Higgs mass in gauge-Higgs unification

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with N. Haba (Tokushima), T. Yamashita (Kyoto), PRD71('05)025006-1, hep-ph/0411250

Y.Hosotani(Osaka), S.Noda(Osaka), hep-ph/0410193 (Phys.Lett.B607('05)276)

I. Introduction

Gauge theory in higher dimensions; new approaches in high energy physics;

Presence of the extra dimension $\Rightarrow$ Compactification on "topological" manifold
$(S^1, S^1/Z_2, T^2, T^2/Z_2, \cdots)$

Boundary Conditions of fields for the compactified direction: Twisted B.C.

⋆ New mechanism, origin of symmetry (gauge, SUSY) breaking

• Spontaneous SUSY breaking from extra dimensions, $\phi(y+L) = \mathrm{e}^{i\alpha}\phi(y)$

$\langle\delta\psi\rangle = i\sqrt{2}\sigma^{\hat{\mu}}\bar{\xi}\partial_{\hat{\mu}}\langle\phi_{vac}(y)\rangle + \sqrt{2}\xi\langle F\rangle \neq 0$ [Sakamoto, Tachibana, K.T.,'99]

• Gauge-Higgs unification, $A_{\hat{\mu}} = (A_\mu, A_y)$ [Hatanaka, Inami, Lim, '98]

Dynamical gauge symmetry breaking through the Wilson line phases,
(Hosotani mechanism)

Important to understand consequence, feature and nature of the mechanism

II. Gauge symmetry breaking through the Wilson line phases

[Y.Hosotani '83]

space-time; $M^{D-1} \times S^1$, $x^{\hat{\mu}} = (x^\mu, y)$, $L = 2\pi R$,

gauge potential, $A_{\hat{\mu}} = (A_\mu, A_y)$, $A_y \cdots$ component gauge field for the S^1 direction

> $A_y = A_y^a T^a \sim$ a scalar field belonging to the adjoint representation under the gauge group from $D-1$ dimensional point of view

$\langle A_y^{(0)} \rangle \cdots\cdots$ cannot be gauged away, physically meaningful, reflecting the topology of the extra dimension

[e.g.] 5 dim., P.B.C. for matter field

$U(1)$ gauge trf. in 4 dim. $\quad \bar{g} L A_y^{(0)}(x) \to \bar{g} L A_y^{(0)}(x) + 2\pi m.$ $(U = \mathrm{e}^{i2\pi m y/L})$

(A) $\bar{g} L \langle A_y^{(0)} \rangle = 2\pi n \leftrightarrow \langle A_y^{(0)} \rangle = 0$; gauge equivalent

(B) $\bar{g} L \langle A_y^{(0)} \rangle = \theta (\neq 2\pi n) \leftrightarrow \langle A_y^{(0)} \rangle = 0$; inequivalent

$\nwarrow \qquad \nearrow$

physically distinct configuration $(\theta$; mod $2\pi)$

generalize to nonabelian gauge theory

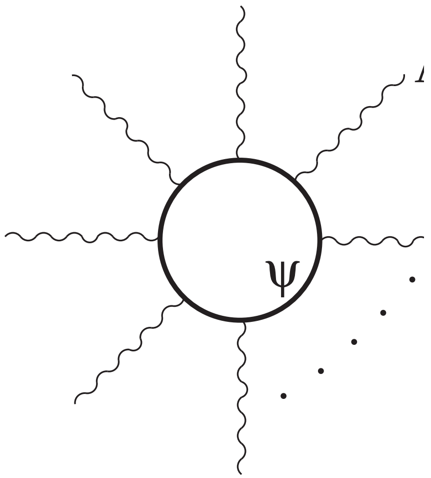
If $\langle A_y^{(0)} \rangle \neq 0$, then, $\mathrm{tr}(F_{\mu y})^2 \sim g^2 \mathrm{tr}[\langle A_y^{(0)} \rangle, A_\mu]^2 \Rightarrow$ gauge symmetry breaking

Effective potential for the Wilson line phases $gL\langle A_y \rangle$; $W = \mathcal{P}\exp(ig\oint dy A_y)$

(t'Hooft-Feynman gauge ($\xi = 1$))

$M^{D-1} \times S^1$

$G = SU(N)$, fermion contributions (P.B.C.)

V(θ)= 

$$V(\theta) = \frac{N_F 2^{[D/2]}\Gamma(D/2)}{L^D \pi^{D/2}} \sum_{i,j=1}^{N} \sum_{n=1}^{\infty} \frac{1}{n^D} \cos[n(\theta_i - \theta_j)] + \cdots$$

♢ $V(\theta)$; radiatively induced potential, ($\partial V(\theta)/\partial\theta = 0 \Rightarrow \theta$; determined dynamically)

♢ $\left.\frac{\partial^2 V(\theta)}{\partial\theta_i \partial\theta_j}\right|_{vac} \sim$ mass term for $A_y^{(0)}$ (the Coleman-Weinberg mechanism)

5 dim. ($R \to \infty \Longleftrightarrow$ 5-D Lorentz inv.) gauge invariance(massless $A_{\hat{\mu}}$),

no 5-D Lorentz invariance by the compactification $1/L$ (only the 4 D Lorentz inv.);

not necessarily massless A_y (the mass scale $1/L$)

- The order parameter for gauge symmetry breaking

  For $\langle A_y \rangle = \frac{1}{gL}\text{diag}(\theta_1, \theta_2, \cdots, \theta_N) \left(= \frac{-i}{g} V^\dagger \partial_y V\right)$,

$$W \equiv \mathcal{P} \text{ exp} \left(ig \oint_{S^1} dy \langle A_y \rangle \right) = \text{diag}(\mathrm{e}^{i\theta_1}, \mathrm{e}^{i\theta_2}, \cdots, \mathrm{e}^{i\theta_N}), \ (\theta_i; \ \text{ mod. } 2\pi),$$

  Then, the residual (physical) gauge symmetry; $H = \{h \in G \ | \ hW = Wh\}$.

  different values of $\theta_i \Rightarrow$ different theory

- summing up all the K-K modes, n

  (original gauge invariance, quantum correction in extra dimension)

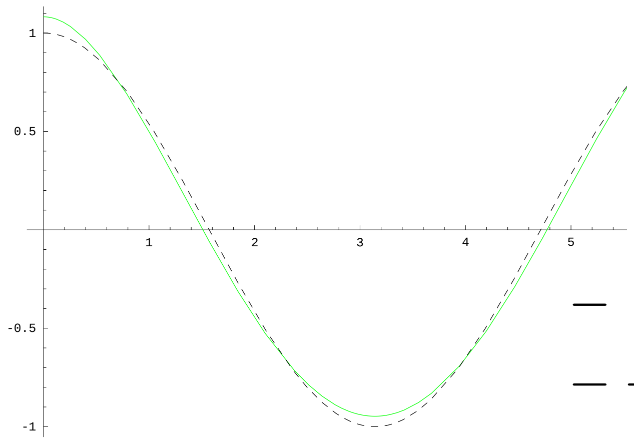
- no dependence of $V(\theta)$ on g at one-loop level

  ($V(\theta) \Leftarrow$ gauge interactions ($gA_{\hat{\mu}} \to A_{\hat{\mu}}$). Two loop $\sim g^2$)

- $F_D(\theta) = \sum_{n=1}^{\infty} \frac{1}{n^D} \cos(n\theta)$.

$$D = 4, \quad F_4(\theta) = \sum_{n=1}^{\infty} \frac{1}{n^4} \cos(n\theta) = \frac{-1}{48}\theta^2(\theta - 2\pi)^2 + \frac{\pi^4}{90}, \qquad (0 \le \theta \le 2\pi)$$

$F_4(\theta)$ $v.s.$ $\cos\theta$



θ

— $F_4(\theta)$

– – – $\cos\theta$ ($\sim F_{10}(\theta)$)

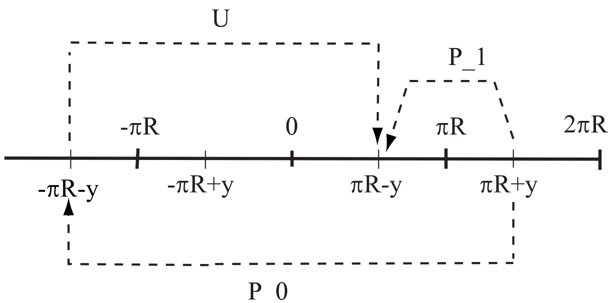
The minimum is located at $\theta = \pi$ (irrelevant to D)

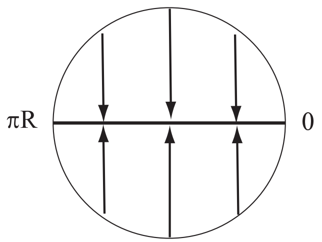
III. Higgs mass in gauge-Higgs unification

S^1 compactification $\to A_y^{(0)}$ adjoint Higgs scalars at low energies

the Higgs field Φ in the standard model; an $SU(2)$ doublet

$\Phi \sim$ a part of the zero mode in $A_y^{(0)} \Longrightarrow S^1/Z_2$ (orbifold), ($y \sim y+2\pi R$ and $y \sim -y$)





Specify boundary conditions of fields for the S^1 direction and at the fixed points;

symmetry degrees of freedom $\Rightarrow$ twisted B.C's.

$$S^1 \ ; \qquad A_{\hat{\mu}}(x, y+L) = U A_{\hat{\mu}}(x,y) U^\dagger,$$

$$y = 0, \pi R \ ; \quad \begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, z_i - y) = P_i \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, z_i + y) P_i^\dagger$$

$z_0 = 0,\ z_1 = \pi R,\ U, P_{0,1} \in G,\ \ P_{0,1}^2 = \mathbf{1}, P_{0,1}^\dagger = P_{0,1},$

the consistency condition yields $U = P_1 P_0$.

$G = SU(3)$; Take $P_{0,1} = \text{diag}(1,1,-1)$, $(P_0, P_1) \equiv$ parity under $P_{0,1}$

$$A_\mu = \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ \hline (-,-) & (-,-) & (+,+) \end{array}\right), [T^{a=1,2,3,8},\ P_{0,1}] = 0,$$

$$A_y = \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array}\right), \{T^{b=4,5,6,7},\ P_{0,1}\} = 0.$$

$A_\mu^{(0)} \cdots SU(2) \times U(1)$ by the orbifolding, P_0 and P_1

$A_y^{(0)} \cdots$ an $SU(2)$ doublet $\Rightarrow$ the Higgs doublet, Φ (= the Wilson line phase)

$$\Phi \equiv \sqrt{2\pi R}\frac{1}{\sqrt{2}}\begin{pmatrix} A_y^4 - iA_y^5 \\ A_y^6 - iA_y^7 \end{pmatrix} \Longrightarrow \langle\Phi\rangle = \sqrt{2\pi R}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \langle A_y^6\rangle \end{pmatrix}, \text{ where } \langle A_y^6\rangle = \frac{a}{gR}$$

Defining $g_4 \equiv g/\sqrt{2\pi R}$ and denoting $v \sim 246$ GeV,

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \quad \Longrightarrow \quad \frac{a_0}{g_4 R} = v$$

$$a_0 << 1 \qquad \text{for} \qquad \frac{1}{R} > v$$

What we want to study is that

$$SU(3) \overset{\text{orbifolding}}{\Longrightarrow} SU(2) \times U(1) \overset{V_{eff(a)}}{\Longrightarrow} \quad ?$$

$$\uparrow$$

Hosotani mechanism

Depending on the values of a_0 (mod $2, \quad 0 \leq a \leq 1$),

$$\text{gauge symmetry breaking patterns} = \begin{cases} SU(2) \times U(1) & \text{for} \quad a_0 = 0, \\ U(1)' \times U(1) & \text{for} \quad a_0 = 1, \\ \tilde{U}(1) & \text{for} \quad \text{otherwise} \end{cases}$$

Field redefinition by $\Omega(y)$ s.t. $\langle A_y \rangle' = \Omega(\langle A_y \rangle - \frac{i}{g}\partial_y)\Omega^\dagger = 0$ with $\Omega(y) = \exp(i\frac{a_0}{2gR}\lambda^6 y)$

Accordingly,

$$P_0' = \Omega(-y)P_0\Omega^\dagger(y) = P_0,\ \ P_1' = \Omega(\pi R - y)P_1\Omega^\dagger(\pi R + y) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a_0) & -i\sin(\pi a_0) \\ 0 & i\sin(\pi a_0) & -\cos(\pi a_0) \end{pmatrix}$$

The residual (physical) gauge symmetry; $\{T^a\}$ such that $[P'_{0,1},\ \ T^{a=1,2,3,8}] = 0$.

[e.g.] $SU(6) \to SU(3) \times \boxed{SU(2) \times U(1)} \times U(1)' \to SU(3) \times \boxed{U(1)''} \times U(1)'$

An example in SUSY models

SUSY models, $V_{eff}(a) = V_{boson}(a) + V_{fermion}(a) = 0$, a; the Wilson line phase

A framework(B.C. of fields);   the Sherk-Schwarz (SS) SUSY breaking,

$$\text{supermultiplet}, \quad (A_{\hat{\mu}}, \lambda); \qquad \lambda(x, y + 2\pi R) = \mathrm{e}^{2\pi i \beta} \lambda(x, y)$$

$\Longrightarrow V_{eff}(a, \beta) = V_{boson}(a) + V_{fermion}(a, \beta) \neq 0$ [K.T., '98]

SUSY breaking mass β/R, then,

$$\frac{\beta}{R} > \frac{a}{R} (\sim m_W) \Longrightarrow \beta > a$$

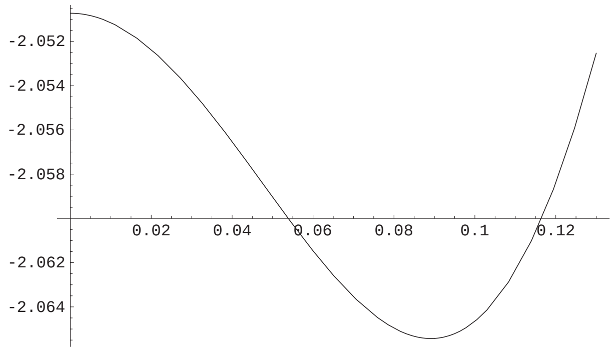
$\star$ 5D vector multiplet, $(A_{\hat{\mu}}, \Sigma, \lambda_D)$;
$\langle \Sigma \rangle$; order parameter for the gauge symmetry breaking, $V_{eff}(\langle A_y \rangle, \langle \Sigma \rangle)$
$\langle \Sigma \rangle = 0$ in many cases   [Haba, K.T, Yamashita,'04]

Introduce $(N_{adj}^{(+)}, N_{adj}^{(-)}, N_{fd}^{(+)}, N_{fd}^{(-)})$, $\qquad (\pm) \rightarrow \eta = \eta_0\eta_1 = \pm 1$;

$$\left(\phi(x,-y) = \eta_0 \ P_0\phi(x,y)P_0^\dagger, \quad \phi(x,\pi R+y) = \eta_1 \ P_1\phi(x,\pi R-y)P_1^\dagger\right)$$

$$\begin{aligned}
V_{eff}(a) &\equiv 2C\bar{V}_{eff}(a) \qquad C = 3/(64\pi^7R^5) \\
&= 2C\sum_{n=1}^{\infty}\frac{1}{n^5}(1-\cos(2\pi n\beta)) \\
&\times \Bigg[(N_{adj}^{(+)}-1)\Big(\cos[2\pi na]+2\cos[\pi na]\Big) + N_{fd}^{(+)}\cos[\pi na] \\
&+ N_{adj}^{(-)}\Big(\cos[2\pi n(a-\frac{1}{2})]+2\cos[\pi n(a-1)]\Big) \\
&+ N_{fd}^{(-)}\cos[\pi n(a-1)]\Bigg],
\end{aligned}$$

V_{eff}



a

$$(N_{adj}^{(+)}, N_{adj}^{(-)}, N_{fd}^{(+)}, N_{fd}^{(-)}) = (2,2,0,2),\ \beta = 0.1$$

$$\Rightarrow a_0 = 0.0891$$

"extra" matter, $N_{adj}^{(\pm)}, N_{fd}^{(-)}$
for $SU(2) \times U(1) \to U(1)$ with $a_0 << 1$

The second derivative of V_{eff} at $a = a_0$
$\Longrightarrow$ the Higgs mass (tends to be light; the Coleman-Weinberg mechanism)

$$\frac{m_H}{g_4^2} \simeq \frac{\sqrt{3}}{4\pi^3}\left(\frac{\partial^2 \bar{V}_{eff}}{\partial a^2}\right)^{\frac{1}{2}}\Bigg|_{a_0}\left(\frac{v}{a_0}\right) = \begin{cases} 95 & \text{(GeV) for} \quad a_0 = 0.0891, \quad \beta = 0.10, \\ 117 & \text{(GeV) for} \quad a_0 = 0.0574, \quad \beta = 0.13, \\ 130 & \text{(GeV) for} \quad a_0 = 0.0379, \quad \beta = 0.14. \end{cases}$$

[N.B.] $\frac{1}{R} \simeq g_4 \times (2.8 \sim 6.5)$ (TeV), $m_{\eta=-1}^{(0)} \simeq 0.5/R,$ $m_{\eta=1}^{(0)} \simeq a_0/R \sim m_W$

- $$\sum_{n=1}^{\infty} \frac{1}{n^5}[1-\cos(2\pi n\beta)]\cos(\pi n a) \qquad (\eta = +1 \text{ type})$$

$$\overset{a<<\beta}{\sim} \quad -\beta^2\pi^4\left(\frac{3}{2}-\ln(2\pi\beta)\right)a^2+\pi^4\left(\frac{25}{288}-\frac{1}{24}\ln(\frac{a}{2\beta})\right)a^4+\cdots,$$

- $$\sum_{n=1}^{\infty} \frac{1}{n^5}[1-\cos(2\pi n\beta)]\cos[\pi n(a-1)] \qquad (\eta = -1 \text{ type})$$

$$\overset{a<<\beta}{\sim} \quad +\beta^2\pi^4(\ln 2)a^2-\pi^4\left(\frac{\pi^2\beta^2}{48}\right)a^4+\cdots$$

(Higher representations, $\mathbf{6}, \mathbf{8}, \mathbf{10}$ under $SU(3)$, $\cos(\pi n a) \to \cos(m\pi n a), m \in$ integer)

$\star$ small VEV, $a_0 << 1$; Almost cancellation of the quadratic terms,

$$a^2 \quad \cdots \quad [6(N_{adj}^{(+)}-1)+N_{fd}^{(+)}]\left(-\frac{3}{2}+\ln(2\pi\beta)\right)+[6N_{adj}^{(-)}+N_{fd}^{(-)}]\ln 2 \sim 0.$$

$$\beta_c \simeq 0.14866\cdots \quad (0.141533\cdots)$$

⋆ Larger β and smaller VEV $a_0 \Rightarrow$ large values for $-\ln(\frac{a_0}{\beta}) \Rightarrow$ large m_H

$$\frac{m_H}{g_4^2} \simeq v\,\frac{\sqrt{3}}{4\pi}\sqrt{-\frac{4}{24}\left(18(N_{adj}^{(+)}-1)+N_{fd}^{(+)}\right)\ln\left(\frac{a_0^2}{\beta^2}\right)+\text{const.}}$$

[Haba, K.T, Yamashita,'04]

⋆ The adjoint matter is important for larger m_H (overcome the loop factor)

Enhancement by the bulk field with the higher representation and with bulk mass

[C.Scrucca, M.Serone and L.Silvestrini,'03]

⋆ Smaller a_0 by fine tuning, $\beta \rightarrow \beta_c = 0.14866\cdots \Longrightarrow$ larger m_H

$[e.g.]$

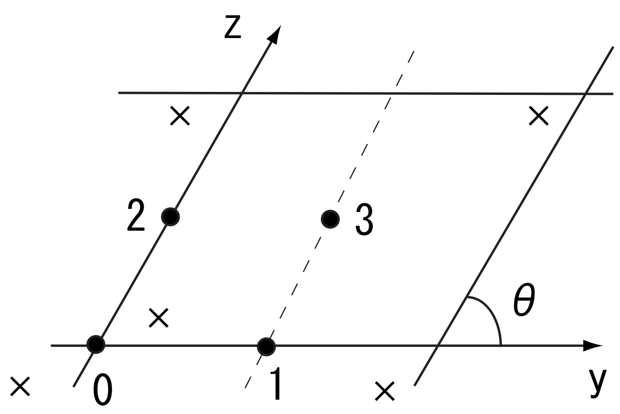
$\beta \simeq 0.14865,\quad a_0 \simeq 0.000872,\quad m_H/g_4^2 \simeq 208$ (GeV), $\quad 1/R \simeq 2.8\times 10^5 \times g_4$ (GeV)

$\beta \simeq 0.148655,\quad a_0 \simeq 0.000588,\quad m_H/g_4^2 \simeq 234$ (GeV), $\quad 1/R \simeq 4.2\times 10^5 \times g_4$ (GeV)

$\beta/a_0 \sim O(10^2)$

♠ Two Higgs doublets, $M^4 \times T^2/Z_2$

Two extra dimensions $\Longrightarrow A_I \equiv (A_y, A_z)$



$\vec{y} \equiv (y^1, y^2) \equiv (y, z)$, $\vec{l}_1 = (2\pi R_1, 0),\ \vec{l}_2 = (0, 2\pi R_2)$

Four fixed points:

$\vec{z}_0 = \vec{0},\ \vec{z}_{1,2} = \vec{l}_{1,2}/2,\ \vec{z}_3 = (\vec{l}_1 + \vec{l}_2)/2,$

Boundary conditions;

$T^2:\ y^I \sim y^I + 2\pi R_I$

$Z_2:\ y^I \sim -y^I$

$$g_{ij} = \begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix}$$

$$\begin{aligned} A_M(x, \vec{y} + \vec{l}_a) &= U_a A_M(x, \vec{y}) U_a^\dagger, \quad (a = 1, 2) \\ \begin{pmatrix} A_\mu \\ A_{y^I} \end{pmatrix}(x, \vec{z}_i - \vec{y}) &= P_i \begin{pmatrix} A_\mu \\ -A_{y^I} \end{pmatrix}(x, \vec{z}_i + \vec{y}) P_i^\dagger, \end{aligned}$$

One can show that $[U_1, U_2] = 0,\ P_i^2 = 1,\ U_a = P_a P_0,\ P_3 = P_1 P_0 P_2 = P_2 P_0 P_1.$

The vacuum structure depends on θ.

$G = SU(3)$ and choose orbifolding boundary conditions as,

$$P_0 = P_1 = P_2 = \text{diag}(1, 1, -1)$$

The zero modes;

$$A_y = \left(\begin{array}{cc|c} & & A_y^{(0)4} - iA_y^{(0)5} \\ & & A_y^{(0)6} - iA_y^{(0)7} \\ \hline \text{c.c.} & \text{c.c.} & \end{array}\right) = \left(\begin{array}{c|c} & \Phi_1 \\ \hline \Phi_1^\dagger & \end{array}\right)$$

$$A_z = \left(\begin{array}{cc|c} & & A_z^{(0)4} - iA_z^{(0)5} \\ & & A_z^{(0)6} - iA_z^{(0)7} \\ \hline \text{c.c.} & \text{c.c.} & \end{array}\right) = \left(\begin{array}{c|c} & \Phi_2 \\ \hline \Phi_2^\dagger & \end{array}\right)$$

The two Higgs doublets = the Wilson line phases, $\langle \Phi_{a=1,2} \rangle$; dynamically determined.

$$SU(3) \to SU(2) \times U(1) \to \ ?$$

The scalar potential at the tree-level,

$$\begin{aligned} V_{tree} &= \mathrm{tr}(F_{yz}^2) \\ &= \frac{g^2}{2}\left((\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \left|\Phi_1^\dagger\Phi_2\right|^2 - (\Phi_1^\dagger\Phi_2)^2 - (\Phi_2^\dagger\Phi_1)^2\right) \end{aligned}$$

(no quadratic terms, less numbers of quartic couplings, (c.f.) MSSM, two Higgs doublet models)

Quartic couplings at the tree level, but

$$[\langle A_y\rangle, \langle A_z\rangle] = 0 \Longrightarrow \langle\Phi_1\rangle \propto \langle\Phi_2\rangle; \quad \text{flat direction.}$$

Then, we parametrize

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\frac{\alpha_1}{gR_1} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\frac{\alpha_2}{gR_2} \end{pmatrix}, \quad \text{where } v_1 \equiv \frac{\alpha_1}{gR_1}, \; v_2 \equiv \frac{\alpha_2}{gR_2}.$$

$$\Phi_a \to \langle \Phi_a \rangle + \tilde{\Phi}_a = \begin{pmatrix} H_a \\ 2^{-1/2}(v_a + \phi_a + i\chi_a) \end{pmatrix} \quad (a = 1, 2)$$

⋆ a charged, a CP-odd Higgs (massive at the tree level)

$$V_{tree} = \frac{g^2}{4}(H_1^\dagger, H_2^\dagger) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} + \frac{g^2}{2}(\chi_1, \chi_2) \begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \cdots \begin{cases} 0 \\ \frac{g}{2}(v_1^2 + v_2^2)^{\frac{1}{2}} = m_W, \end{cases} \qquad \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} \cdots \begin{cases} 0 \\ g(v_1^2 + v_2^2)^{\frac{1}{2}} = 2m_W \end{cases}$$

$$(v_1 = \tfrac{\alpha_1}{gR_1},\ v_2 = \tfrac{\alpha_2}{gR_2})$$

⋆ two CP-even Higgses (flat direction, massive at one-loop level)

$$\mathcal{L}_{eff}^{CP-even} = \frac{1}{2} g^{ij} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{2} \phi_j \mathcal{M}^{jk} \phi_k, \quad \mathcal{M}^{jk} \equiv g^2 R_j R_k \frac{\partial^2 V_{eff}}{\partial \alpha_j \partial \alpha_k}\bigg|_{min}$$

$U(3)_c \times U(3)_w$ (NonSUSY) model [Antoniadis, et.al.,'01]

$$U(3)_c \times U(3)_w = (SU(3)_c \times U(1)_3) \times (\boxed{SU(3)_w} \times U(1)_2)$$

$$\Downarrow$$

$$\boxed{SU(2)_w \times U(1)_1} \text{ by the orbifolding, } P_{0,1,2}$$

$\Longrightarrow SU(3)_c \times SU(2)_w \times U(1)_Y$ (one anomaly free combination among $U(1)_i$'s)

$\Longrightarrow SU(3)_c \times U(1)_{em}$ by the Hosotani mechanism $(\sin^2\theta_w = 1/(4 + \frac{2g_w^2}{3g_c^2}))$

Quarks and Leptons (+ Mirror fermions)

$$L_L^{1,2,3} = \begin{pmatrix} l \\ \tilde{e} \end{pmatrix}_L, L_R^{1,2,3} = \begin{pmatrix} \tilde{l} \\ e \end{pmatrix}_R,$$

$$Q_L^{1,2} = \begin{pmatrix} q \\ \tilde{u} \end{pmatrix}_L, Q_L^3 = \begin{pmatrix} \tilde{q}^c \\ u^c \end{pmatrix}_L, Q_R^{1,2} = \begin{pmatrix} \tilde{q} \\ u \end{pmatrix}_R, Q_R^3 = \begin{pmatrix} q^c \\ \tilde{u}^c \end{pmatrix}_R.$$

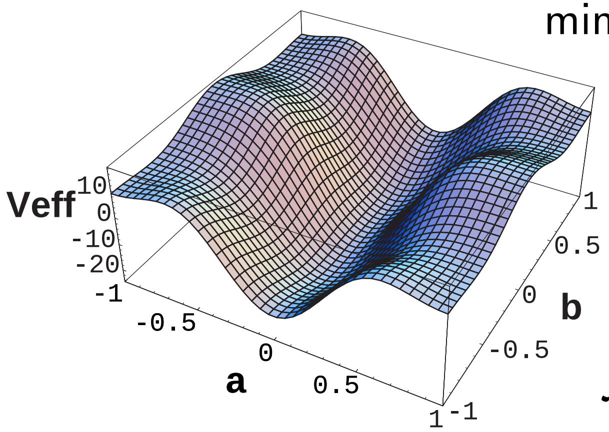
B.C. $\psi^{D=6}(x, \vec{z}_j - \vec{y}) = \eta_j T[p_j](i\Gamma^4\Gamma^5)\psi^{D=6}(x, \vec{z}_j + \vec{y})$

η-parity $(+/-)$; $(\eta_0\eta_1, \eta_0\eta_2) = (1,1), (-1,1), (1,-1), (-1,-1)$

$U(3)_c \times U(3)_w$ (NonSUSY) model [Hosotani, Noda, K.T., '04]

$$(N_{fd}^{(+,+)}, N_{fd}^{(+,-)}, N_{fd}^{(-,+)}, N_{fd}^{(-,-)}, N_{adj}^{(-,+)}) = (3,3,3,3,1)$$

min.; $(\alpha_1,\ \alpha_2) = (0,\ 0.269)$, $(SU(2)_L \times U(1)_Y \to U(1)_{EM})$



$R_1 = R_2 \equiv R \sim (0.6\ \text{TeV})^{-1}$

The CP-even Higgs mass ($g_4^2 = \frac{g_w^2}{2\pi^2 R^2}$).

$$\mathcal{M}^2_{jk};\ \Longrightarrow \begin{pmatrix} 0.871 \\ 3.26 \end{pmatrix} \times (g_4^2/4\pi)^{1/2} \times m_W \ \ (\text{GeV})$$

Another example, $(N_{fd}^{(+,+)}, N_{fd}^{(+,-)}, N_{fd}^{(-,+)}, N_{fd}^{(-,-)}, N_{adj}^{(-,+)}) = (9,9,9,9,0)$

min.; $(\alpha_1,\ \alpha_2) = (0.32, 0.32)$, $(SU(2)_L \times U(1)_Y \to U(1)_{EM})$

$R_1 = R_2 \equiv R \sim (0.35\ \text{TeV})^{-1}$

The CP-even Higgs mass

$$\mathcal{M}^2_{jk};\ \Longrightarrow \begin{pmatrix} 0.799 \\ 1.174 \end{pmatrix} \times (g_4^2/4\pi)^{1/2} \times m_W \ \ (\text{GeV})$$

$\star$ The location of the absolute minimum depends on θ;

$$(\alpha_1, \alpha_2) = \begin{cases} (\pm 0.013, \pm 0.224) & \text{for} \quad \cos\theta = 0.1, \\ (0,0) & \text{for} \quad \cos\theta > 0.133. \end{cases}$$

For $\cos\theta = 0.133$, $(\pm 0.0135, \pm 0.158)$, $(0,0)$ are degenerate.

The electroweak symmetry breaking, $SU(2)_L \times U(1)_Y \to U(1)_{em}$ for $\cos\theta \leq 0.133$

with $(N_{fd}^{(+,+)}, N_{fd}^{(+,-)}, N_{fd}^{(-,+)}, N_{fd}^{(-,-)}, N_{adj}^{(-,+)}) = (3,3,3,3,1)$.

$\star$ two CP-even Higgses;

$$\mathcal{M}^2_{jk}; \quad \overset{\sqrt{\text{eigenvalues}}}{\Longrightarrow} \quad \begin{cases} \begin{pmatrix} 0.871 \\ 3.26 \end{pmatrix} \times (g_4^2/4\pi)^{1/2} \times m_W \ \ \text{(GeV)} \cdots \cos\theta = 0, \\ \begin{pmatrix} 0.799 \\ 4.01 \end{pmatrix} \times (g_4^2/4\pi)^{1/2} \times m_W \ \ \text{(GeV)} \cdots \cos\theta = 0.1. \end{cases}$$

♠ Effects of bare mass $(m^2|\phi|^2)$ [K.T.,'03, Haba,K.T,Yamashita,'04]

$$V(\theta) = \frac{3 \times 4N_F}{4\pi^2 L^5} \sum_{n=1}^{\infty} [1 - (1 + nz + \frac{(nz)^2}{3}) \mathrm{e}^{-nz} \cos(2\pi n\beta)] \sum_{i,j} \cos[n(\theta_i - \theta_j)],$$

where $z \equiv mL$; m bare mass

the Boltzmann like factor, $\mathrm{e}^{-nz} \Rightarrow$ { gauge symmetry breaking patterns; effects on the Higgs mass }

[E.G.] For fixed flavour number, $(N_{adj}^{(+)}, N_{adj}^{(-)}, N_{fd}^{(+)}, N_{fd}^{(-)}) = (2, 1, 0, 2)$, $\beta = 0.1$,

$$\begin{aligned}
(z_{adj}^{(+)}, z_{adj}^{(-)}, z_{fd}^{(+)}, z_{fd}^{(-)}) &= (0, 0, 0, 0), \\
m_H/g_4^2 &\simeq 42 \text{ (GeV)} \quad (a_0 = 0.2362) \\
(z_{adj}^{(+)}, z_{adj}^{(-)}, z_{fd}^{(+)}, z_{fd}^{(-)}) &= (0.1, 0.1, 0.0, 1.0), \\
m_H/g_4^2 &\simeq 150 \text{ (GeV)} \quad (a_0 = 0.0097)
\end{aligned}$$

IV. Summary

- The Higgs doublet Φ can be embedded in $A^{(0)}_{y(z)}$ by the orbifolding. Introducing extra matter, the EW symmetry is broken to $U(1)$ by the Hosotani mechanism.
- The Higgs mass (the Coleman-Weinberg mechanism).

$$V = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4, \quad m^2 = V''|_{\phi_0=v} = \lambda\ v^2, \quad \lambda \cdots \text{loop effects}$$

$\star$ a few adjoint matter is important to cancel the loop suppression factor.

$\star$ the small VEV ($a_0 << 1$) and large β(SUSY breaking)

$\Rightarrow$ large ln(a_0/β) $\Rightarrow$larger m_H (fine tuning of $\beta \to \beta_c$)

$\star$ higher dimensional couplings, $(g_4 R)^{n-4}$ $(n \geq 6) \sim$ (a few TeV)$^{-1}$

$\star$ two Higgs doublet model, flat direction in the tree-level potential,
the light CP-even Higgs masses $<< m_W$, the other Higgses $\sim O(m_W)$

$\star$ introducing the bare mass $(z \equiv mL)$ also plays a role to enhance m_H

- the Weinberg angle,

$$\star\ 2\int dy \frac{1}{4} F^a_{\mu 5} F^{a\mu 5} \quad = \quad \left| \left(\partial_\mu + i g_4 A^a_\mu \frac{\tau^a}{2} + i\sqrt{3} g_4 \frac{A^8_\mu}{2} \right) \Phi \right|^2 \Rightarrow \sin^2\theta_w = \frac{3}{4} > 0.22$$

$\star$ The model by Antoniadis *et.al.* $U(3)_c \times U(3)_w \cdots \sin\theta_w^2 = 1/(4 + \frac{2g^2}{3g_s^2})$

[N.B.] $\rho = \frac{m_Z^2}{m_W^2 \cos^2\theta_w} \sim \left(1 - (\Omega_{12}\frac{m_W^2}{m_{GS2}^2} + \Omega_{13}\frac{m_W^2}{m_{GS3}^2}) \right)^{-1} \neq 1$ at tree level

- Fermion mass spectrum *et.al.*.

Realistic model

♠ Fine tuning of β,

$$(N_{adj}^{(+)}, N_{adj}^{(-)}, N_{fnd}^{(+)}, N_{fnd}^{(-)}) = (2, 2, 0, 2) \quad \beta_c = 0.14866 \cdots$$

β	a_0	m_H/g_4^2 (GeV)
0.1486	0.00234917	190.88
0.14865	0.000872472	208.012
0.148655	0.000588083	234.158

$G = SU(3)$

$N_{adj}^{(+)}$	$N_{adj}^{(-)}$	$N_{fnd}^{(+)}$	$N_{fnd}^{(-)}$	β	$z_{adj}^{(+)}$	$z_{adj}^{(-)}$	$z_{fnd}^{(+)}$	$z_{fnd}^{(-)}$	a_0	m_H/g_4^2
2	3	0	4	0.05	0.01	0.01	-	0.045	0.0040	164
2	4	2	6	0.05	0	0	0.05	0.05	0.0037	176
2	4	0	6	0.025	0.025	0.025	-	0.025	0.0066	129
1	1	0	2	0.01	1	1	-	1	0.0196	125
2	1	0	2	0.1	0.1	0.1	-	1	0.0097	150
2	2	0	2	0.14	0	0	-	0	0.0379	130

(GeV)

$G = SU(6)$

$N_{adj}^{(+)}$	$N_{adj}^{(-)}$	$N_{fnd}^{(+)}$	$N_{fnd}^{(-)}$	β	$z_{adj}^{(+)}$	$z_{adj}^{(-)}$	$z_{fnd}^{(+)}$	$z_{fnd}^{(-)}$	a_0	m_H/g_4^2
2	0	0	10	0.1	0.05	-	-	0.05	0.0207	139
2	0	0	6	0.15	0.1	-	-	0.1	0.0268	139
2	0	0	16	0.04	0	-	-	0.03	0.0021	173
2	0	0	4	0.07	0.5	-	-	0.5	0.0366	138
2	0	0	2	0.32	0	-	-	0	0.0594	135

(GeV)

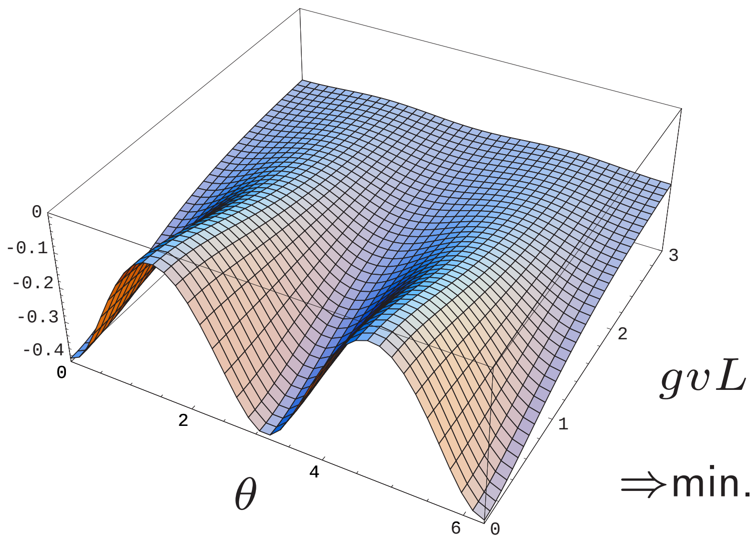
♠ $G = SU(2)$ case, $\langle \Sigma \rangle = \text{diag}(v, -v)$, $M =$ bare mass for λ

$$V_{eff}(v, \theta_i) = -4\left(\frac{3}{4\pi^2}\right) \sum_{n=1}^{\infty} \frac{1}{n^5 L^5} \left[B(v,n) - F(v,n,M)\cos(n\beta)\right] 2\cos[2n\theta].$$

where $B(v,n) \equiv \left(1 + 2gvnL + \frac{4g^2v^2n^2L^2}{3}\right) \mathrm{e}^{-2gvnL}$,

$$F(v,n,M) \equiv \left(1 + nL\sqrt{4g^2v^2 + M^2} + \frac{4g^2v^2 + M^2}{3} n^2 L^2\right) \mathrm{e}^{-\sqrt{M^2+4g^2v^2}nL}.$$

$\beta/2\pi = 0.1, ML = 1.0$



$\Rightarrow$min.$(v, \theta) = (0, 0(\text{mod}\pi))$