ASPECTS OF GAUGE-HIGGS UNIFICATION

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- Introduction
- Gauge-Higgs unification in 5D
- Gauge-Higgs unification in 6D
- A 5D model of flavour
- Conclusions & Outlook

Based on work done in collaboration with G. Martinelli, M. Salvatori, C.A. Scrucca, M. Serone and A. Wulzer

INTRODUCTION

- SM extremely successful in reproducing EW & Flavour data, but
  - NO understanding of flavour structure
  - Higgs EW breaking quadratically sensitive to the UV
- SUSY removes UV sensitivity, but
  - Flavour problem gets much worse
  - SUSY breaking still a mistery (hidden sector)
  - Why are superpartners so heavy?
- Symmetry breaking weakest point:

EW (m_H?), SUSY (L_{soft}?), Flavour

INTRODUCTION (cont'd)

- Gauge-Higgs unification is a promising alternative to SUSY to get UV-insensitive electroweak breaking via the Scherk-Schwarz (SS) compactification on orbifolds Hatanaka, Inami & Lim; Kubo, Lim & Yamashita; Antoniadis, Benakli & Quiros; Arkani-Hamed, Cohen & Georgi; Csaki, Grojean & Murayama; Burdman & Nomura; Hosotani, Haba, Yamashita; talk by Takenaga; ...
- Can introduce a flavour symmetry broken à la SS to solve the flavour problem

SS BREAKING ON ORBIFOLDS

- Interesting possibilities combining orbifold projections and SS breaking. Example: SU(3) →SU(2)xU(1)→U(1)

$$P = e^{i\pi\lambda_3} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T(\alpha) = e^{2i\pi\alpha\lambda_7} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\pi\alpha & \sin 2\pi\alpha \\ 0 & -\sin 2\pi\alpha & \cos 2\pi\alpha \end{pmatrix}$$

$$\psi(-y) = P\gamma_5\psi(y) \qquad A_\mu(-y) = PA_\mu(y)P^\dagger \qquad A_5(-y) = -PA_5(y)P^\dagger$$

$$\psi(y+2\pi R) = U\psi(y) \qquad A_M(y+2\pi R) = UA_M(y)U^\dagger$$

SSB ON ORBIFOLDS (cont'd)

- Under the twist the photon is neutral, $W^{\pm}$ have charge 1 and Z has charge 2, so that $M_W = \alpha/R$ and $M_Z = 2\alpha/R$. KK level spacing is 1/R.
- Doing the Hosotani transformation, this is equivalent to SU(2)xU(1) broken by

$$\langle h \rangle \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} \langle A_5^4 + i A_5^5 \rangle \\ \langle A_5^6 + i A_5^7 \rangle \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\alpha}{R} \end{pmatrix}$$

- h looks like the SM Higgs! Is it any better?

FINITE HIGGS MASS WITHOUT SUSY

Hatanaka, Inami & Lim; Kubo, Lim & Yamashita; Antoniadis, Benakli & Quiros; Arkani-Hamed, Cohen & Georgi;...

- Since A_5 is a gauge field, on S^1 gauge inv. forbids local (divergent) mass terms
- On orbifold fixed points there is a reduced group SU(2)xU(1) ($A_\mu^{4,5,6,7}$ and $\xi^{4,5,6,7}$ odd, vanish at the fp). Is A_5 safe?
- Yes! Since $\xi^{4,5,6,7}$ odd, $\partial_5\xi^{4,5,6,7}$ is even:

  $A_5^{4-7} \longrightarrow A_5^{4-7} + i\partial_5\xi^{4-7}$ forbids local mass terms & divergences! V. Gersdorff, Irges & Quiros
- Not true in D=6: F_{56} at the fp is gauge invariant! Dangerous tadpoles can be present...

Scrucca, Serone, L.S. & Wulzer; Biggio & Quiros

A PROBLEM WITH FERMIONS

Can this "Higgs" generate fermion masses?

Fermion masses must come from gauge couplings to A_5 :

- Family universal, no mixing
- All masses of order M_Z
- Must embed L and R components of SM fermions in the same repr. of SU(3)

  Ex. (Q_L, d_R) in a triplet: OK for d masses, but u remain massless

  Antoniadis, Benakli & Quiros

Doesn't really look reasonable…

A HINT OF A SOLUTION

Csaki, Grojean & Murayama

- Put SM L and R fermions at the two boundaries:
  - No need to embed them in multiplets of the higher gauge group
  - The exchange of massive bulk particles could generate effective mass terms
  - Flavour-violating couplings to bulk particles could generate hierarchies and mixings
- However:
  - In 6D, divergences at fixed points for m_h are present;
  - No concrete realization of fermion mass generation

A 5D MODEL OF EW BREAKING

C.A. Scrucca, M. Serone & L.S. (S^3)

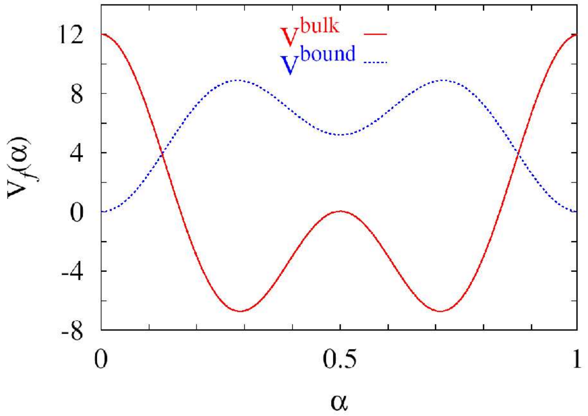
- SU(3)xU(1) on twisted S_1/Z_2:

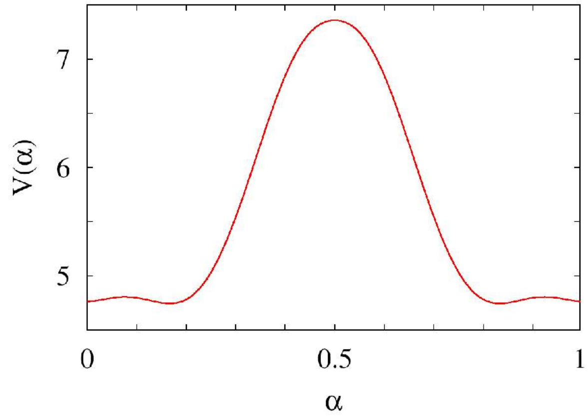
  $SU(3)\times U(1) \rightarrow SU(2)\times U(1)'\times U(1) \rightarrow U(1)_{em}\times U(1)''$
- EW symmetry broken by $\langle A_5\rangle=\alpha$, $M_W=\alpha/R$: α computable & UV insensitive
- Introduce SM fermions at the fixed points plus massive bulk fermions (**3** for d-quark masses and **6** for u-quark masses, same for leptons)

THE EFFECTIVE POTENTIAL

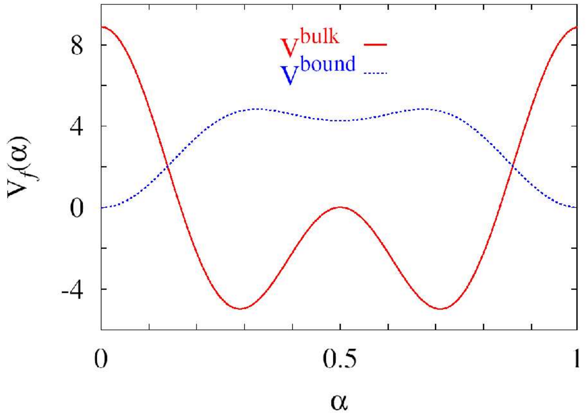
Fermionic contribution

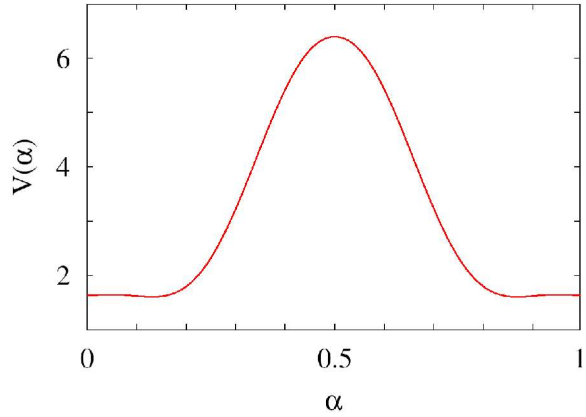
Full potential





All matter in y=0





LH matter in y=0,
RH in $y=\pi R$

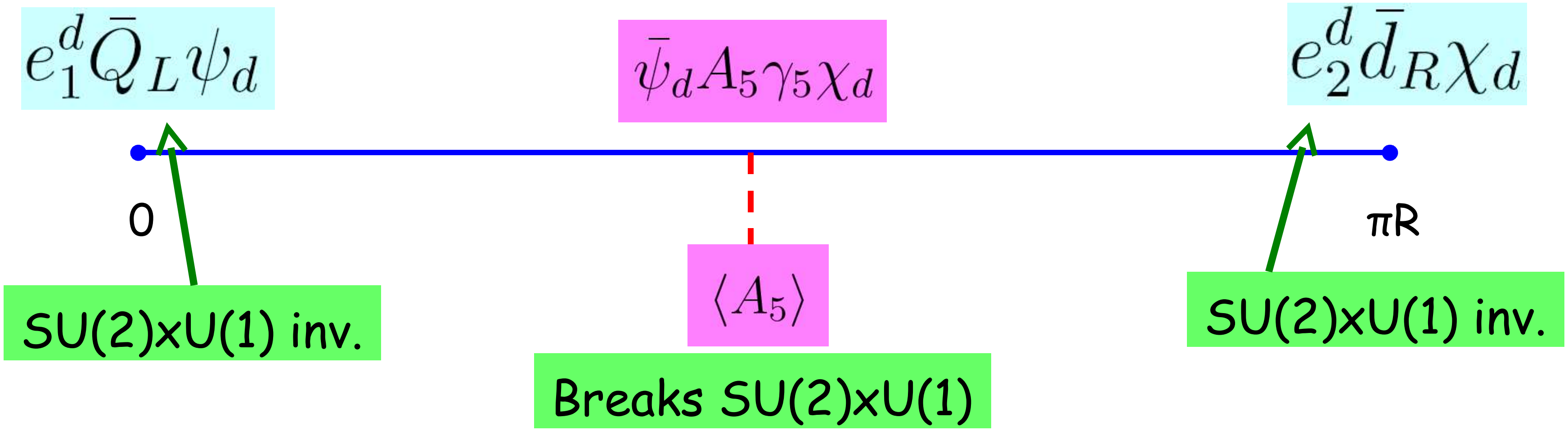
Get α ~ 0.1-0.2, 1/R ~ 400-800 GeV, m_H ~ 30 GeV

POSSIBLE IMPROVEMENTS

- Add bulk matter in large reps of SU(3): get α=1/rank, m_H improves considerably. Cutoff lowered from 100/R to a few 1/R.
- Add brane-localized gauge kinetic terms: bulk gauge coupling g larger, m_H and 1/R grow as g. Mixing with KK modes violates custodial symmetry $\Rightarrow$ need very small α to respect EW constraints

FERMION MASSES

- Take down-type quarks for example:



- This generates a mass term of the form

$$m_d \propto \epsilon_d^1 \epsilon_d^2 x_d e^{-x_d} M_W$$

$$x_d = \pi R M_d$$

$$\epsilon^{1,2} = \sqrt{\pi R} e^{1,2}$$

AN EFFECTIVE THEORY POINT OF VIEW

- Mass generation:

Q_L at y=0 Bulk fermions d_R at y=π

$$\tilde{m}_d \sim \epsilon_d^1 \, \epsilon_d^2 \, e^{-M\pi R} \, M_W$$

- Wave function corrections:

Q_L at y=0 Bulk fermions Q_L at y=o

$$\delta Z_Q \sim \frac{\left(\epsilon_d^1\right)^2}{M\pi R}\left(1 + e^{-2M\pi R}\alpha^2\right)$$

- Vertex corrections:

Q_L at y=0 Bulk fermions Q_L at y=o

Z^0

$$\delta V_Q \sim \left(\delta Z_Q + \frac{\left(\epsilon_d^1\right)^2 \alpha^2}{M\pi R}\right)$$

- Go to canonically normalized fields:

$$m_d = \frac{\tilde{m}_d}{\sqrt{Z_Q Z_d}} \sim \boxed{\frac{\epsilon_d^1\,\epsilon_d^2}{\sqrt{(1+(\epsilon_d^1)^2)(1+(\epsilon_d^2)^2)}}}\, x_d\, e^{-x_d}\, M_W$$

upper bound to fermion masses ~ M_W:

a problem for the top mass...

- Vertex corrections spoil the universality of weak interactions at $O(\alpha^2)$
- Z^0-vertex corrections give FCNC couplings at $O(\alpha^2)$

A KK POINT OF VIEW

- SM particles arise as an admixture of bulk and brane fields: diagonalize full mass matrix. Define mixing angles by

$$\alpha_2^{d;a} = \mathrm{Arctan}\left(\sqrt{1/x_d}\,\epsilon_2^{d;a}\right) \qquad \alpha_1^{d;a} = \mathrm{Arctan}\left(\frac{\sqrt{1/x_d}\,\epsilon_1^{d;a}}{\sqrt{1+1/x_u\,\left(\epsilon_1^{u;a}\right)^2}}\right)$$

- Quark masses are then given by

$$m_d = \sin\alpha_1^d \sin\alpha_2^d x_d e^{-x_d} M_W$$

- Mixing with vector-like bulk fermions $\Rightarrow$ tree-level FCNC

WILSON LINES AGAIN

- Effects of bulk fermions encoded in the functions

$$f_0(x,\alpha) = \sum_{n=-\infty}^{\infty} \frac{1}{x + i\pi(n+\alpha)} = \coth(x + i\pi\alpha) \; ,$$

$$f_1(x,\alpha) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{x + i\pi(n+\alpha)} = \sinh^{-1}(x + i\pi\alpha)$$

- Taylor expansion reveals Wilson lines:

$$f_\delta(x,\alpha) = \sum_{k=-\infty}^{\infty} e^{-|2k+\delta|(x+i\pi\alpha)} \quad \Rightarrow \quad \sum_k e^{-|2k+\delta|x} \quad W_{|2k+\delta|}$$

- Wilson line wrapping 2k times around the extra dimension

PROBLEMS OF S^3

- With this bulk fermion content, difficult to achieve $\alpha < 0.1$

  $\Rightarrow$ 1/R ~ 800 GeV too low!!
- No tree-level higgs quartic coupling

  $\Rightarrow m_H \sim 30$ GeV too low!!
- Breaks universality of weak interactions

  $\Rightarrow$ need $\alpha < 0.01$
- Mixing with bulk fields gives tree-level FCNC $\Rightarrow$ need $\epsilon^2\alpha^2 < 10^{-5}$

GAUGE-HIGGS UNIFICATION IN D=6

- Possible orbifold projections:

  T_2/Z_N with N=2,3,4,6.

- Orbifold action on the complex T^2 plane:

  $z \rightarrow \tau z$, with $\tau = e^{2\pi i/N}$

- Orbifold action on SU(3) gauge sector:

$$P = \tau^{2\,n_p\left(\frac{1}{3}+\frac{1}{\sqrt{3}}t^8\right)} = \begin{pmatrix} \tau^{n_p} & 0 & 0 \\ 0 & \tau^{n_p} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

n_p integer

THE HIGGS SECTOR

- Orbifold projection on gauge fields: define $A_{M\pm1} \sim A_{M4} \mp iA_{M5}$, $A_{M\pm2} \sim A_{M6} \mp iA_{M7}$:

$$A_{\mu 1,2,3,8} : 1\,, \quad A_{z 1,2,3,8} : \tau^{-1}\,, \quad A_{\bar{z}\, 1,2,3,8} : \tau^{+1}\,,$$

$$A_{\mu\pm i} : \tau^{\pm n_p}\,, \quad A_{z\pm i} : \tau^{-1\pm n_p}\,, \quad A_{\bar{z}\pm i} : \tau^{+1\pm n_p}\,.$$

- Get 2 doublets for N=2, 1 doublet for N=3, 1 or 0 doublets for N=4, 6.

THE HIGGS SECTOR (cont'd)

- Case of 2 doublets similar to MSSM: lightest Higgs tends to be too light (detailed analysis in progress)
- For a single Higgs doublet, tree-level quartic coupling is $g^2/2$, $m_W = gv/2$ and $m_H = gv$, so that $\mathbf{m_H = 2\, m_W}$!

LOCALIZED TADPOLES

- In D=6, localized tadpoles can arise at orbifold fixed points:

  F_{56} 4D scalar, gauge covariant

- Violates generalized parity $z \leftrightarrow \bar{z}$, $\tau \rightarrow \bar{\tau}$ takes the form $\mathrm{Im}\,\mathrm{Tr}\, P^k F_{z\bar{z}}$
- Nonvanishing and quadratically divergent for Im P $\neq$ 0 $\Leftrightarrow$ **single Higgs**!

EFFECTS OF LOCALIZED TADPOLES

- Tadpole Lagrangian:

$$\mathcal{L}_{\mathrm{tad}} = -i \sum_{k=1}^{[N/2]} \frac{\mathcal{C}_k}{N_k} \sum_{i_k=1}^{N_k} \delta^{(2)}(z - z_{i_k}) F^8_{z\bar{z}}(z)$$

generates background for A^8_z and quadratic divergence for m_H: unacceptable

- Cannot be canceled locally without adding scalars

GLOBALLY VANISHING TADPOLES

- Integrated tadpole can vanish in Z_4 with a suitable choice of fermion content
- In this case, Higgs still massless but with nontrivial profile in extra-dims
- Constraints from anomaly cancellation (depends on 4D chirality, tadpole not)
- Detailed analysis in progress

BACK TO S^3

- Suppose the problems with the EW breaking can be solved, so that $\alpha \sim 0.01$, $m_H > 115$ GeV and everything works.
- Could we exploit SS breaking for a flavour symmetry, to explain masses and mixing angles in terms of a single breaking parameter?

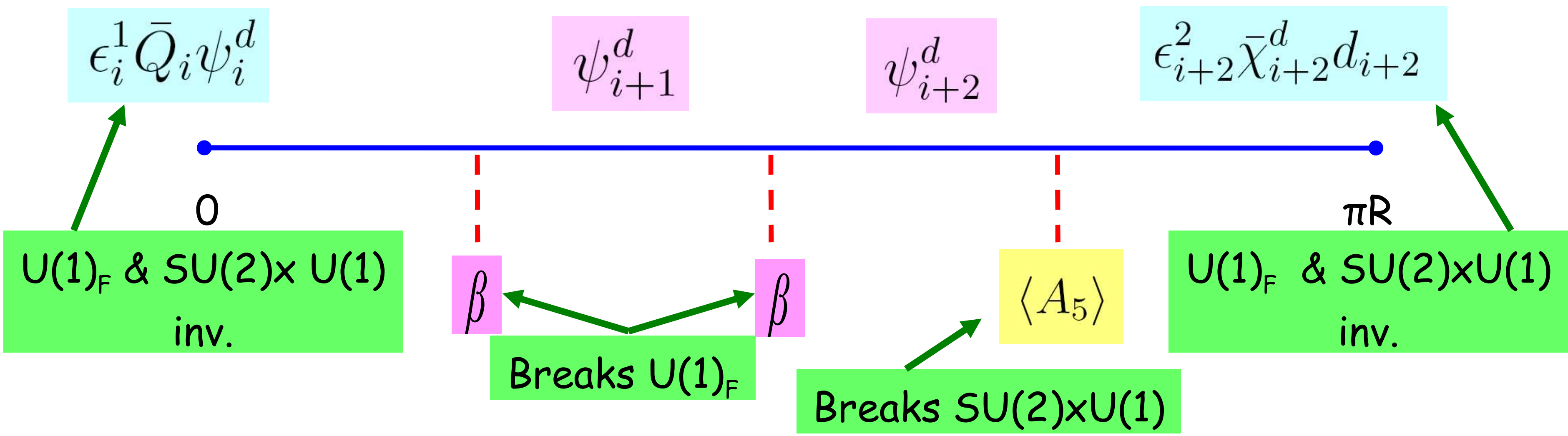
A 5D MODEL OF FLAVOUR

G. Martinelli, C.A. Scrucca, M. Salvatori & L.S.

- Minimal possibility: extend a $U(1)_F$ flavour model to 5D, based on S^3
  - $SU(2) \to U(1) \to$ Nothing via orbifold + SS
  - Effective $U(1)_F$ on the branes for SM fields
- Explain quark masses and mixings in terms of SS ($U(1)_F$-breaking) parameter $\beta \sim \lambda_c$ and bulk fermion masses M_u, M_d
- Result is UV insensitive

FERMION MASSES AGAIN

- Take down-type quarks for example:



- This connects SM fermions with U(1) flavour charges i and i+2

AN EFFECTIVE THEORY POINT OF VIEW

- Mass generation:

Q_L at y=0 | Bulk fermions | d_R at y=π

$$\tilde{m}_d^{ij} \sim \epsilon_i^1 \epsilon_j^2 e^{-M\pi R} \alpha \beta^{|q_i - q_j|}$$

- Wave function corrections:

Q_L at y=0 | Bulk fermions | Q_L at y=o

$$\delta Z_Q^{ij} \sim \frac{\epsilon_i^1 \epsilon_j^1}{M\pi R}\left(\delta_{ij} + e^{-2M\pi R}\alpha^2 \beta^{|q_i - q_j|}\right)$$

- Vertex corrections:

Q_L at y=0 | Bulk fermions | Q_L at y=o

Z^0

$$\delta V_Q^{ij} \sim \frac{\epsilon_i^1 \epsilon_j^1}{M\pi R}\left(\delta_{ij} + e^{-2M\pi R}\alpha^2 \beta^{|q_i - q_j|}\right)$$

Get additional tree-level FCNC, but suppressed by $e^{-2M\pi R}$ ⇒under control!

MINIMAL SU(2) MODEL

- We get an effective U(1) model with

$$Y_{ij} \propto \beta^{|q_i - q_j|}$$

- Since $m_u/m_t \sim O(\lambda_c^{6\text{-}8})$, one needs at least a spin 3 (q_{max}=3) bulk SU(2) representation.
- Bulk-brane mixing angles induce O(1) coefficients for individual Yukawa couplings
- All mass ratios and mixing angles can be generated with the correct order of magnitude by a spin 7/2 representation, with just three free parameters: β, M_u & M_d

Resulting Yukawa couplings prop. to:

$$\widetilde{\mathcal{M}}^d = -4i\,\lambda_c \times \begin{pmatrix} i\frac{\sqrt{7}}{4}\lambda_c{}^5 & \frac{\sqrt{21}}{4}\lambda_c{}^4 & i\frac{\sqrt{35}}{4}\lambda_c{}^3 \\ \frac{3}{2}\lambda_c{}^4 & i\frac{5\sqrt{3}}{4}\lambda_c{}^3 & \sqrt{5}\,\lambda_c{}^2 \\ \sqrt{5}\,\lambda_c{}^2 & i\frac{\sqrt{15}}{2}\lambda_c & 1 \end{pmatrix}$$

$$\widetilde{\mathcal{M}}^u = 4i\,\lambda_c \times \begin{pmatrix} \frac{1}{4}\lambda_c^6 & -i\,\frac{\sqrt{7}}{4}\,\lambda_c^5 & -i\,\frac{\sqrt{35}}{4}\,\lambda_c^3 \\ -i\frac{\sqrt{7}}{4}\,\lambda_c^5 & \frac{3}{2}\lambda_c^4 & \sqrt{5}\,\lambda_c^2 \\ -i\,\frac{\sqrt{35}}{4}\,\lambda_c^3 & \sqrt{5}\,\lambda_c^2 & 1 \end{pmatrix}$$

Half-integer spin rep. do not contain U(1)-neutral states. Mass scale λ_c x M_W gives

$m_t \sim 15$ GeV

$\rightarrow$ need integer spin to raise top mass

A FULL MODEL OF FLAVOUR

- Integer spin representation have large Clebsch-Gordan coefficients which tend to modify the power expansion
- Top (and possibly bottom) quarks are U(1) neutral, so also a bulk flavour singlet could generate their masses
- With a bulk singlet and a bulk 7-plet for up and down quarks (5 parameters), all masses and mixings can be reproduced
- Loop effects expected to be calculable and small due to nonlocal symmetry breaking: explicit computation in progress

CONCLUSIONS

- We have built a 5D model in which:
  - EW breaking is UV insensitive
  - The observed flavour structure can be explained by a flavour symmetry
  - Flavour breaking is UV insensitive
- But of course we inherit the problems of gauge-Higgs unification in 5D:
  - Difficult to get a vev $\alpha \sim 10^{-1}$ - 10^{-2} so that $1/R$ is a few TeV and KK modes do not pose problems.
  - The radiatively induced Higgs mass is too small (~ 30 GeV), since no tree-level potential is present.

OUTLOOK

- The easiest possibility is to include large brane kinetic terms in the 5D model ($\rightarrow$warping) but need custodial symmetry
- 6D models could in principle solve all problems:
  - Are single-Higgs-doublet models viable?
  - Can a smaller α be achieved?
- The 5D bulk masses (free parameters) could be constrained by a larger 6D flavour symmetry
- The dynamical problem of bulk-brane mixing could be solved in a more fundamental theory