
sum	692.38 ± 5.88	275.52 ± 1.85	696.15 ± 5.68	276.90 ± 1.77

Table 5: Contributions to the dispersion relations (4) and (5) from the individual channels.

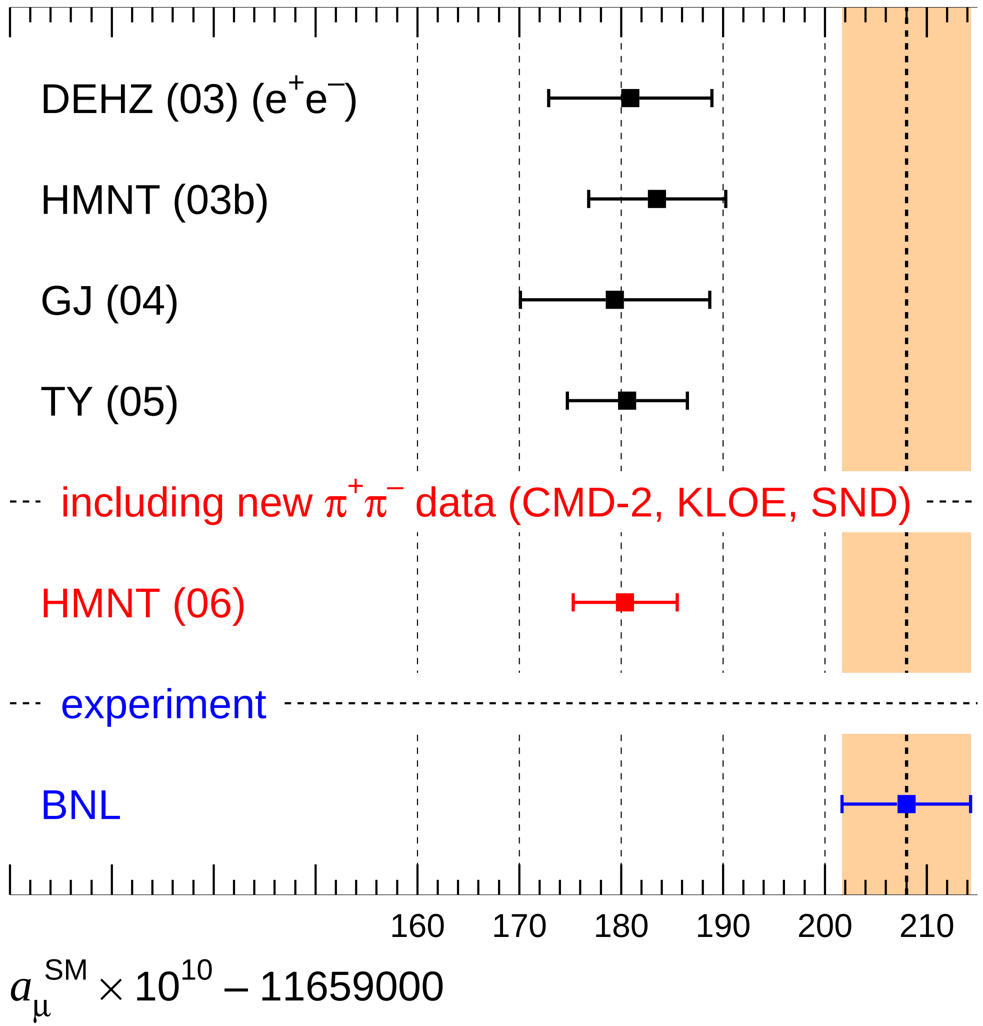
Our Evaluation of $a_\mu^{\text{had,LO}}$ and Breakdown

energy range (GeV)	$a_\mu^{\text{had,LO}} \times 10^{10}$	comments
$m_\pi \ldots 0.32$	2.49 ± 0.05	chiral PT
$0.32 \ldots 1.43$	602.03 ± 3.19	sum of exclusive data
$1.43 \ldots 2.00$	32.05 ± 2.43	inclusive measurements
$2.00 \ldots 11.09$	42.75 ± 1.08	inclusive measurements
J/ψ and $\psi(2S)$	7.90 ± 0.16	narrow width approx.
$\Upsilon(1S - 6S)$	0.10 ± 0.00	narrow width approx.
$11.09 \ldots \infty$	2.11 ± 0.00	pQCD
$\sum$ of all	$689.44 \pm 4.17_{\text{exp}}$	

★ The sum is dominated by the contribution from low energies, $\sqrt{s} \lesssim 1.4\text{GeV}$. (Roughly 600 out of 700)

★ $a_\mu^{\text{had, NLO}}$ can be evaluated similarly. Our result: $a_\mu^{\text{had, NLO}} = (-9.79 \pm 0.09) \times 10^{-10}$.

$a_\mu^{\text{had,LO}}$ combined with the other contributions to a_μ^{SM}



HMNT, hep-ph/0611102

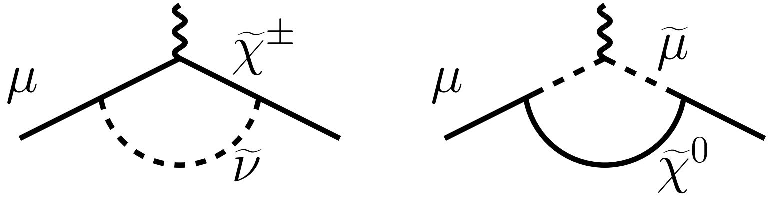
- Our results: consistent with previous results with smaller error

✓ $\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (27.6 \pm 8.1) \times 10^{-10}$: **3.4$\sigma$ discrepancy**

SUSY Contribution to Muon $g-2$

IF the 3.4σ deviation is due to SUSY,...

Dominant **SUSY contributions**:



which is, very roughly, given by

$$a_{\mu}^{\rm SUSY} = ({\rm sgn}\ \mu)\frac{\alpha(M_Z)}{8\pi\sin^2\theta_W}\frac{m_{\mu}^2}{\widetilde{m}^2}\tan\beta,$$

where $\widetilde{m}$ is the SUSY scale. (Many people have published papers more or less related to this.)

Numerically,

$$\begin{aligned} a_{\mu}^{\rm SUSY} =& ({\rm sgn}\mu)\times 13\times 10^{-10} \\ & \times\left(\frac{100{\rm GeV}}{\widetilde{m}}\right)^2\tan\beta \end{aligned}$$

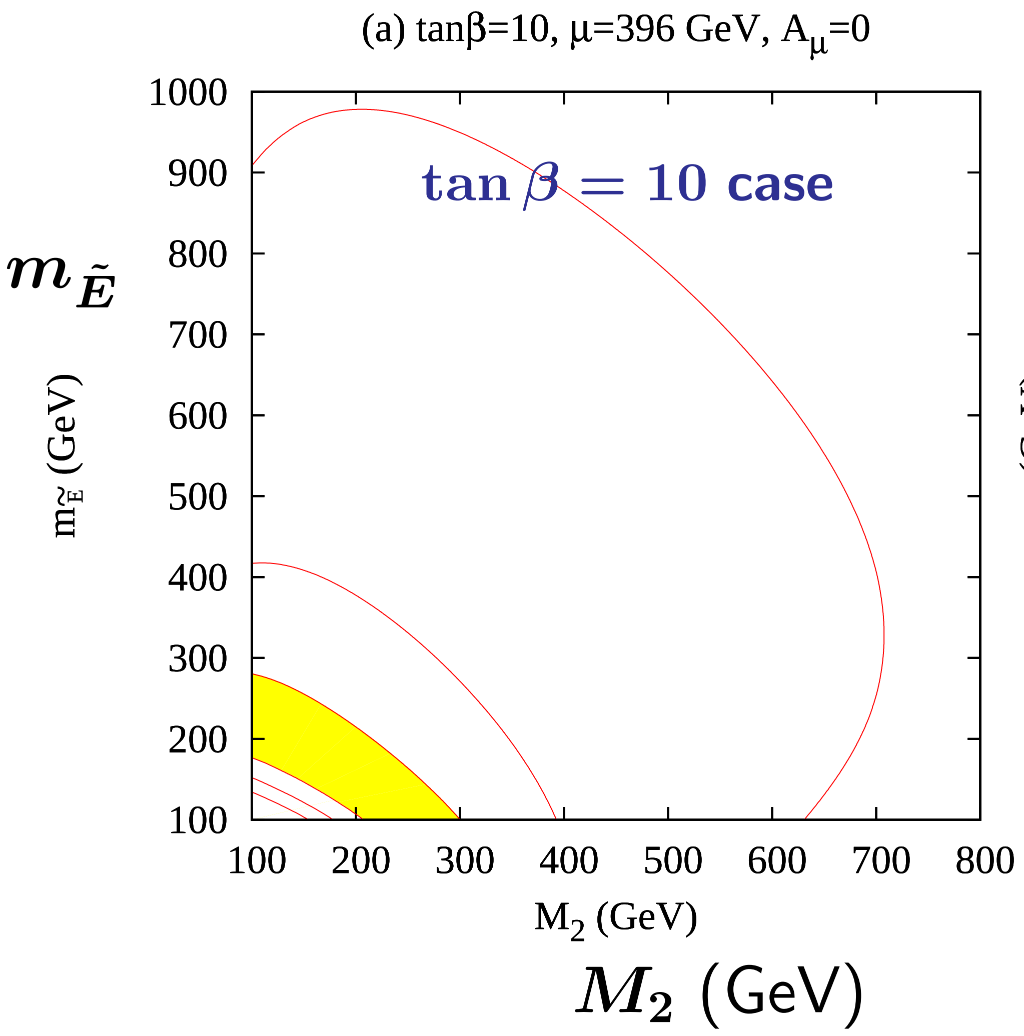
In order for this to be $19.5 \le a_{\mu}^{\rm SUSY}\times 10^{10} \le 35.7$ (1-σ range),

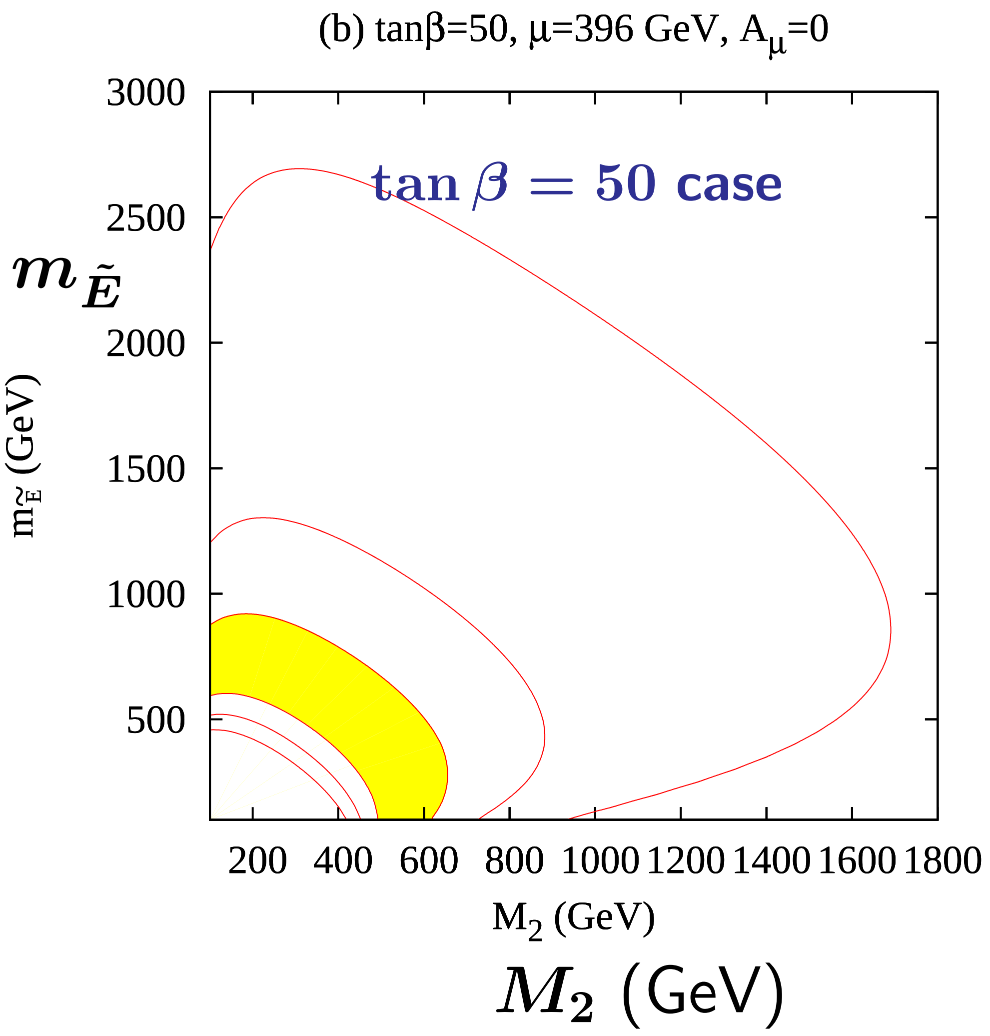
$$\boxed{\widetilde{m} = \mathbf{190-580}\ \text{GeV}}$$

for $\tan\beta = 10-50$. (**Very rough estimate!**)

SUSY Contribution to Muon $g-2$ (II)

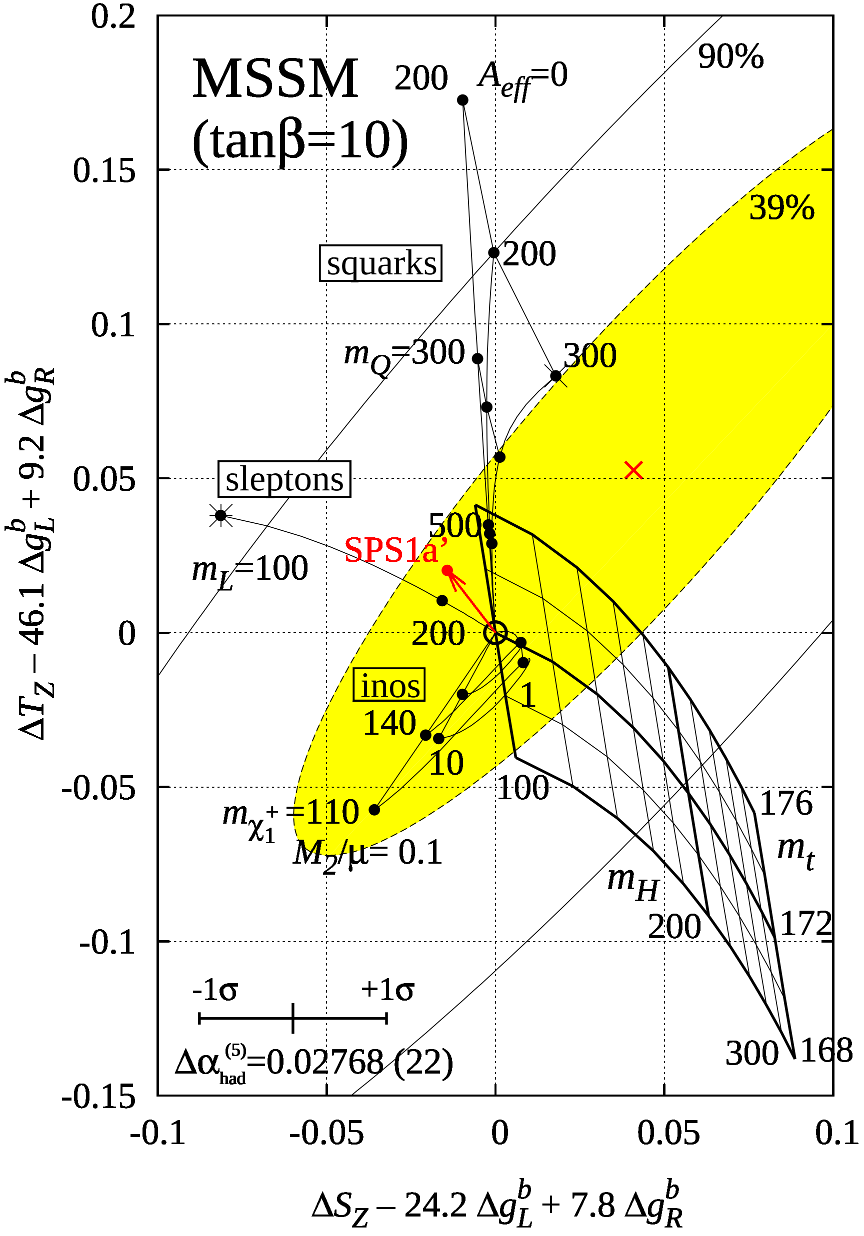
Favored parameter region in the M_2-$m_{\tilde{E}}$ plane





Favored slepton mass: **$<$ 300 GeV** for $\tan\beta = 10$, and **$<$ 900 GeV** for $\tan\beta = 50$ (1-σ range. **Rough estimate!**)

Electroweak Precision Data vs MSSM

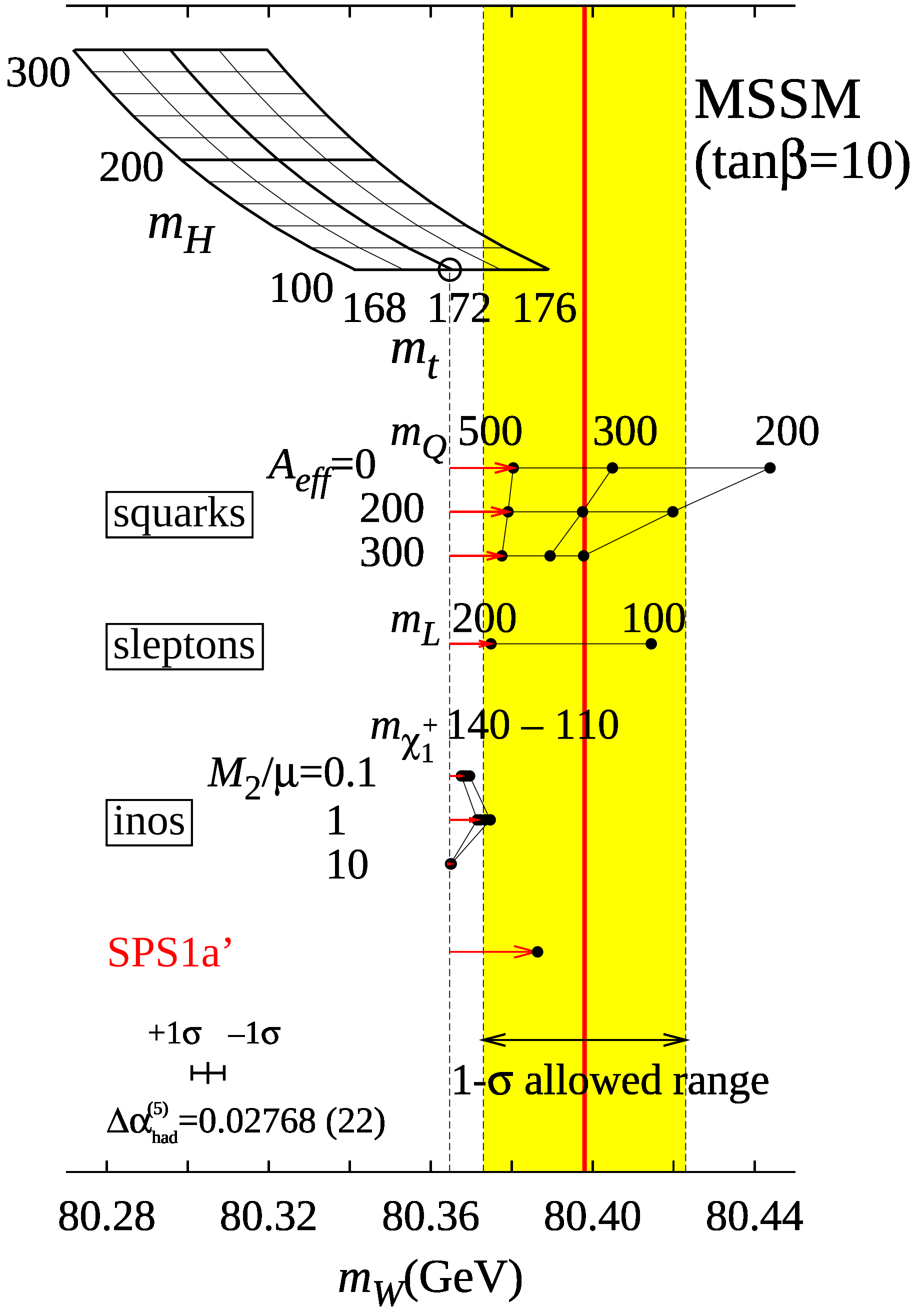


Using the final LEP EW precision data, we can give a constraint on MSSM contributions to S and T.

Our Results:

✓ The SM with $m_H \sim 100$ GeV gives a good description.

Electroweak Precision Data vs MSSM (II), M_W



Our Results:

✓ The MSSM with $\mathcal{O}(100)$ GeV SUSY masses gives a good description.

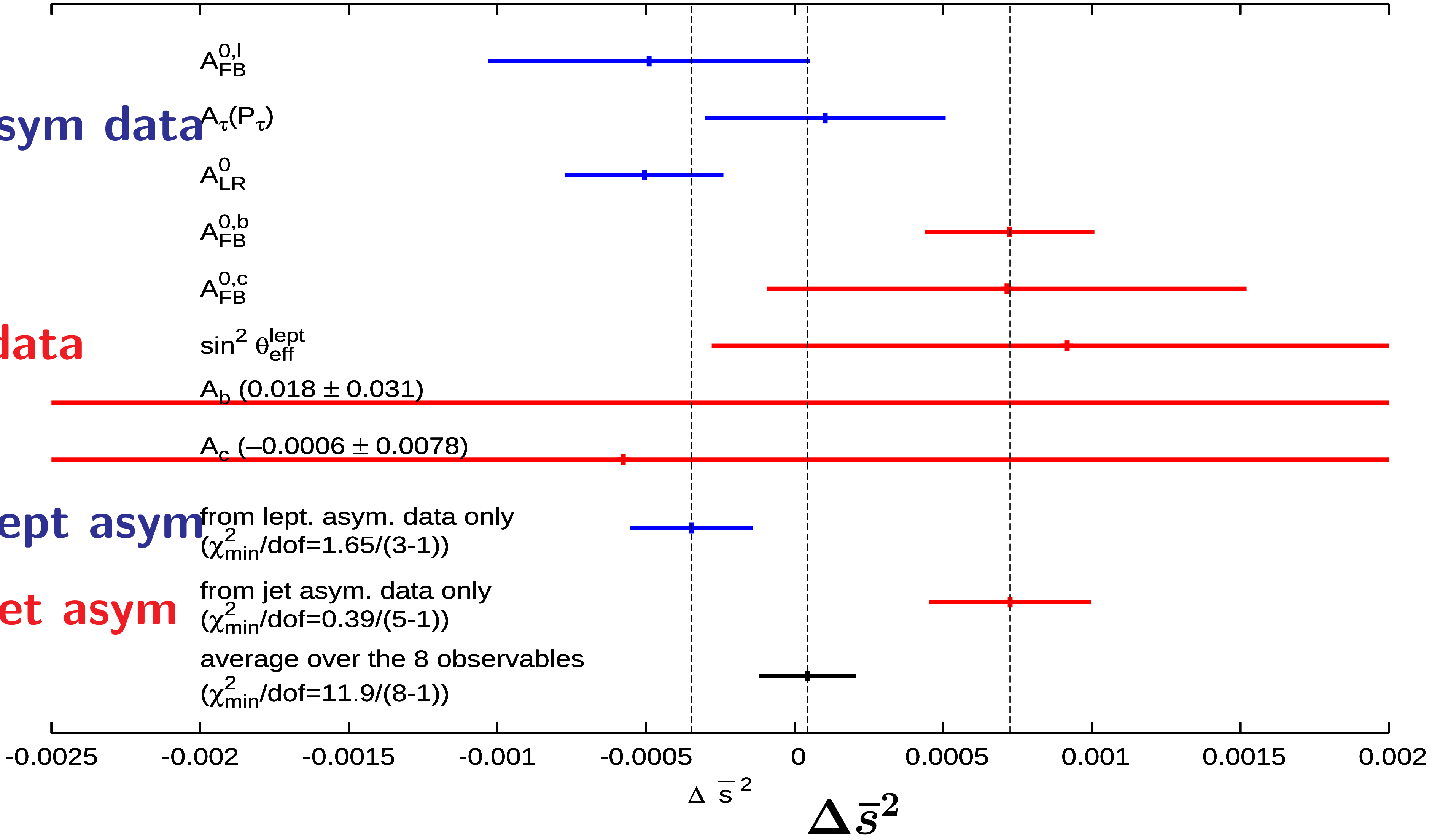
Problem in Jet Asymmetry Data?

leptonic asym data

jet asym data

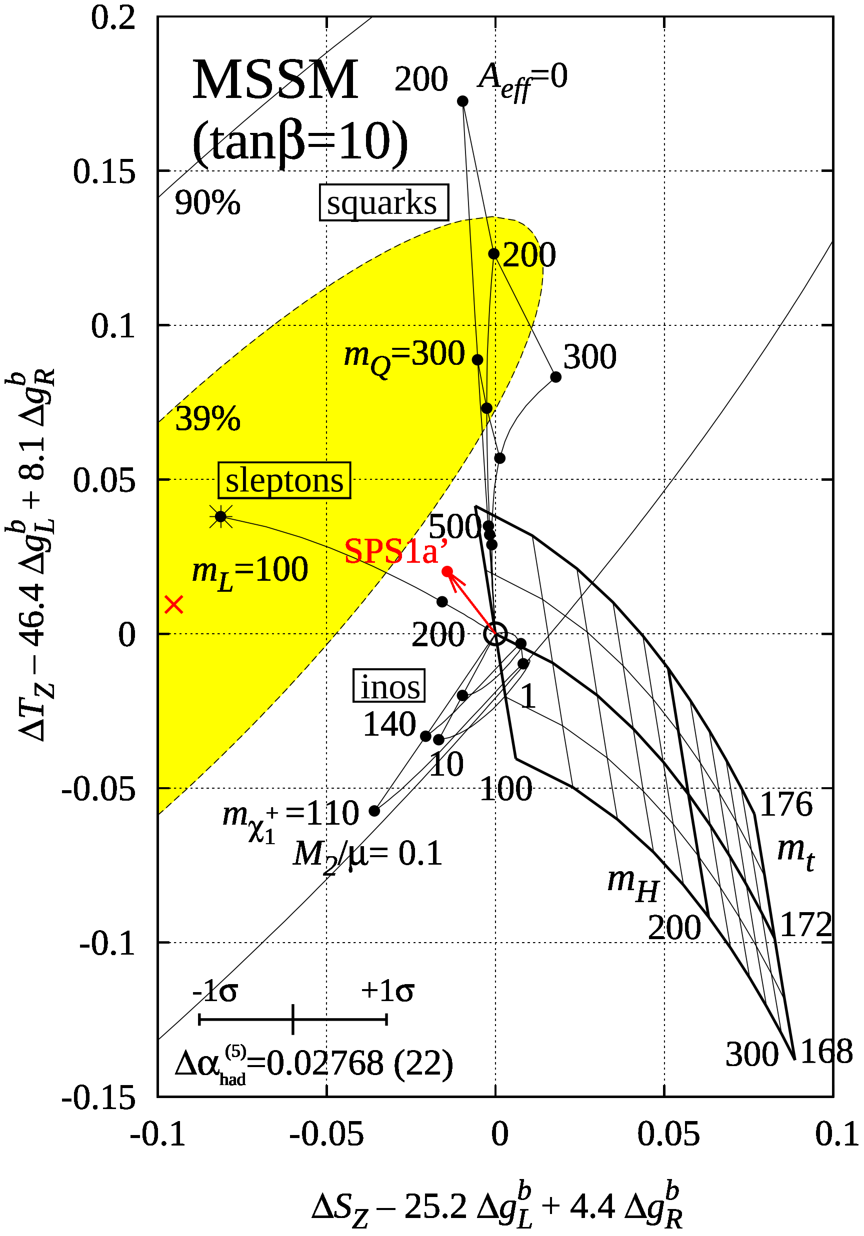
avg over lept asym

avg over jet asym



The value of the effective mixing angle $\bar{s}^2$ determined only from leptonic asymmetry data and that determined only from jet asym. data do not agree very well $\Longrightarrow$ **problem in jet asym. data (or in the analysis of them)?**

Electroweak Precision Data w/o Jet Asym. Data vs MSSM



If we do not use the jet asym. data, light sleptons tend to be **favored**.

Cho-Hagiwara-Matsumoto-DN, in preparation

Summary

Muon $g-2$:

✓ The largest uncertainty in a_μ: still from the **LO hadronic** contribution.

★ **Our results**: **3.4 σ** deviation from experiment. $\Longrightarrow$ **SUSY contribution?**

▶ Waiting for new precise data from the radiative return at BaBar and Belle.

▶ New data on the pion form factor appeared from **KLOE**, but there is some inconsistency in shape with CMD-2 and SND data, which is yet to be understood.

▶ proposal at **BNL**: If approved, another factor of **2** improvement expected.

▶ planned measurement of a_μ at J-PARC: Another factor of 5-10 improvement expected.

EW Precision Fit:

We performed the EW precision fit in the MSSM, using the LEP final EW precision data.

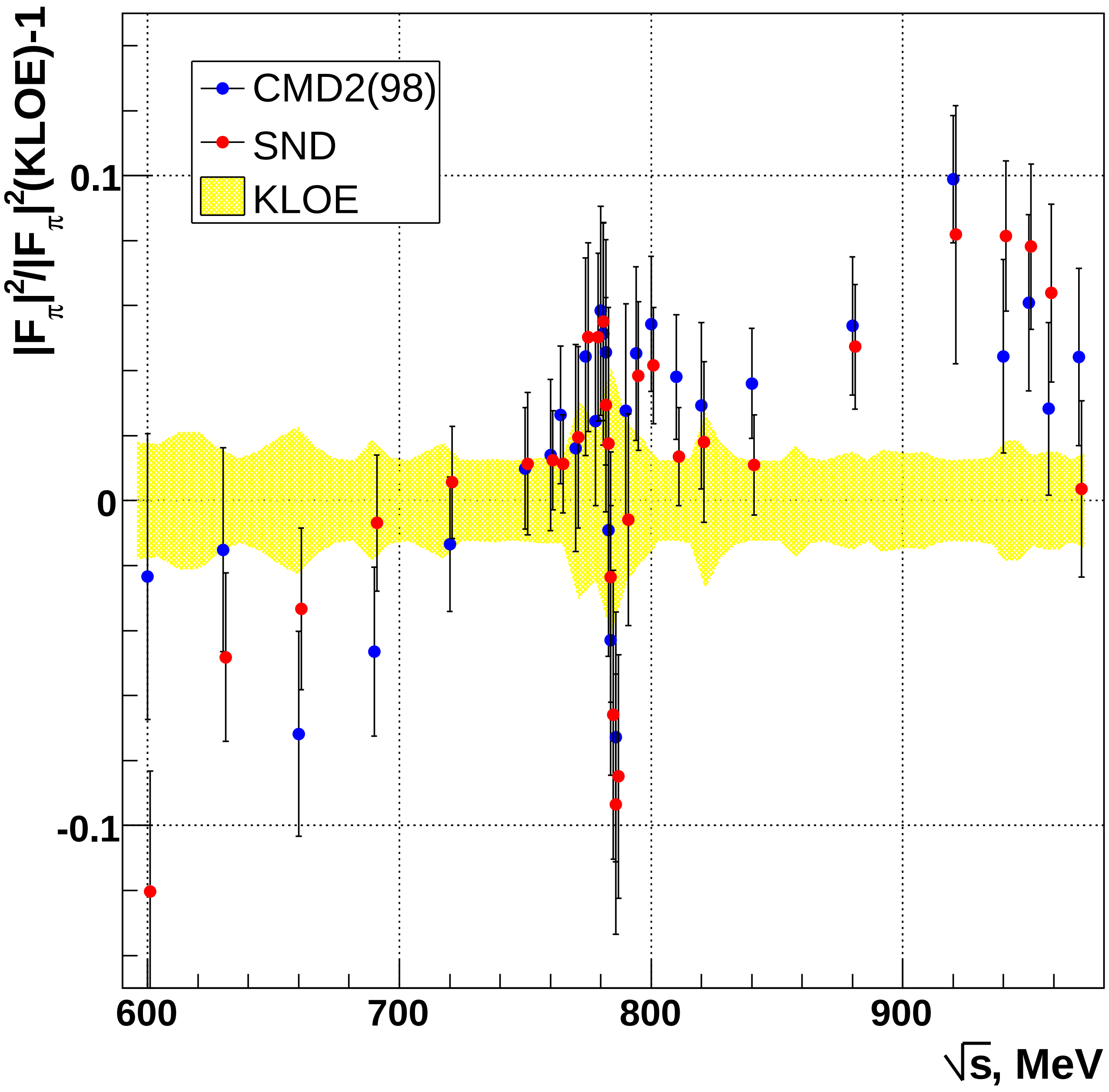
If all the data are used, the SM with a light Higgs gives a good description. Light sfermions tend to be disfavored.

However, there is a slight discrepancy ($\sim 3\sigma$) between the leptonic asymmetry data and the jet asymmetry data.

If we neglect the jet asymmetry data, light sleptons are favored, which can explain the muon $g-2$ anomaly more easily.

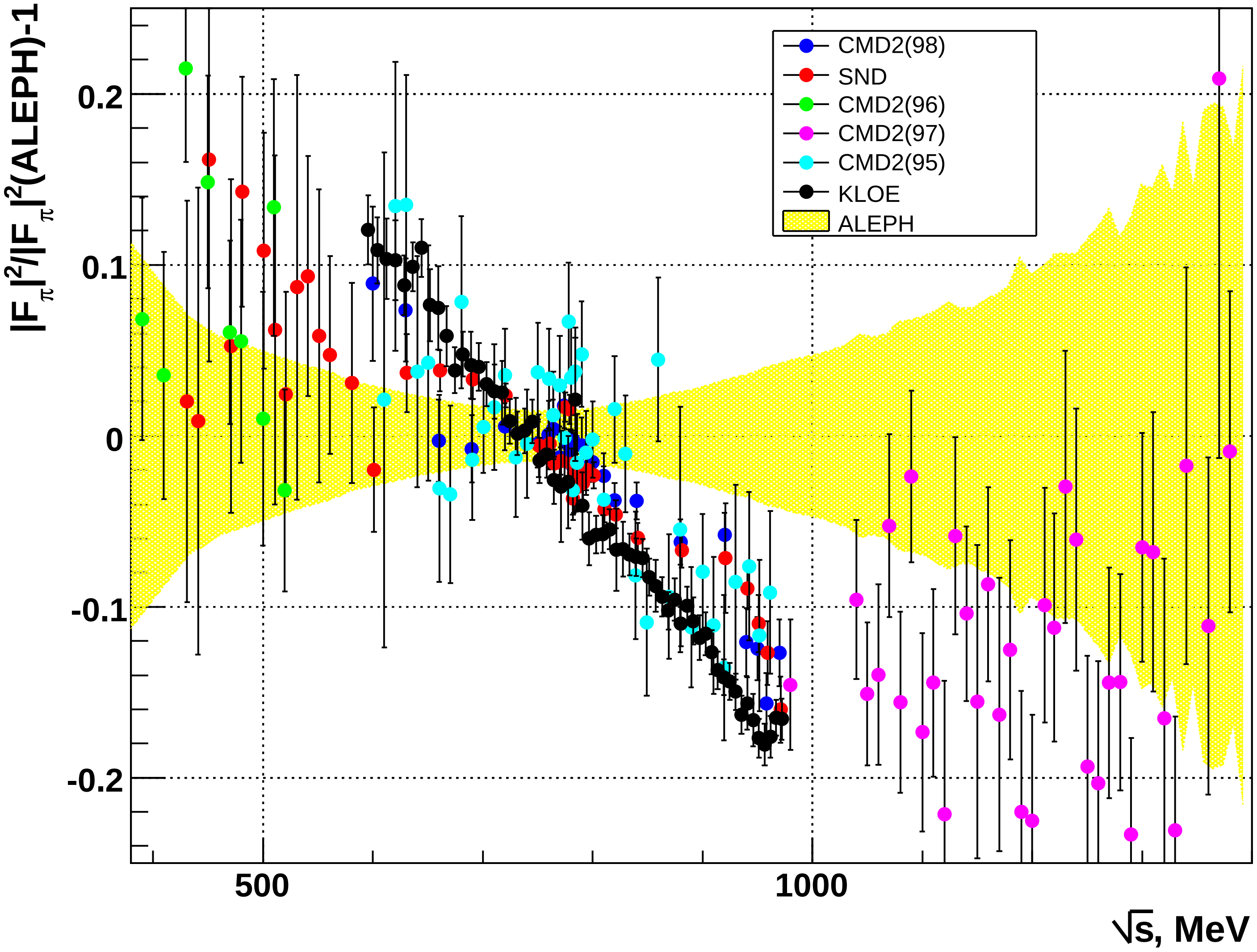
Backup Slides

KLOE vs e^+e^-



$|F_\pi|^2/|F_\pi|^2(\text{KLOE})-1$
0.1
0
-0.1
600
700
800
900
$\sqrt{s}$, MeV
CMD2(98)
SND
KLOE

KLOE vs e^+e^- vs τ



Comparison between $a_\mu^{\text{had, LO}}$ and $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$