An Attempt To Solve The Gauge Hierarchy Problem In Gravity-Gauge-Higgs Unification Scenario

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PLB604 (2004) 133 [hep-ph/0408028]

02/14/2005

“Windows on New Paradigm in Particle Physics”
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PLAN

- INTRODUCTION
- A TOY MODEL &
  EXPLICIT 1-LOOP CALCULATION
  of the GRAVISCALAR MASS
- SUMMARY & DISCUSSION

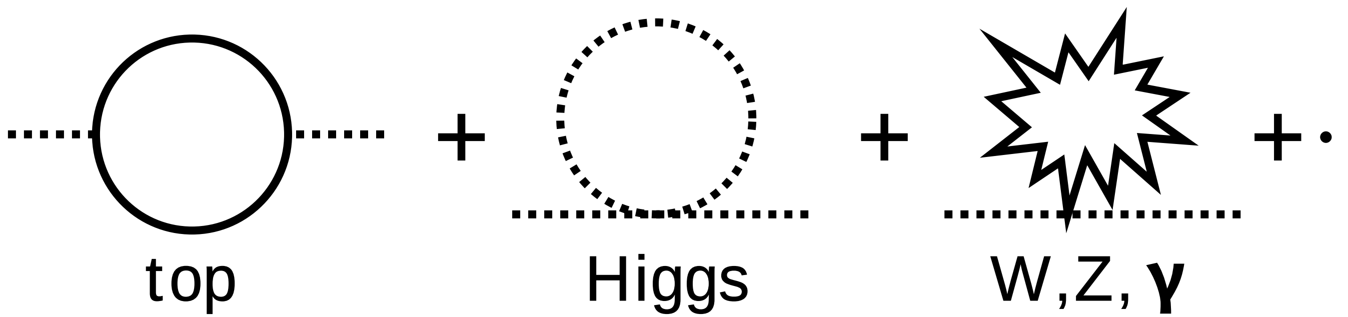
INTRODUCTION

Gauge hierarchy problem is one of the guiding principles for exploring the theory beyond the Standard Model

The problem is ...

the scalar mass is sensitive to UV physics !!

$$\delta m^2_{scalar} =$$



$$\sim \frac{1}{16\pi^2}\Lambda^2 \left(\Lambda : \text{UV cutoff}\right)$$

Many Solutions have been proposed so far

- Technicolor
- Supersymmetry
- Pseudo NG-Boson (Little Higgs)
- Extra Dimension
  - Large Extra Dimensions
  - Warped Extra Dimensions
  - (Gravity-)Gauge-Higgs Unification
  - Higgsless Models
- ...

LESSONS FROM GAUGE-HIGGS UNIFICATION

Hatanaka, Inami & Lim (98)

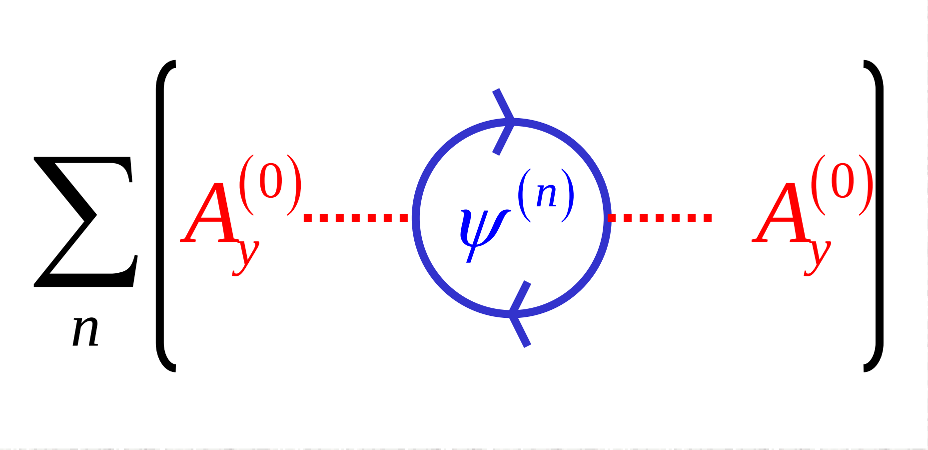
5D QED on $M^4 \times S^1$

GAUGE FIELD: $A_M = (A_\mu, A_y)$

0-mode is identified as "Higgs"

Higgs mass @tree level
is forbidden by gauge symmetry

$$A_y \to A'_y = A_y - \partial_y \alpha(y)$$



- FINITE
  (Quadratic div. cancelled)
- Due to Wilson loop
- Non-local gauge inv.

Here,

QUANTUM CORRECTIONS TO THE MASS OF GRAVISCALAR ("RADION") FROM THE HIGHER DIMENSIONAL GRAVITY

is calculated

We will clarify the structure of

QUADRATIC DIVERGENCE CANCELLATION

& **FINITE MASS GENERATION**

(due to 5D GENERAL COORDINATE TRANSFORMATION INVARIANCE)

in a 5D Gravity coupled with A Scalar field compactified on a CIRCLE

A Toy Model & Explicit 1-Loop Calculation Of The Graviscalar Mass

Consider

5D Gravity Theory coupled with a Scalar field Φ

compactified on S^1 with the radius "R"

"Physical" radius, which is **the general coordinate transformation (g.c.t.) invariant**, should be defined by

$$2\pi\hat{R} \equiv \int_0^{2\pi R} dy\sqrt{-g_{55}} = 2\pi R\sqrt{-g_{55}}\ (\text{for y - independent})$$

Action:

$$S = \frac{1}{16\pi G_5}\int d^4x dy \sqrt{g}\,\mathcal{R} + \frac{1}{2}\int d^4x dy \sqrt{g}\, g^{MN}\partial_M\Phi\partial_N\Phi$$

(M,N = 0,1,2,3,5; μ, ν = 0,1,2,3)

Metric: $$ds^2 = g_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu - e^\varphi\left(dy + A_\mu dx^\mu\right)^2$$

Consider an infinitesimal g.c.t. $x^\mu \to x^\mu,\ y \to y + \beta(x,y)$

For the metric

$$ds^2 = g_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu - e^\varphi \left(dy + A_\mu dx^\mu\right)^2$$

to be invariant

We note that each field transforms as:

$$g_{\mu\nu} \to g_{\mu\nu},\ A_\mu \to \left(1 + \partial_y \beta\right) A_\mu - \partial_\mu \beta,\ \varphi \to \varphi - 2\partial_y \beta$$

A_μ & φ transform inhomogeneously, as in the ordinary local gauge transformation

➡ Mass terms of A_μ and φ are **forbidden**

(In fact, the gauge trf. is reproduced for y-independent **β**:

$$A_\mu \to A_\mu - \partial_\mu \beta,\ \varphi \to \varphi$$
)

Now, we calculate 1-loop corrections of the 2-point function of φ

Action of scalar from 4D viewpoint:

$$S_s = -\frac{1}{2}\int d^4x dy \sqrt{g}\,\Phi(x,y)\, g^{MN}\partial_M\partial_N\Phi(x,y)$$

$$= -\frac{1}{2}\int d^4x dy\, e^{\varphi/2}\Phi(x,y)\left(\eta^{\mu\nu}\partial_\mu\partial_\nu - e^{-\varphi}\partial_y^2\right)\Phi(x,y) \leftarrow g_{\mu\nu} = diag\left(\eta_{\mu\nu}, -e^{\varphi}\right)$$

$$= \frac{1}{2}\int d^4x \sum_n \Phi_n(x)\left(-\eta^{\mu\nu}\partial_\mu\partial_\nu - e^{-\varphi}\left(\frac{n}{R}\right)^2\right)\Phi_n(x)$$

$$\Phi(x,y) = \sum_n \Phi_n(x)\frac{e^{iny/R}}{\sqrt{2\pi\hat{R}}},\quad \int_0^{2\pi R}\sqrt{-g_{55}}\,dy\left(\frac{e^{imy/R}}{\sqrt{2\pi\hat{R}}}\right)\left(\frac{e^{iny/R}}{\sqrt{2\pi\hat{R}}}\right) = \delta_{m+n,0}$$

To get Feynman rules,
the fluctuation around
the classical solution is considered

$$\varphi \rightarrow \varphi_0 + h$$

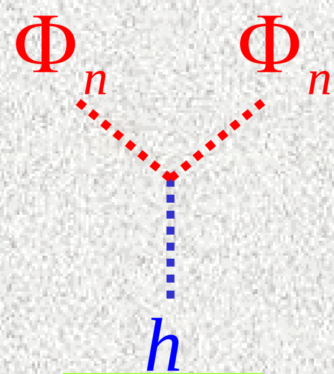
Feynman rule:

$$S_s = S_s^{(\text{free})} + S_s^{(\text{int})}$$

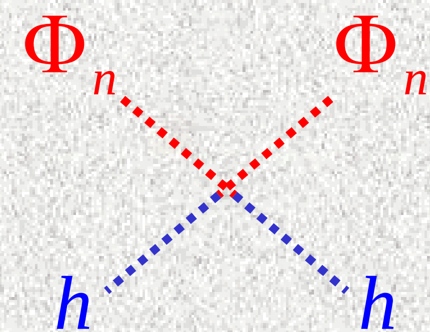
$$S_s^{(\text{free})} = \int d^4x \sum_n \frac{1}{2} \Phi_n(x) \left\{ -\eta^{\mu\nu} \partial_\mu \partial_\nu - \left(\frac{n}{\hat{R}_0} \right)^2 \right\} \Phi_n(x)$$

$$S_s^{(\text{int})} = -\frac{1}{2} \int d^4x \sum_n \left(-h + \frac{1}{2} h^2 \right) \left(\frac{n}{\hat{R}_0} \right)^2 \Phi_n(x)^2$$

$$\frac{i}{k_\mu k^\mu - \left(n / \hat{R}_0 \right)^2}$$



$$i \left(\frac{n}{\hat{R}_0} \right)^2$$



$$-i \left(\frac{n}{\hat{R}_0} \right)^2$$

Remark:

Finiteness of m_h^2 is guaranteed from 5D viewpoint,
all KK modes should be taken into account
⇒ in summing KK modes, some care are necessary

$$\frac{1}{2\pi\hat{R}}\sum_n$$

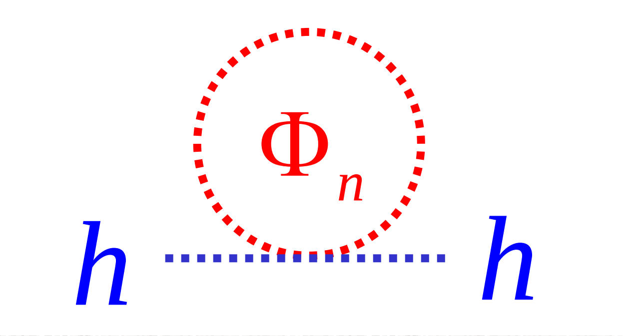
should be adopted

$$\int\frac{dk_y}{2\pi}\left(\hat{R}\to\infty\right)$$

$$R\,e^{(\varphi_0+h)/2}$$

(Nontrivial “h” dependence)

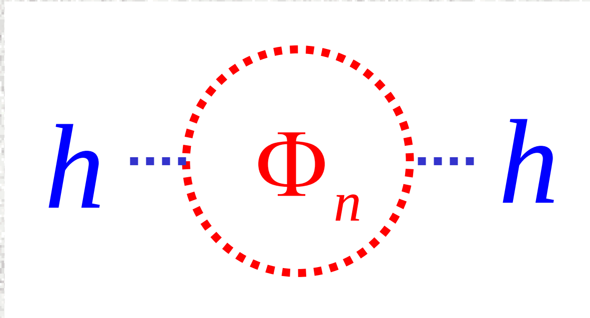
(Natural 5D interpretation)



$$= \frac{1}{2\pi\hat{R}} \sum_n \int \frac{d^4k}{(2\pi)^4} i \frac{1}{2}\frac{1}{2}(-i)\left(\frac{n}{\hat{R}_0}\right)^2 \frac{i}{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2} h^2$$

$$= \frac{1}{2\pi R} e^{-\varphi_0/2}\left(1 - \frac{h}{2} + \frac{h^2}{8} - \cdots\right) \sum_n \int \frac{d^4k}{(2\pi)^4} i \frac{1}{4}(-i)\left(\frac{n}{\hat{R}_0}\right)^2 \frac{i}{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2} h^2$$

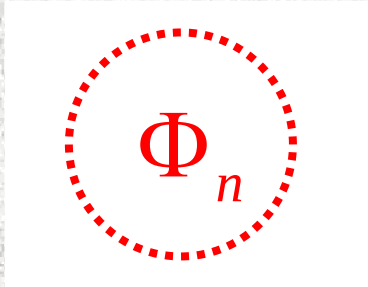
$$\supset \frac{1}{2\pi\hat{R}_0} \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{i}{4} \frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2} h^2$$



$$= \frac{1}{2\pi\hat{R}}\sum_n\int\frac{d^4k}{(2\pi)^4}\, i\left(\frac{1}{2}\right)^2\left\{i\left(\frac{n}{\hat{R}_0}\right)^2\right\}^2\left\{\frac{i}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}\right\}^2 h^2$$

$$= \frac{1}{2\pi R}e^{-\varphi_0/2}\left(1-\frac{h}{2}+\frac{h^2}{8}-\cdots\right)\sum_n\int\frac{d^4k}{(2\pi)^4}\, i\,\frac{1}{4}\left\{i\left(\frac{n}{\hat{R}_0}\right)^2\right\}^2\left\{\frac{i}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}\right\}^2 h^2$$

$$\supset \frac{1}{2\pi\hat{R}_0}\sum_n\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{4}\left\{\frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}\right\}^2 h^2$$

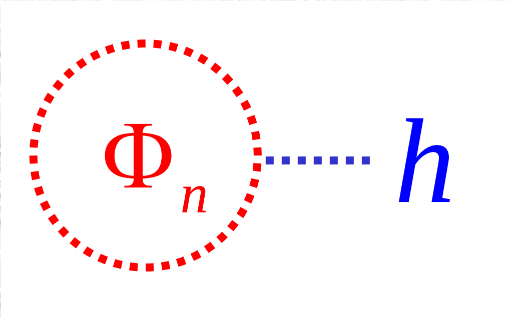


"Bubble"

$$=\frac{1}{2\pi\hat{R}}\sum_n\int\frac{d^4k}{(2\pi)^4}\left(-\frac{i}{2}\right)\ln\left[-k_\mu k^\mu+\left(\frac{n}{\hat{R}_0}\right)^2\right]$$

$$=\frac{1}{2\pi R}e^{-\varphi_0/2}\left(1-\frac{h}{2}+\frac{h^2}{8}-\cdots\right)\sum_n\int\frac{d^4k}{(2\pi)^4}\left(-\frac{i}{2}\right)\ln\left[-k_\mu k^\mu+\left(\frac{n}{\hat{R}_0}\right)^2\right]$$

$$\supset\frac{1}{2\pi\hat{R}_0}\sum_n\int\frac{d^4k}{(2\pi)^4}\left(-\frac{i}{2}\right)\frac{1}{8}\ln\left[-k_\mu k^\mu+\left(\frac{n}{\hat{R}_0}\right)^2\right]h^2$$



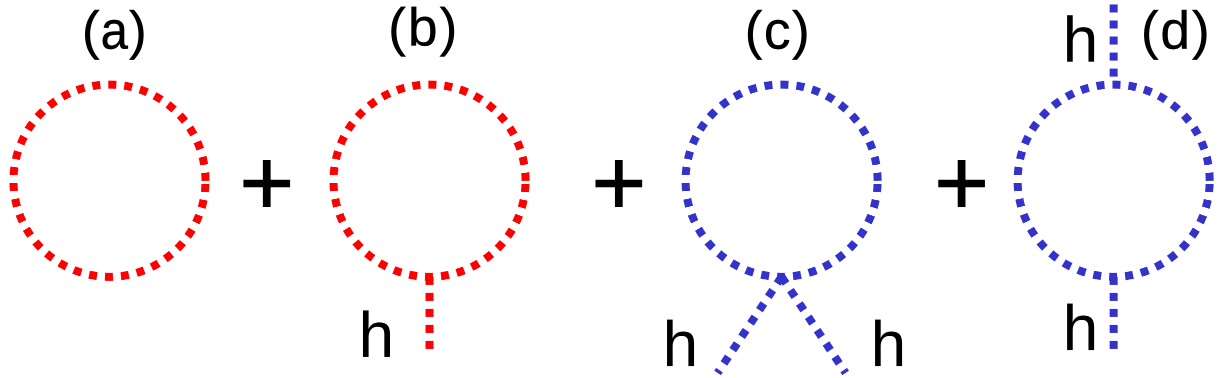
“Tadpole”

$$=\frac{1}{2\pi\hat{R}}\sum_n\int\frac{d^4k}{(2\pi)^4}i\,\frac{1}{2}\,i\left(\frac{n}{\hat{R}_0}\right)^2\frac{i}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}h$$

$$=\frac{1}{2\pi R}e^{-\varphi_0/2}\left(1-\frac{h}{2}+\frac{h^2}{8}-\cdots\right)\sum_n\int\frac{d^4k}{(2\pi)^4}i\,\frac{i}{2}\left(\frac{n}{\hat{R}_0}\right)^2\frac{i}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}h$$

$$\supset\frac{1}{2\pi\hat{R}_0}\sum_n\int\frac{d^4k}{(2\pi)^4}\frac{i}{4}\frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}h^2$$

$$\frac{1}{2} m_h^2 h^2 =$$



$$= -\frac{i}{2} \frac{1}{2\pi \hat{R}_0} \frac{1}{M_5^3} \sum_n \int \frac{d^4 k}{(2\pi)^4} \times$$

$$\left[\underbrace{\frac{1}{4} \ln\left(-k_\mu k^\mu + \left(n/\hat{R}_0 \right)^2 \right)}_{(a)} - \underbrace{\frac{\left(n/\hat{R}_0 \right)^2}{k_\mu k^\mu - \left(n/\hat{R}_0 \right)^2}}_{(b)} - \underbrace{\frac{\left(n/\hat{R}_0 \right)^2}{k_\mu k^\mu - \left(n/\hat{R}_0 \right)^2}}_{(c)} - \underbrace{\frac{\left(n/\hat{R}_0 \right)^4}{\left\{ k_\mu k^\mu - \left(n/\hat{R}_0 \right)^2 \right\}^2}}_{(d)} \right]$$

$$\times \frac{1}{2} h^2$$

Before considering the finite radius case,
it is instructive to take the infinite radius limit

$$m_h^2 \xrightarrow[\substack{n/\hat{R}\to k_y \\ \frac{1}{2\pi\hat{R}}\sum_n \to \int \frac{dk_y}{2\pi}}]{\hat{R}\to\infty} -\frac{i}{2}\int \frac{d^4k dk_y}{(2\pi)^5}\left\{\frac{1}{4}\ln\left[-k_\mu k^\mu + k_y^2\right] + 2\frac{k_y^2}{-k_\mu k^\mu + k_y^2} - \frac{k_y^4}{\left[-k_\mu k^\mu + k_y^2\right]^2}\right\}$$

Infinite radius limit of m_φ^2 can be in fact shown to be zero

$$m_h^2\left(\hat{R}\to\infty\right) = \left[\frac{1}{4} + \frac{1}{\alpha}\frac{d}{d\alpha} + \frac{1}{4}\frac{1}{2\alpha}\frac{d}{d\alpha}\frac{1}{2\alpha}\frac{d}{d\alpha}\right] I(\alpha)\Big|_{\alpha=1}$$

$$= \left[\frac{1}{4\alpha} - \frac{1}{\alpha^3} + \frac{3}{4\alpha^5}\right]_{\alpha=1} \tilde{I} = \left[\frac{1}{4} - 1 + \frac{3}{4}\right]\tilde{I} = 0!!$$

using a useful expression

$$I(\alpha) \equiv -\frac{i}{2}\int \frac{d^4k dk_y}{(2\pi)^5}\ln\left[-k_\mu k^\mu + \alpha^2 k_y^2\right] = -\frac{i}{2}\frac{1}{\alpha}\int \frac{d^4k d\tilde{k}_y}{(2\pi)^5}\ln\left[-k_\mu k^\mu + \tilde{k}_y^2\right] \equiv \frac{1}{\alpha}\tilde{I}$$

We expect that the calculated mass should be finite

More precisely,

the mass should vanish in the limit of $\hat{R} \to \infty$

since the local operator h^2 is forbidden

by the general coordinate transformation invariance

The difference between the finite and

the infinite radius cases appears in the infrared region

of 5^{th} momentum

i.e. UV divergence is insensitive to

the finiteness of the radius

For the finite radius of interest,

the mass can be calculated as

$$m_h^2 = \left[\frac{1}{4}\hat{I} + \frac{1}{\alpha}\frac{\partial}{\partial \alpha}\hat{I} + \frac{1}{4}\left(\frac{1}{\alpha}\frac{\partial}{\partial \alpha}\right)^2 \hat{I} \right]_{\alpha=1} = \underbrace{\left(\frac{1}{4} - 1 + \frac{3}{4}\right)}_{0!!} \frac{\Lambda_{UV}^5}{M_5^3} \underbrace{- \frac{75}{512\pi^7}\frac{\zeta(5)}{M_5^3 \hat{R}^5}}_{\mathit{finite}!!}$$

$$\hat{I}\left(\alpha, \hat{R}\right) \equiv -\frac{i}{2}\frac{1}{2\pi\hat{R}}\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4} \ln\left[-k_\mu k^\mu + \alpha^2 \left(\frac{n}{\hat{R}}\right)^2\right] = \frac{1}{\alpha}\left\{\Lambda_{UV}^5 - \frac{3\alpha^5}{128\pi^7}\frac{\zeta(5)}{\hat{R}^5}\right\}$$

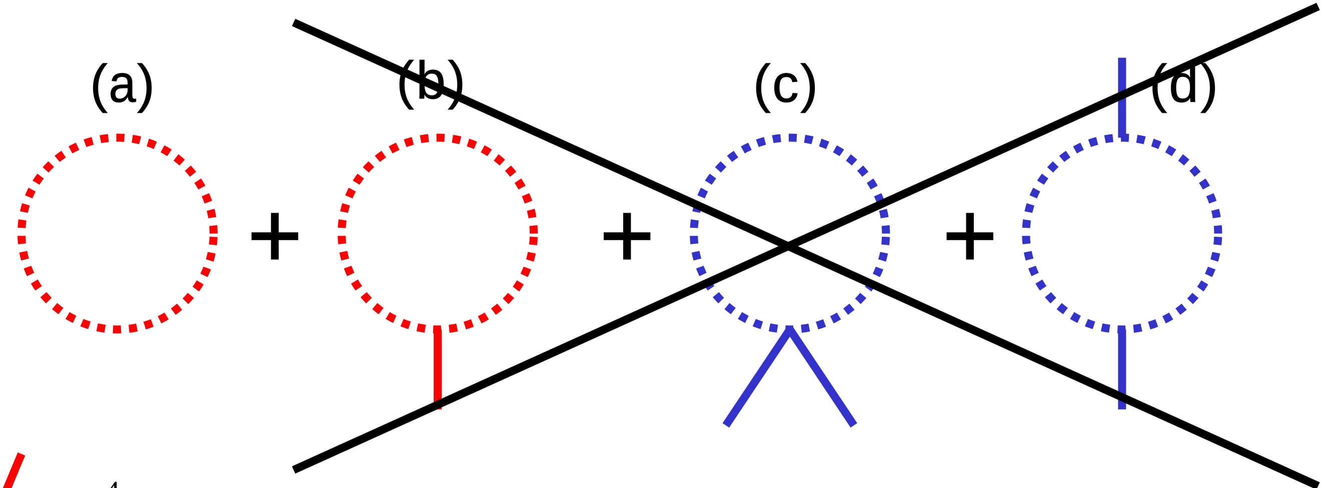
UV divergent constants, corresponding to the infinite radius limit, are **exactly canceled** as expected

For canonically normalized “H”:

$$m_H^2 = 2\pi \hat{R}_0 \times 32\pi G \times m_h^2 = -\frac{75\zeta(5)}{8\pi^5 M_4^2 \hat{R}_0^4}$$

I f only zero mode (n=0) is considered …

$$\frac{1}{2} m_h^2 h^2 =$$



$$= -\frac{i}{2}\frac{1}{2\pi\hat{R}_0}\frac{1}{M_5^3}\sum_n \int \frac{d^4k}{(2\pi)^4}\times$$

$$\left[\underbrace{\frac{1}{4}\ln\left(-k_\mu k^\mu + \left(n/\hat{R}_0\right)^2\right)}_{(a)} - \underbrace{\frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2}}_{(b)} - \underbrace{\frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2}}_{(c)} - \underbrace{\frac{\left(n/\hat{R}_0\right)^4}{\left\{k_\mu k^\mu - \left(n/\hat{R}_0\right)^2\right\}^2}}_{(d)}\right] \times \frac{1}{2}h^2$$

UV Divergence due to a 0-mode is reproduced
“Bubble” diagram is important!!

$$m_h^2 = -\frac{i}{2}\left(\frac{1}{2\pi \hat{R}_0}\right)\int \frac{d^4k}{(2\pi)^4}\frac{1}{4}\ln\left[-k_\mu k^\mu\right] = \infty$$

UV Divergence due to a 0-mode is reproduced

"Bubble" diagram is important!!

Consistent with the fact that 5D general coordinate transformation invariance generated by the KK mode sum ensured the finiteness of Higgs mass

The above result can be obtained more systematically

Effective potential: Massless scalar contribution

$$V_{eff}(\varphi) = -\frac{i}{2}\left(\frac{1}{2\pi\hat{R}}\right)\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\ln\left[-k_\mu k^\mu + \left(\frac{n}{\hat{R}}\right)^2\right]$$

Appelquist & Chodos (83)

Easy to see that Higgs mass vanishes in the $\hat{R} \to \infty$ limit

$$m_\varphi^2 = \frac{\partial^2 V_{eff}(\varphi)}{\partial\varphi^2} \xrightarrow[\substack{n/\hat{R}\to k_y \\ \frac{1}{2\pi\hat{R}}\sum_n \to \int\frac{dk_y}{2\pi}}]{\hat{R}\to\infty} \frac{\partial^2}{\partial\varphi^2}\left\{-\frac{i}{2}\int\frac{d^4k\,dk_y}{(2\pi)^5}\ln\left[-k_\mu k^\mu + k_y^2\right]\right\} = 0!!$$

Higgs mass for the finite radius case is readily obtained

$$m_h^2 = \frac{\partial^2}{\partial\phi^2}\left\{-\frac{i}{2}\left(\frac{1}{2\pi\hat{R}}\right)\sum_n\int\frac{d^4k}{(2\pi)^4}\ln\left[-k_\mu k^\mu+\left(\frac{n}{\hat{R}}\right)^2\right]\right\}\Bigg|_{\phi=\phi_0}$$

$$= \left(-\frac{i}{2}\right)\left(\frac{1}{2\pi\hat{R}_0}\right)\sum_n\int\frac{d^4k}{(2\pi)^4}$$

$$\times\left\{\frac{1}{4}\ln\left[-k_\mu k^\mu+\left(\frac{n}{\hat{R}_0}\right)^2\right]-2\frac{\left(n/\hat{R}_0\right)^2}{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2}-\frac{\left(n/\hat{R}_0\right)^4}{\left\{k_\mu k^\mu-\left(n/\hat{R}_0\right)^2\right\}^2}\right\}$$

Completely agree with Feynman diagram calculations

$$\Rightarrow \quad m_H^2 = -\frac{75\zeta(5)}{8\pi^5 M_4^2\hat{R}_0^4}$$

Straightforward to calculate the effective potential for other massless fields by taking into account the DOF of the physical polarization Appelquist & Chodos (83)

Ponton & Poppitz (01)

$$V_{\text{graviton}}(\varphi) = 5V_{\text{scalar}}(\varphi)$$

$$V_{\text{vector}}(\varphi) = 3V_{\text{scalar}}(\varphi)$$

$$V_{\text{gravitino}}(\varphi) = -8V_{\text{scalar}}(\varphi)$$

$$V_{\text{fermion}}(\varphi) = -4V_{\text{scalar}}(\varphi)$$

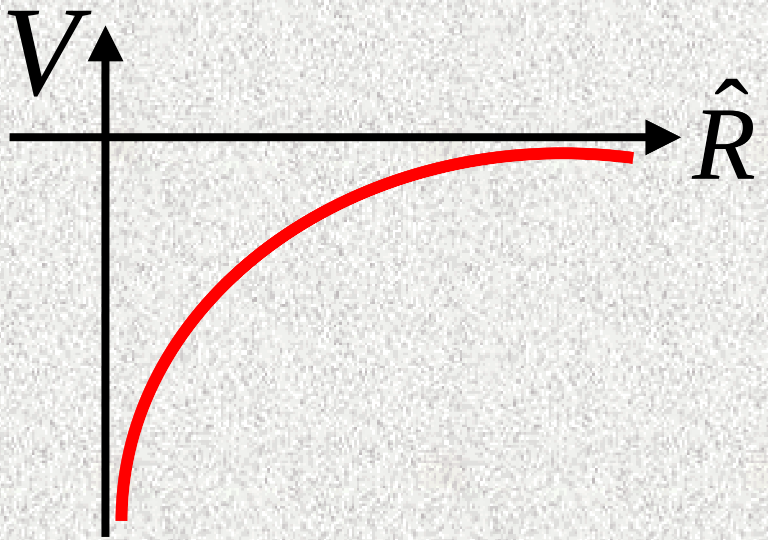
$$m_H^2 = -\left\{\left(5n_{\text{graviton}} + 3n_{\text{vector}} + n_{\text{scalar}}\right) - \left(8n_{\text{gravitino}} + 4n_{\text{fermion}}\right)\right\} \frac{25\zeta(5)}{4\pi^5 M_4^2 \hat{R}_0^4}$$

Comments on Radius Stabilization

Effective potential shows that the radius shrinks to 0

$$V_{\text{Massless scalar}} = -\frac{3}{128\pi^7 \hat{R}^5}\zeta(5)$$

$$V_{\text{Graviton}} = 5V_{\text{Massless scalar}}$$



Ex. Massive scalar with mass “m” Ponton & Poppitz (01)

$$V^{\pm}_{\text{Massive scalar}} = -\frac{3}{128\pi^7 \hat{R}^5}\left[Li_5\left(\pm e^{-2\pi m\hat{R}}\right) + 2\pi m\hat{R}\, Li_4\left(\pm e^{-2\pi m\hat{R}}\right) + \frac{4}{3}\pi^2 m^2 \hat{R}^2 Li_3\left(\pm e^{-2\pi m\hat{R}}\right)\right]$$

$$Li_n(x) \equiv \sum_{k=1}^{\infty}\frac{x^k}{k^n},\quad \pm\text{: periodicity of B.C.}$$

Minimum for anti-periodic B.C. $\Rightarrow$ $\hat{R} \sim \mathcal{O}(0.1-1)\, m^{-1}$ (for $m\hat{R} < 1$)

Summary & Discussion

- We have explicitly calculated
  1-loop Corrections to the Graviscalar Mass
  in 5D Gravity coupled with a Scalar field
  compactified on a circle

- **Quadratic Divergences** are cancelled &
  (Non-local, g.c.t. inv) Finite Mass is induced

- Unclear that this finite mass can be understood
  due to Wilson loop as in the Gauge-Higgs Unification
  (line integral of Christoffel's gamma)

Phenomenological problem

$$m_H^2 = -\frac{25\zeta(5)}{4\pi^5 M_4^2 \hat{R}_0^4} \approx M_W^2 \Rightarrow \frac{1}{\hat{R}_0} \sim \sqrt{M_4 M_W} \sim 10^{10}\,\text{GeV}$$

4D Gauge coupling: $\mathcal{O}\left(\frac{1}{M_4 \hat{R}_0}\right) \sim \mathcal{O}\left(10^{-8}\right)$

too small to account for the magnitude $e = \sqrt{4\pi\alpha}$

For 4D gauge coupling "e" to be correct order,

$$\hat{R}_0^{-1} \sim \sqrt{4\pi\alpha} M_4 \sim 0.1 M_4$$

In the present S^1 case,

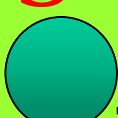
the gauge group is abelian & the graviscalar is neutral

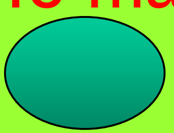
⇒ Symmetry breaking does not occur by VEV of the graviscalar

In order to obtain charged graviscalar as Higgs,
Nonabelian extension is needed ⇒ more than 6D

In progress with Hasegawa & Lim

- 6D theory compactified on S^2 ⇒ SU(2)
- S^2 → Elliptic manifold ⇒ SU(2) → U(1)





- $m_h^2 = 0$ is expected since any loop on S^2 can be shrinked to a point

Backup Slides

Dimensional reduction

Y dependence of fields are simply dropped & integration of y

$$S_g = \frac{1}{16\pi G_5}\int d^4x dy \sqrt{g}\,\mathcal{R}_{(5)}, \quad g_{MN} = \begin{pmatrix} g_{\mu\nu} - e^{\varphi} A_\mu A_\nu & -e^{\varphi} A_\mu \\ -e^{\varphi} A_\nu & -e^{\varphi} \end{pmatrix}$$

$$\Rightarrow \frac{2\pi R}{16\pi G_5}\int d^4x \sqrt{-g_{(4)}}\, e^{\varphi/2}\left[\mathcal{R}_{(4)} - \frac{1}{4} e^{\varphi} F^{\mu\nu} F_{\mu\nu}\right] \left(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\right)$$

$$\Downarrow \quad \begin{cases} \text{gauge conditions}: \partial^\mu h_{\mu\nu} = 0,\ \eta^{\mu\nu} h_{\mu\nu} - h = 0,\ \partial^\mu A_\mu = 0 \\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},\ e^{\varphi} = e^{\varphi_0}(1+h) \end{cases}$$

$$= \frac{1}{16\pi G_4}\int d^4x \left[\frac{1}{4}\left(\partial_\mu h_{\alpha\beta}\right)\left(\partial^\mu h^{\alpha\beta}\right) - \frac{e^{\varphi_0}}{2}\left(\partial_\mu A_\nu\right)\left(\partial^\mu A^\nu\right) + \frac{3}{8}\left(\partial_\mu h\right)\left(\partial^\mu h\right)\right]$$

$$G_4 \equiv G_5 \Big/ \left(e^{\varphi_0/2}\, 2\pi R\right)$$

Canonically normalized Higgs "H"

$$H = \sqrt{\frac{6}{8\pi G_4}}\,\frac{h}{4}$$

Classical Solution of Einstein equation:

$$g_{\mu\nu} = \eta_{\mu\nu}, A_\mu = 0, \varphi = \text{const}$$

Consistent with that
the background space-time is $M^4 \times S^1$

At the classical level,
the constant φ or the radius of S^1 is not fixed,
but determined by the quantum effects