Relaxing Constraints on Inflation Models with the Curvaton

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Ref. Moroi, T.T. , Toyoda, [hep-ph/0501007]

1. Introduction

Inflation : a superluminal expansion at the early universe [Guth 1980, Sato 1980]

one of the most promising ideas to solve
the horizon problem and the flatness problem

The potential energy of a scalar field (inflaton) causes inflation.

★ Inflation can also provide the seed of cosmic density perturbations today.

- Quantum fluctuation of the inflaton field can be the origin of today's cosmic fluctuation.

→ Details of fluctuation (such as scale dependence) depend on inflation models (or the form of the potential).

Usually, the scale dependence of the fluctuation is expressed with the initial power spectrum: $P_{\mathcal{R}} \propto k^{n_s - 1}$

★ n_s (the spectral index) depends on models.

→ n_s has information of the model.

Current cosmological observations such as

- Cosmic microwave background (CMB)
- Large scale structure

$\vdots$

can give constraints on n_s. → Information on models.

(The gravity wave is also generated. The size of it can give information of inflation models.)

★ In particular, after WMAP, observations can give severe constraints on inflation models.

However, fluctuation can be provided by some sources other than fluctuation of the inflaton.

From the viewpoint of particle physics,
there can exist scalar fields other than the inflaton.

(late-decaying moduli, Affleck-Dine fields, right-handed sneutrino...)

What if another scalar field acquires primordial fluctuation?

Such a field can also be the origin of today's fluctuation.

↑

Curvaton field (since it can generate the curvature perturbation.)

[Enqvist & Sloth, Lyth & Wands, Moroi & TT 2002]

★Generally, fluctuation of inflaton and curvaton can be both responsible for the today's density fluctuation.

This talk ➡ What is the implication of the curvaton for constraints on models of inflation?

2. Constraints on models of inflation: Standard case

- Primordial fluctuation: the standard case

The quantum fluctuation of the inflaton $\delta\chi \sim \dfrac{H}{2\pi}$

$H \equiv \dfrac{\dot{a}}{a}$:Hubble parameter

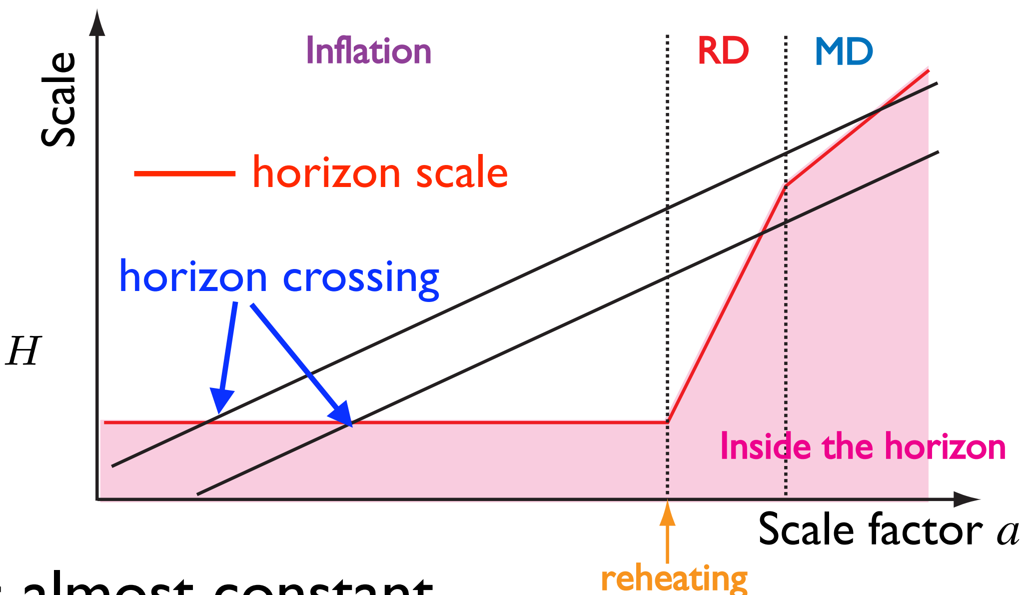
$\dot{\chi} \equiv \dfrac{d\chi}{dt}$

→ The curvature perturbation $\mathcal{R} \sim -\dfrac{H}{\dot{\chi}}\delta\chi$

The power spectrum of $\mathcal{R}$.

(The initial power spectrum)

$$P_{\mathcal{R}}(k) \sim <\mathcal{R}^2> \sim \left(\frac{H}{\dot{\chi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 \Bigg|_{k=aH}$$



During inflation, the Hubble parameter is almost constant

→ Almost scale-invariant spectrum

Generally, it can be written as a power-law form; $P_{\mathcal{R}} \propto k^{n_s - 1}$

What is the discriminators of models of inflation?

- The initial power spectrum ($P_{\mathcal{R}} \propto k^{n_s - 1}$) depends on models.
- During inflation, the gravity waves are also produced.

➡ The size of gravity waves (tensor mode) also depends on models.

$\boxed{\text{The observable quantities}}$

- Scalar spectral index n_s $\left(n_s - 1 \equiv \left.\frac{d \ln P_{\mathcal{R}}}{d \ln k}\right|_{k=aH} \right)$
- Tensor-scalar ratio $r \equiv \frac{P_{\rm grav}}{P_{\mathcal{R}}}$
- (• Overall normalization of $P_{\mathcal{R}}$)

These parameters can be determined from observations.

(such as CMB, large scale structure and so on.)

➡ Constraints on models of inflation.

The observable quantities can be written using the slow-roll approximation.

where $\dot{\chi} \equiv \frac{d\chi}{dt}$

Equation of motion for a scalar field: $$\ddot{\chi} + 3H\dot{\chi} + \frac{dV(\chi)}{d\chi} = 0$$

When it slowly rolls down the potential, $$3H\dot{\chi} + \frac{dV(\chi)}{d\chi} \simeq 0$$

(Slow-roll approximation)

Slow-roll parameters, $$\epsilon \equiv \frac{1}{2M_{\rm pl}^2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv \frac{1}{M_{\rm pl}^2}\frac{V''}{V}$$

where $V'(\chi) \equiv \frac{dV(\chi)}{d\chi}$

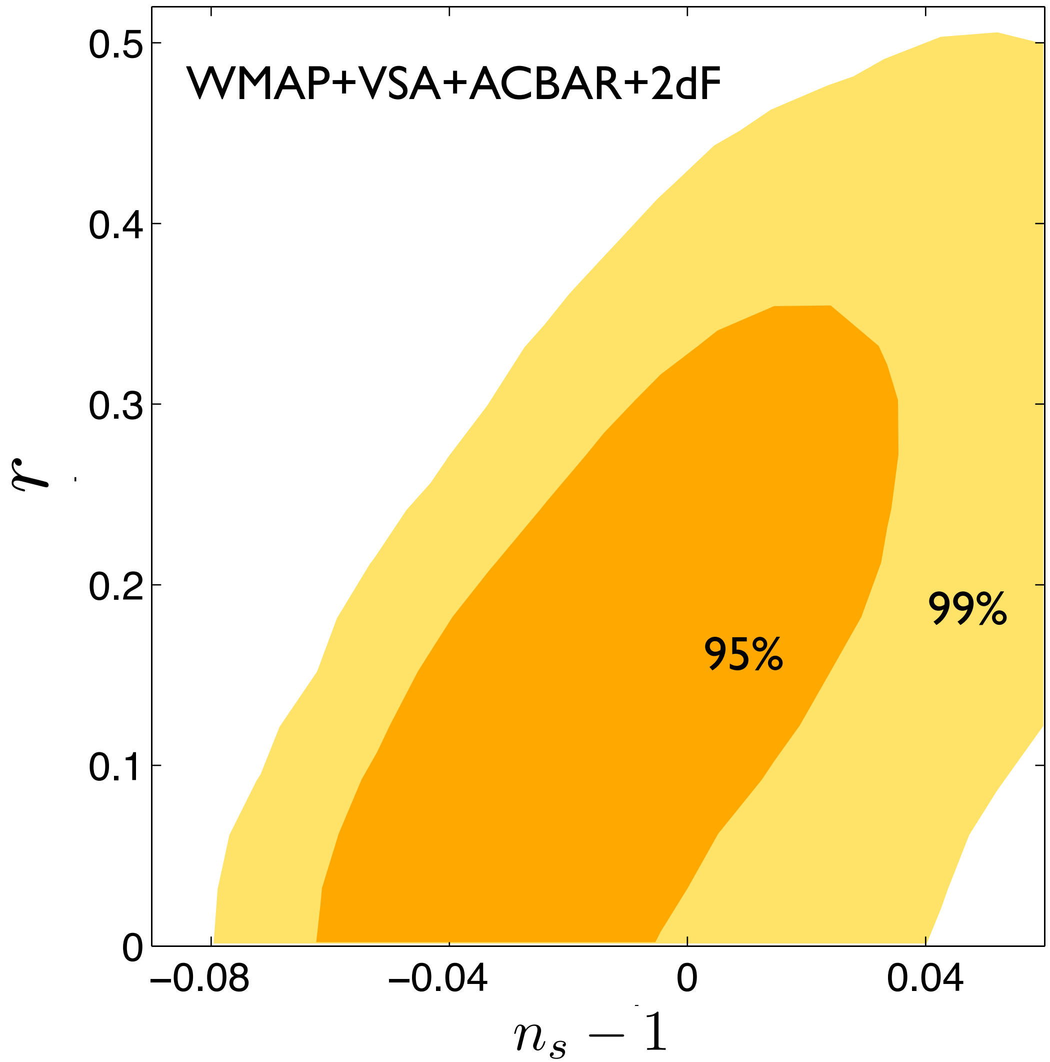
(The slow-roll approximation is valid when $\epsilon, |\eta| \ll 1$.)

<u>The scalar spectral index</u> $$n_s - 1 = 4\eta - 6\epsilon$$

<u>The tensor-scalar ratio</u> $$r = \frac{P_{\rm grav}}{P_{\mathcal{R}}} = 16\epsilon$$

Constraints from current observations [Leach, Liddle 2003]

[Data from CMB (WMAP, VSA, ACBAR) and large scale structure (2dF) are used.]



(The reference scale $k = 0.01\mathrm{Mpc}^{-1}$)

A worked example: the chaotic inflation [Linde 1983]

Potential: $V(\chi) = \lambda M_{\rm pl}^4 \left(\frac{\chi}{M_{\rm pl}} \right)^\alpha$

$$\epsilon = \frac{1}{2} M_{\rm pl}^2 \left(\frac{V'}{V} \right)^2 = \frac{1}{2} \alpha^2 \frac{M_{\rm pl}^2}{\chi_*^2} \qquad \eta = M_{\rm pl}^2 \frac{V''}{V} = \alpha(\alpha - 1) \frac{M_{\rm pl}^2}{\chi_*^2}$$

where $\chi_* = \chi|_{k=aH}$

χ_* can be written with the number of e-foldings during inflation.

The number of e-foldings: $N_e \equiv \ln \frac{a_{\rm end}}{a_*}$

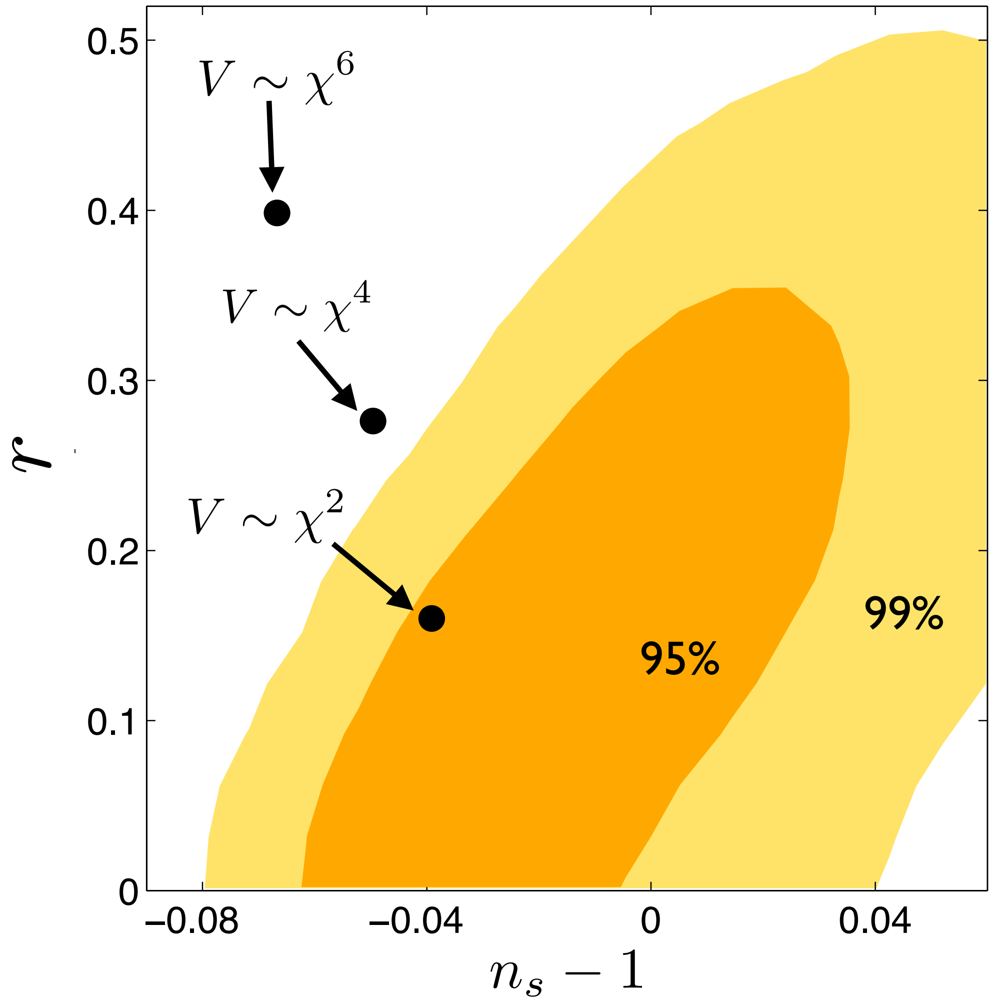
a_* :Scale factor at horizon crossing
$a_{\rm end}$:Scale factor at the end of inflation

Using the slow-roll approximation,

$$N_e = \frac{1}{M_{\rm pl}^2} \int_{\chi_{\rm end}}^{\chi_*} \frac{V_{\rm inf}}{V'_{\rm inf}} d\chi \simeq \frac{1}{2\alpha M_{\rm pl}^2} \chi_*^2.$$

The scalar spectral index: $n_s - 1 = -\frac{\alpha + 2}{2N}$

The tensor-scalar ratio: $r = \frac{4\alpha}{N}$



$n_s - 1$ r

$(-0.04, 0.16)$ For $\alpha = 2$

(Assuming N=50)

$(-0.05, 0.27)$ For $\alpha = 4$

(Assuming N=60)

Almost excluded

However, the curvaton can liberate the model!

3. Curvaton mechanism [Enqvist & Sloth, Lyth & Wands, Moroi & TT 2002]

Curvaton: a light scalar field which is partially or totally responsible for the density fluctuation.

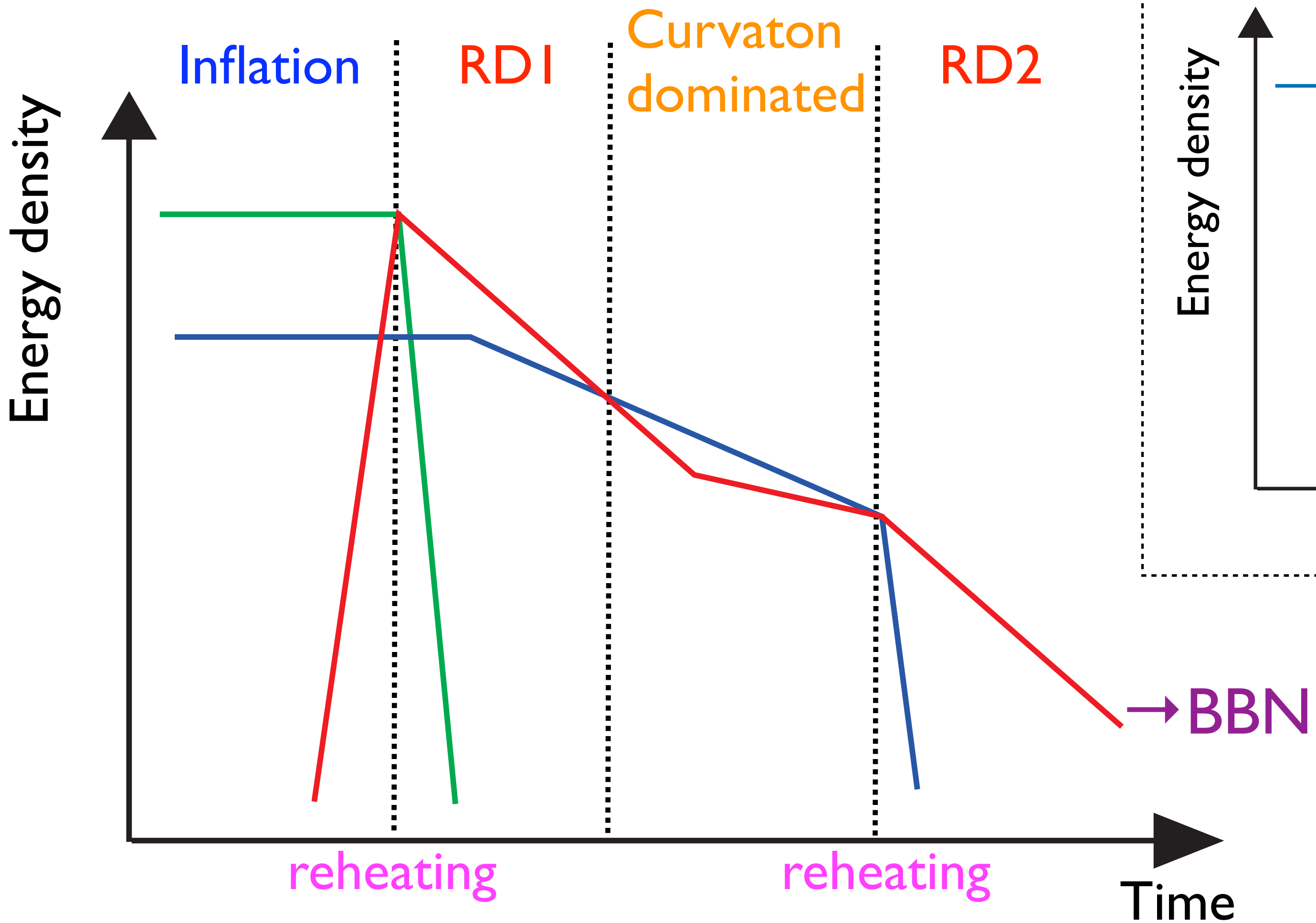
(Usually, quantum fluctuation of the inflaton is assumed to be responsible for that.)

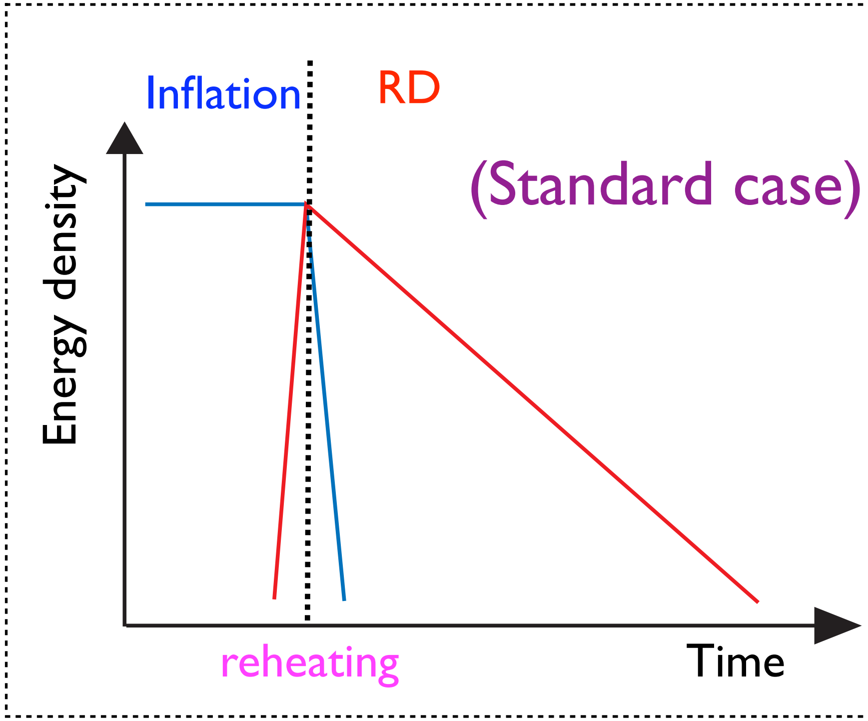
Curvaton scenario

Inflaton ➡ causes the inflation,
not fully responsible for the cosmic fluctuation

Curvaton ➡ is not responsible for the inflation,
is fully or partially responsible for the cosmic fluctuation

Thermal history of the universe with the curvaton



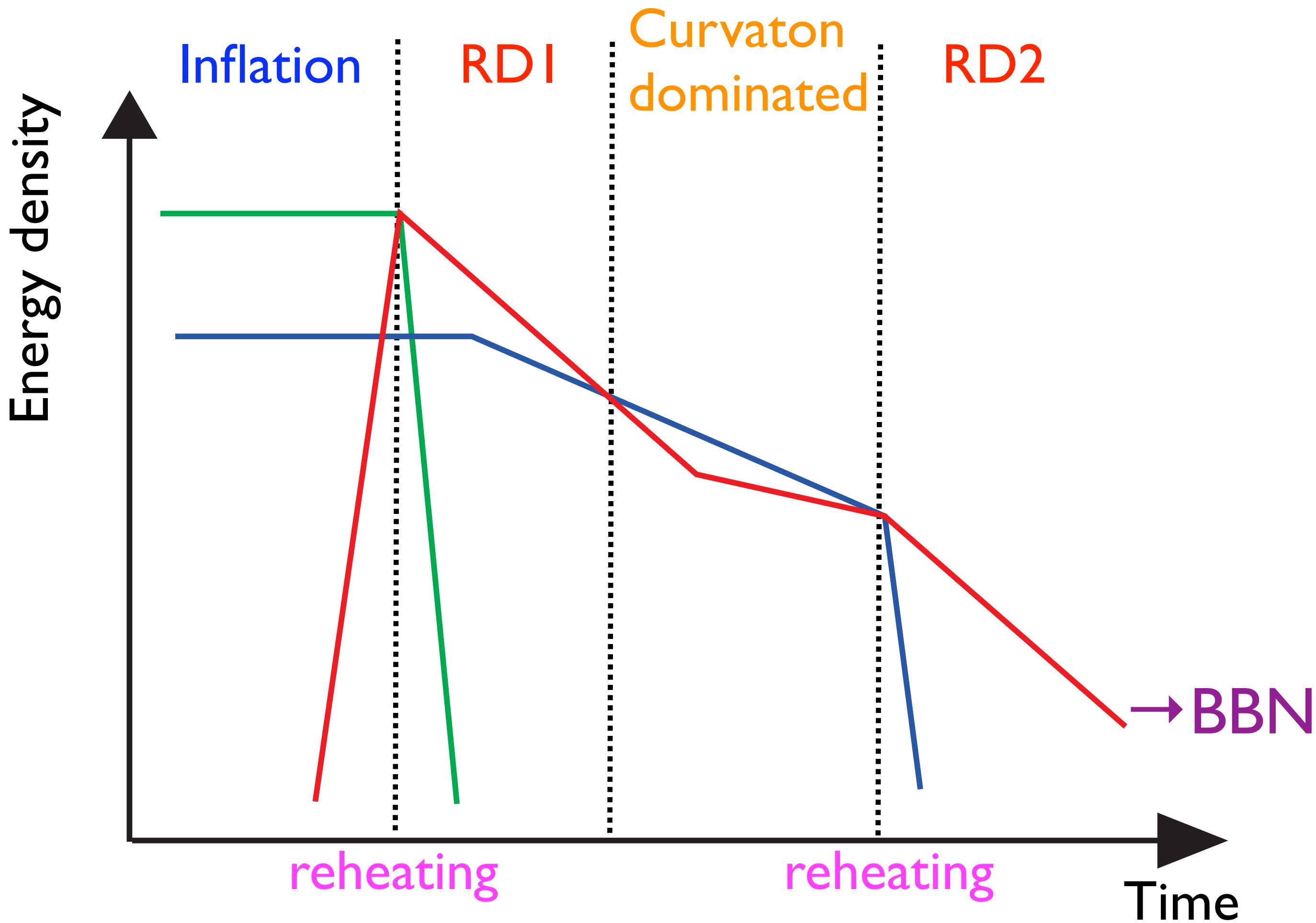


(For simplicity, an oscillating inflaton period is omitted.)

The potential of the curvaton is assumed as $V_{\mathrm{curvaton}} = \frac{1}{2} m_\phi^2 \phi^2$

➡ The oscillating field behaves as matter.

Thermal history of the universe with the curvaton



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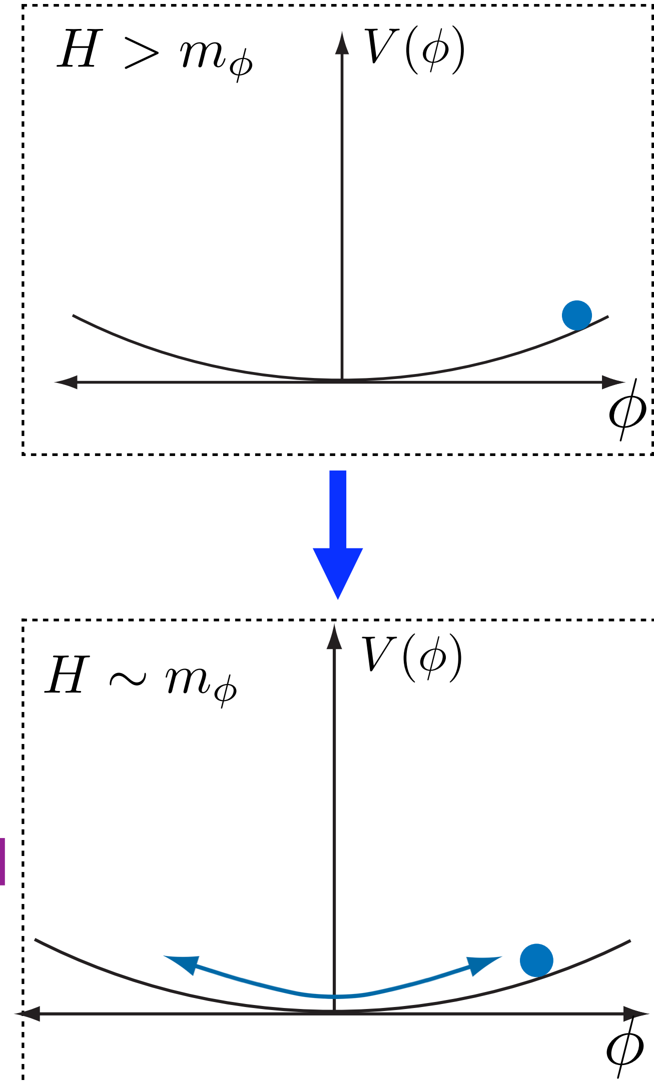


The potential of the curvaton is assumed as $V_{\rm curvaton} = \frac{1}{2} m_\phi^2 \phi^2$

→ The oscillating field behaves as matter.

Density Perturbation

- Fluctuation of the inflaton ➡ Curvature perturbation

$$\left(\delta\chi \sim \frac{H}{2\pi}\right) \qquad \mathcal{R} \sim -\frac{H}{\dot{\chi}}\delta\chi$$

- Fluctuation of the curvaton ➡ No curvature perturbation

$$\left(\delta\phi \sim \frac{H}{2\pi}\right)$$

(The curvaton is subdominant component during inflation.)

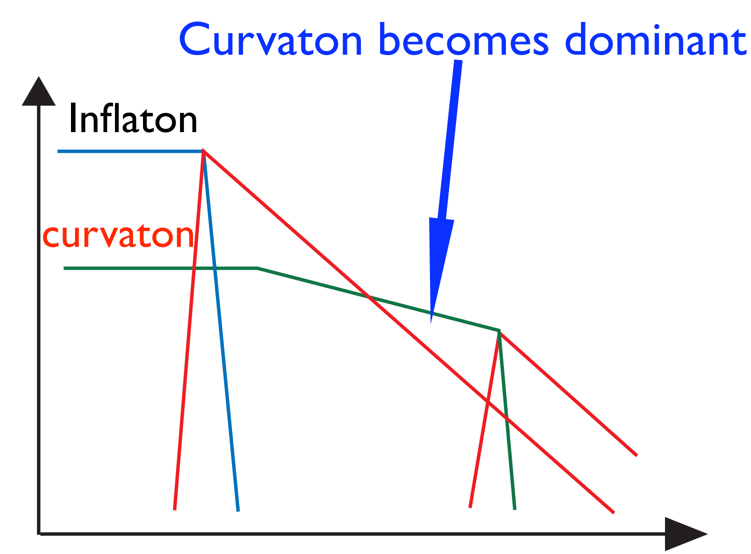
➡ However, isocurvature fluctuation can be generated.

$$S_{\phi\chi} \sim \delta_\chi - \delta_\phi = \frac{2\delta\phi_{\rm init}}{\phi_{\rm init}}$$

where $\delta_i \equiv \dfrac{\delta\rho_i}{\rho_i}$

At some point, the curvaton dominates the universe.

➡ the isocurvature fluc.becomes adiabatic (curvature) fluc.



After the decay of the curvaton, the curvature fluctuation becomes

[Langlois, Vernizzi, 2004]

$$\mathcal{R}_{\rm RD2} = \underbrace{\frac{1}{M_{\rm pl}^2}\frac{V_{\rm inf}}{V'_{\rm inf}}\delta\chi_{\rm init}}_{\text{From inflaton}} \underbrace{\boxed{+\frac{3}{2}f(X)\frac{\delta\phi_{\rm init}}{M_{\rm pl}}}}_{\text{From curvaton}}$$

where

$X = \phi_{\rm init}/M_{\rm pl}$

From inflaton

From curvaton

$f(X)$: represents the size of contribution from the curvaton

The power spectrum (scalar mode) ➡ $P_{\mathcal{R}} = \frac{9}{4}\left[1+\tilde{f}^2(X)\epsilon\right]\frac{V_{\rm inf}}{54\pi^2 M_{\rm pl}^4\epsilon}$

where $\tilde{f} = (3/\sqrt{2})f$

The tensor mode spectrum is not modified. ➡ $P_{\rm grav} = \frac{2}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$

The scalar spectral index and tensor-scalar ratio with the curvaton

$$n_s - 1 = -2\epsilon + \frac{2\eta - 4\epsilon}{1+\tilde{f}^2\epsilon}\,, \qquad r = \frac{16\epsilon}{1+\tilde{f}^2\epsilon}$$

(Notice: the standard case $n_s - 1 = 4\eta - 6\epsilon, \quad r = 16\epsilon$)

Contribution from the curvaton

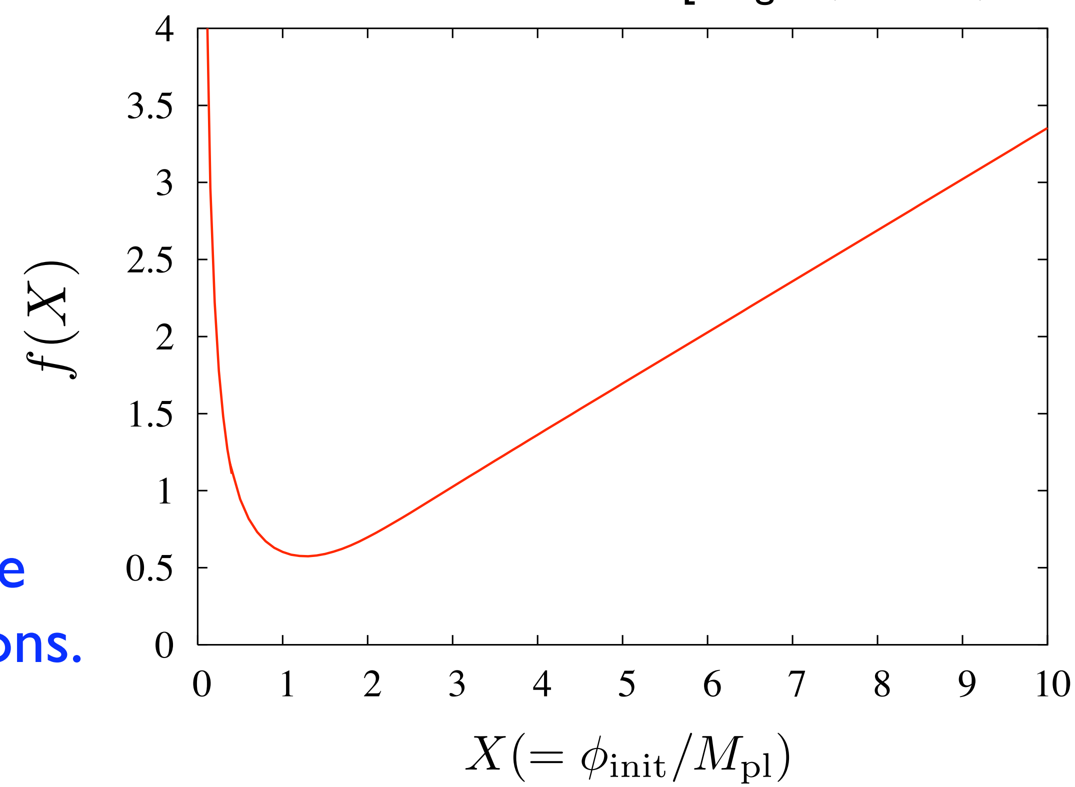
$$\mathcal{R}_{\rm RD2} = \frac{1}{M_{\rm pl}^2}\frac{V_{\rm inf}}{V'_{\rm inf}}\delta\chi_{\rm init} + \frac{3}{2}f(X)\frac{\delta\phi_{\rm init}}{M_{\rm pl}}$$

$f(X)$ can be obtained solving a linear perturbation theory.

For small and large limit,

$$f(X) \simeq \begin{cases} \dfrac{4}{9X} & : \quad \phi_{\rm init} \ll M_{\rm pl} \\[2ex] \dfrac{1}{3}X & : \quad \phi_{\rm init} \gg M_{\rm pl} \end{cases}$$

[Langlois, Vernizzi, 2004]



★ Small and large initial amplitude give large curvaton contributions.

4. Relaxing constraints on models of inflation with curvaton mechanism

[Moroi, TT & Toyoda, hep-ph/0501007]

Fluctuation of the curvaton can be fully or partly responsible for density fluctuation today.

→ Affects constraints on inflation models.

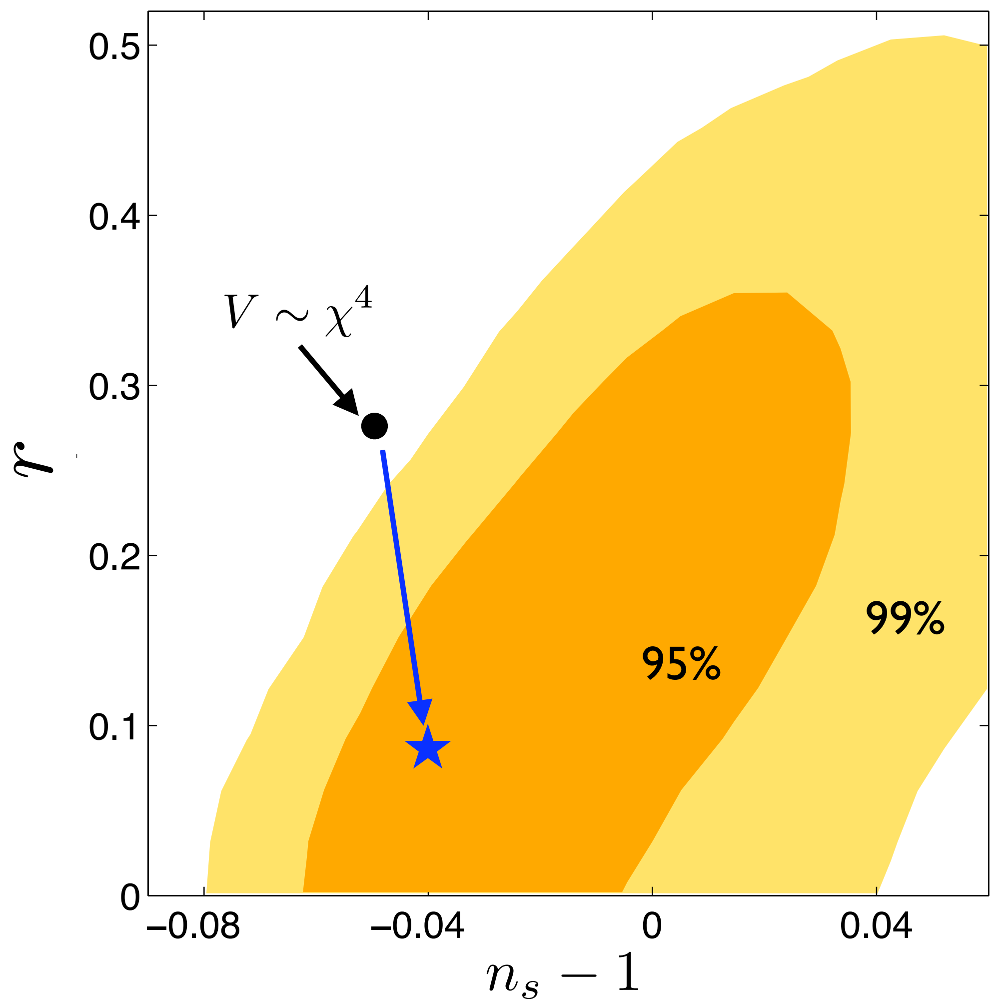
★ We investigated this subject for various inflation models.

- Chaotic inflation with several monomials.
- Natural inflation
- New inflation

Case with the chaotic inflation

Potential: $V_{\rm inf} = \lambda\chi^4$

For $\phi_{\rm init} = 0.1 M_{\rm pl}$ Including curvaton ➡ $n_s - 1 \sim -0.04,\ r \sim 0.08$



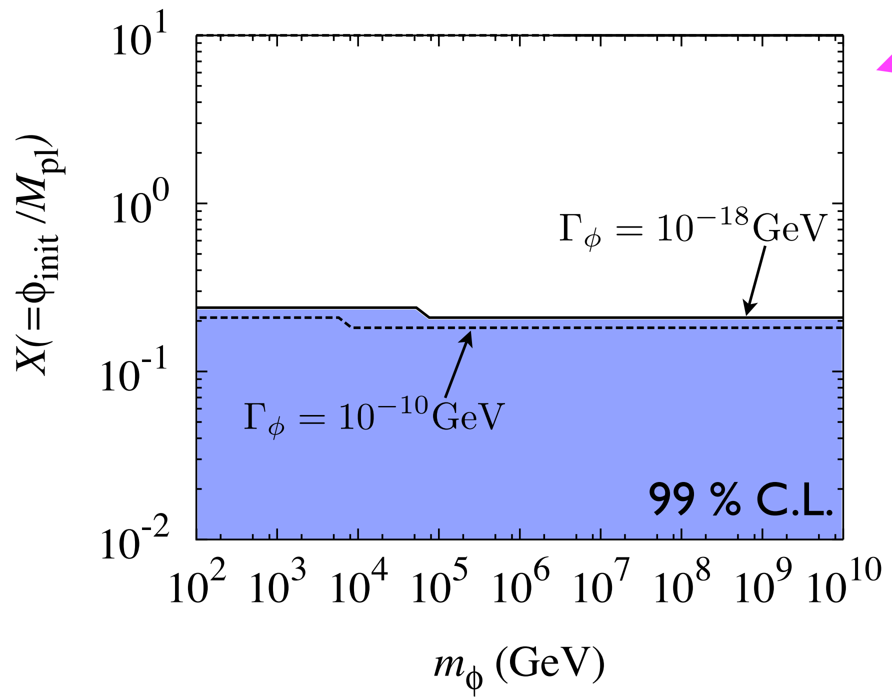
The model becomes viable due to the curvaton!

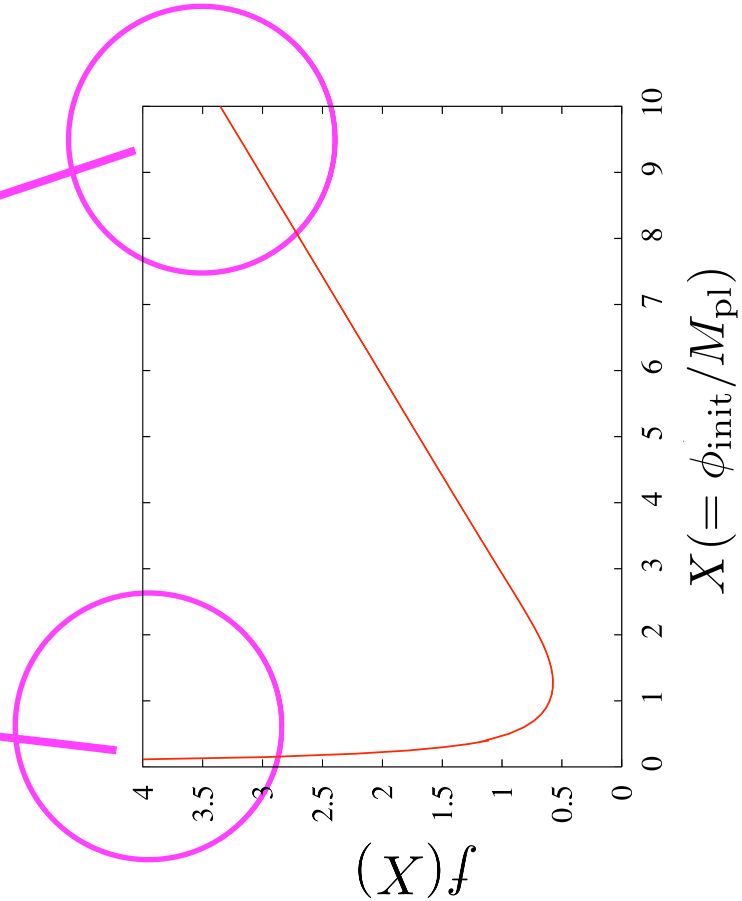
Case with the chaotic inflation

Potential: $V_{\rm inf} = \lambda\chi^4$

★ Model parameter dependence

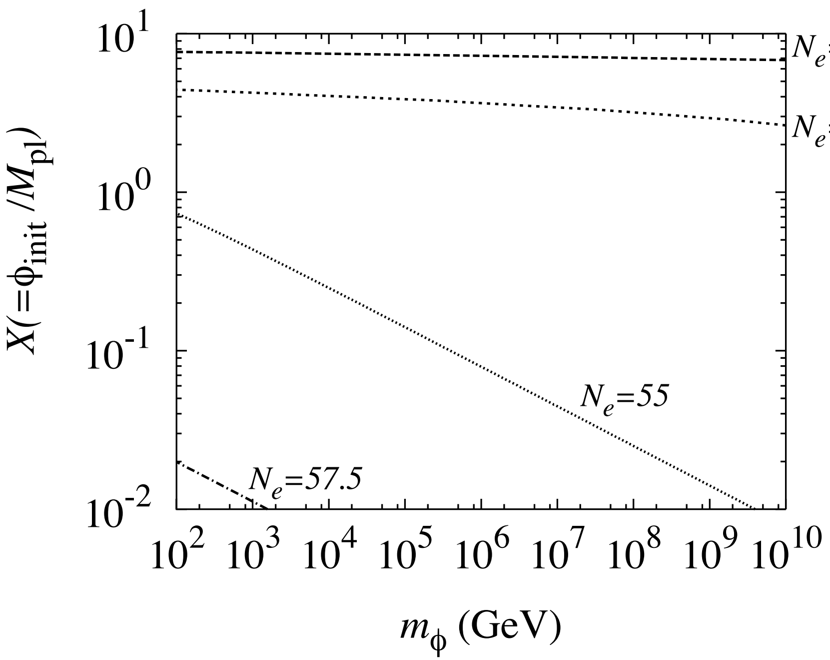
- Initial amplitude of ϕ: $\phi_{\rm init}$
- Mass of ϕ: m_ϕ
- Decay rate of ϕ: Γ_ϕ

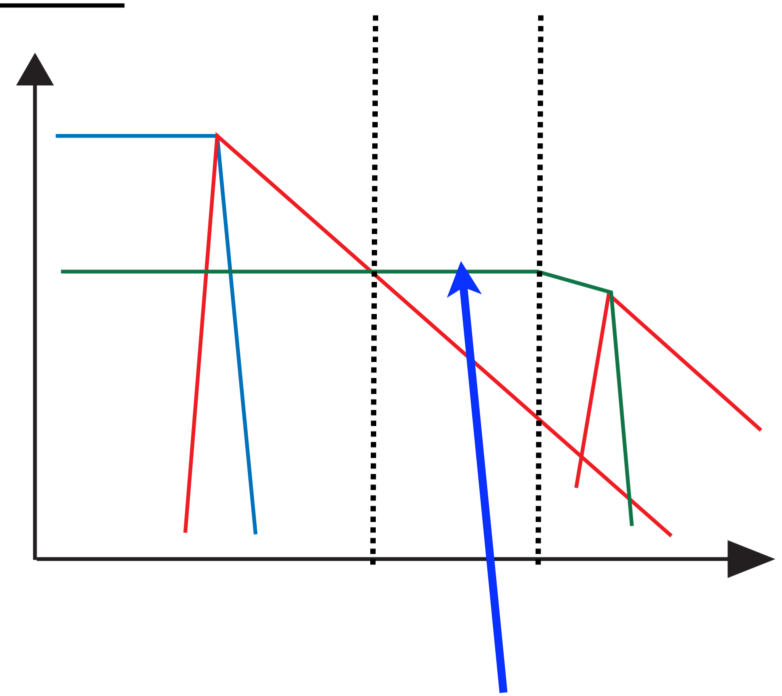




★ The case with large initial amplitude cannot liberate the model, although $f(X)$ is large.

The number of e-folding during inflation





For large initial curvaton amplitudes, the curvaton can cause the 2nd inflation.

The second inflation reduces N_e.

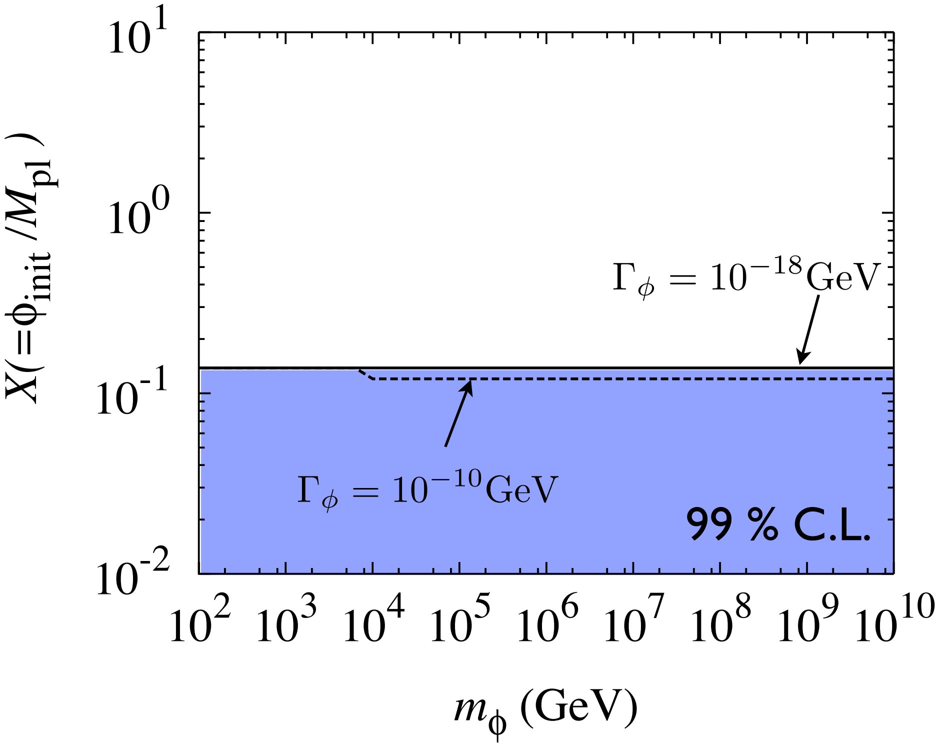
→ modify the spectral index.

Notice: $n_s^{(\rm inf)} - 1 = -\dfrac{\alpha + 2}{2N}$

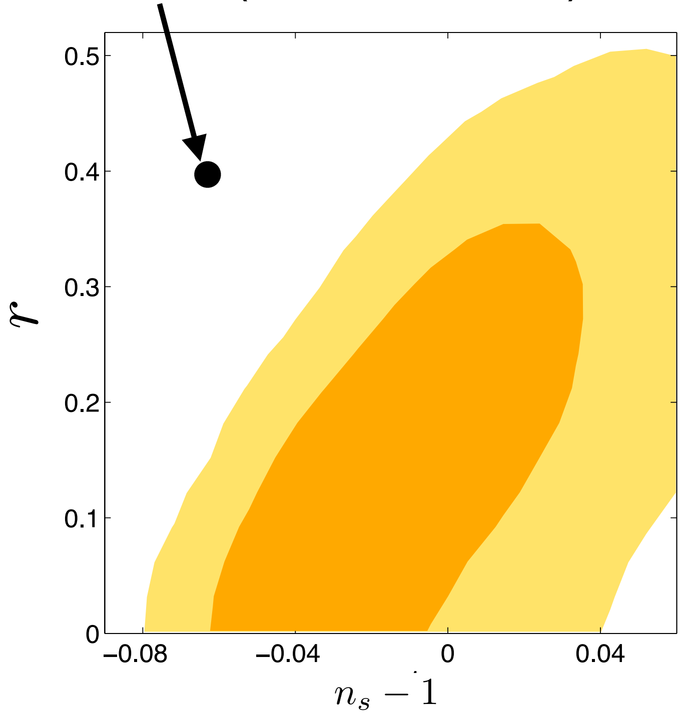
$$N(k)^{(\rm curvaton)} = N(k)^{(\rm standard)} - N_2 + \frac{1}{12}\ln 3 - \frac{1}{6}\ln\frac{m_\phi}{\Gamma_\phi} - \frac{1}{6}\ln\frac{\phi_{\rm init}}{M_{\rm pl}}$$

For other chaotic inflation models

Potential: $V_{\rm inf} = \lambda \frac{\chi^6}{M_{\rm pl}^2}$



$V \sim \chi^6$ (w/o curvaton)



★ The situation is almost the same as the quartic case.

For larger monomial models

Remind that the spectral index is modified as

$$n_s - 1 = -2\epsilon + \frac{2\eta - 4\epsilon}{1 + \tilde{f}^2 \epsilon}$$

Even if $f(X) \to \infty$ case, (Pure curvaton case)

the spectral index becomes $n_s - 1 \sim -2\epsilon = \dfrac{\alpha}{2N_e}$

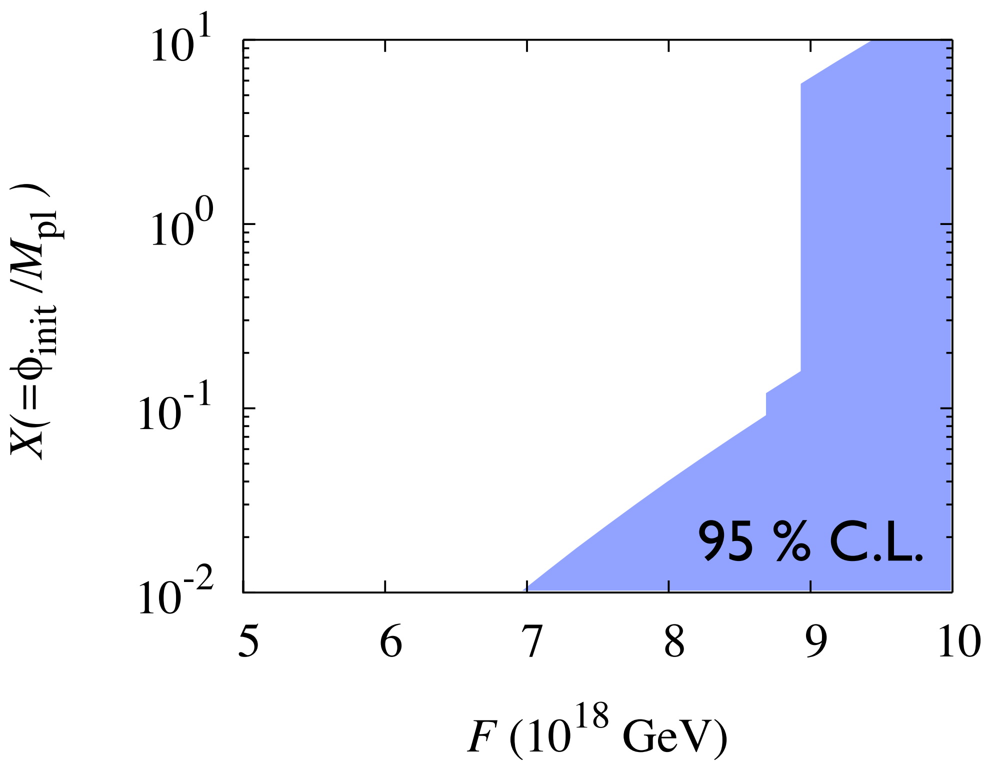
(For $V(\chi) = \lambda M_{\rm pl}^4 \left(\dfrac{\chi}{M_{\rm pl}} \right)^{\alpha}$)

→ Large α cases cannot be made viable even even with the curvaton.

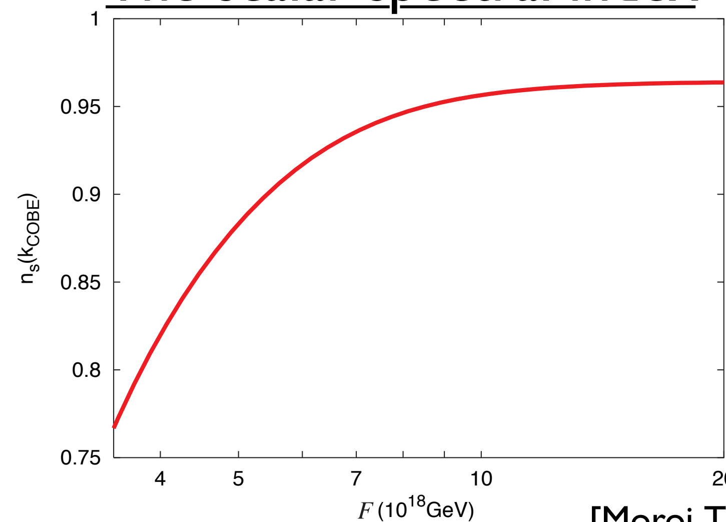
Case with the natural inflation

[Freese, Friemann, Olint, Adams,Bond, Freese, Frieman,Olint,1990]

Potential: $V_{\rm inf} = \Lambda^4 \left[1 - \cos\left(\frac{\chi}{F} \right) \right]$



The scalar spectral index



[Moroi,TT 2000]

$$\epsilon = \frac{1}{2}\left(\frac{M_{\rm pl}}{F}\right)^2 \frac{1}{\tan^2(\chi/2F)}$$

$$\eta = \frac{1}{2}\left(\frac{M_{\rm pl}}{F}\right)^2 \left[\frac{1}{\tan^2(\chi/2F)} - 1\right]$$

Case with the new inflation

[Linde; Albrecht, Steinhardt 1982]

Potential: $V_{\rm inf} = \lambda^2 v^4 \left[1 - 2\left(\frac{\chi}{\sqrt{2}v}\right)^n + \left(\frac{\chi}{\sqrt{2}v}\right)^{2n}\right]$

[Kumekawa, Moroi, Yanagida 1994; Izawa, Yanagida 1997]

The scalar spectral index

$$n_s - 1 = -2\frac{n-1}{n-2}\frac{1}{N_e}$$

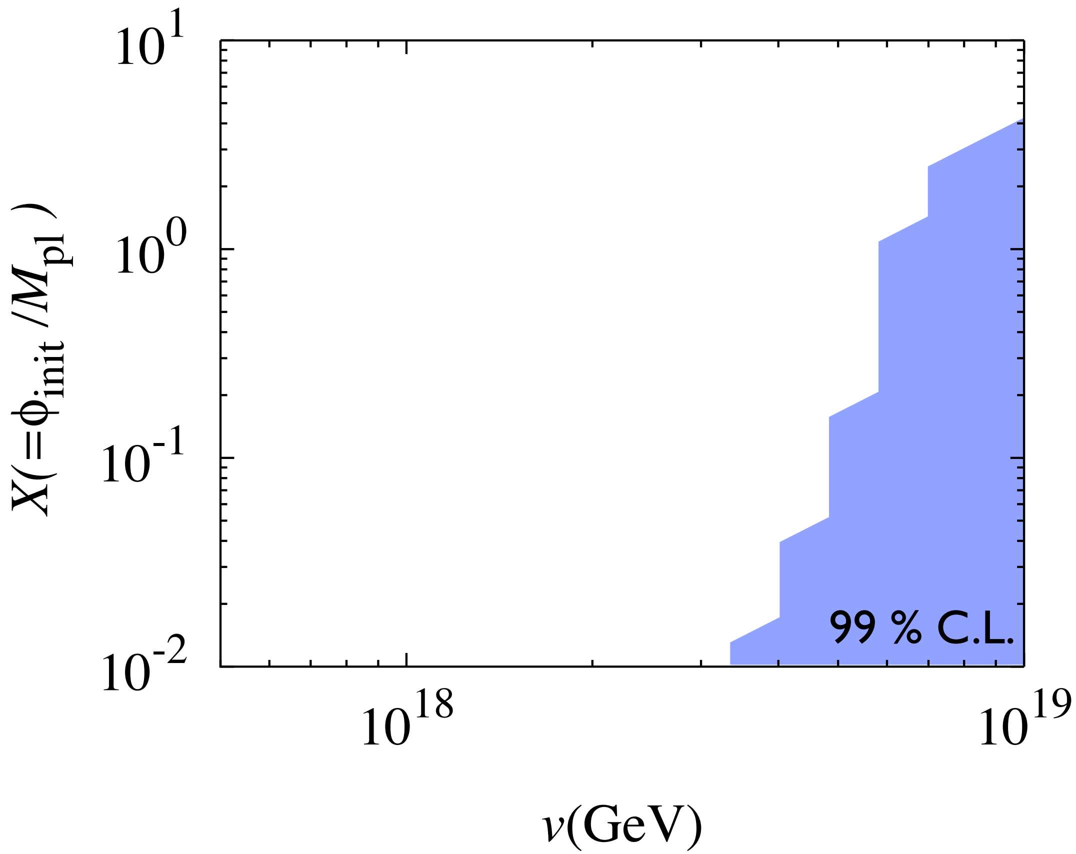
The tensor-scalar ratio

$$\epsilon = n^2 \left(\frac{M_{\rm pl}}{v}\right)^2 \left[\frac{1}{n(n-2)}\left(\frac{v}{M_{\rm pl}}\right)^2 \frac{1}{N_e}\right]^{\frac{2(n-1)}{n-2}}$$

for $n = 3$, $\epsilon = \frac{1}{9N_e^4}\left(\frac{v}{M_{\rm pl}}\right)^6$

r is small for $v \ll M_{\rm pl}$

→ Curvaton cannot liberate the model, when ϵ is very small.



Case with Curvaton

$$n_s - 1 = -2\epsilon + \frac{2\eta - 4\epsilon}{1 + \tilde{f}^2\epsilon}$$

5. Summary

★ Current cosmological observation such as CMB, LSS give severe constraints on inflation models.

★ However, if we introduce the curvaton, a late-decaying scalar condensate, primordial fluctuation can be provided by the curvaton field.

★ As a consequence, constraints on inflation models cannot be applied in such a case.

★ We studied this issue in detail using some concrete inflation models, and showed that how constraints on inflation models can be liberated.

★ We found that in what case inflation models are made viable by the curvaton.