Higgsless models
with and without an extra dimension

— Structure of corrections to electroweak interactions —

February 15, 2005

Windows to New Paradigm in Particle Physics @ Sendai

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- R.S. Chivukula, M. Kurachi and M. T., JHEP **0406**, 004 (2004), [arXiv:hep-ph/0403112].
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. T., Phys. Rev. **D70**, 075008 (2004), [arXiv:hep-ph/0406077]; Phys. Lett. **B603**, 210 (2004), [arXiv:hep-ph/0408262]; Phys. Rev. **D71**, 035007 (2005), [arXiv:hep-ph/0410154].
- R.S. Chivukula, E.H. Simmons, H.-J. He, M. Kurachi and M. T., in preparation.

§.0. Introduction

- **5D Higgsless model** $\Rightarrow$ talk by C.Grojean

  An interesting alternative to the SM Higgs, achieving perturbative unitarity at $\simeq$ 1TeV.

- **Deconstructed (latticized) Higgsless** (my talk)

  Advantages

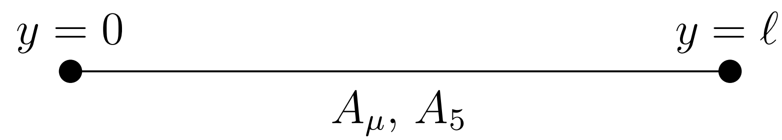
  - Familiar language of spontaneous gauge symmetry breaking in 4D (gauged nonlinear σ model).
  - Easier to understand the physics behind the delay of unitarity violation.
  - Easier to calculate corrections to electroweak interactions.
  - Allows for arbitrary background 5D geometry, spatially dependent gauge couplings, and brane kinetic terms.

c.f., R.Foadi, S.Gopalakrishna, and C.Schmidt, JHEP 03 (2004) 042.

§.1. Boundary condition vs. Deconstruction

Gauge symmetry breaking from boundary conditions

Example: 5D gauge theory with an extra dimension compactified on an **interval**.



Dirichlet or Neumann BC?

1. $A_\mu(x,y)|_{y=0} = 0$ (D), $A_\mu(x,y)|_{y=\ell} = 0$ (D). [DD]
2. $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N). [NN]
3. $A_\mu(x,y)|_{y=0} = 0$ (D), $\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N). [DN]
4. $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $A_\mu(x,y)|_{y=\ell} = 0$ (D). [ND]

Spectrum and 4D gauge symmetry

In addition to a tower of massive spin-1 KK-modes, we have

1. [DD]: massless spin-0 particle. 4D gauge sym. is all broken.
2. [NN]: massless spin-1 particle. unbroken 4D gauge sym.
3. [DN]: no massless particle. 4D gauge sym. is all broken.
4. [ND]: no massless particle. 4D gauge sym. is all broken.

Deconstruction of an interval

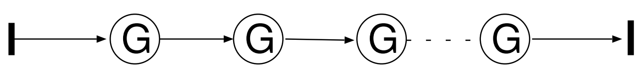
- N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001).
- C. T. Hill, S. Pokorski and J. Wang, Phys. Rev. D **64**, 105005 (2001).

Lattice spacing a.

- $A_\mu^j(x) = A_\mu(x, y = ja)$: Gauge field at the site j.
- $U_j(x) = \exp(i \int_{(j-1)a}^{ja} dy A_5(x, y))$: Link field between sites $j-1$ and j. Nonlinear σ model field.

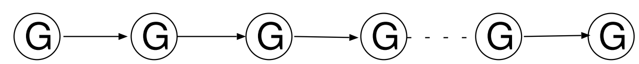
Deconstruction of an interval in "moose" notation:

[DD] case



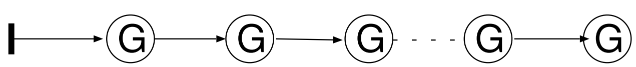
$\#(U_j) = \#(A_\mu^j) + 1.$

[NN] case



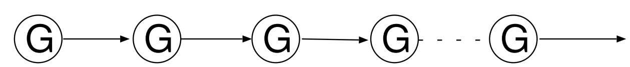
$\#(U_j) = \#(A_\mu^j) - 1.$

[DN] case



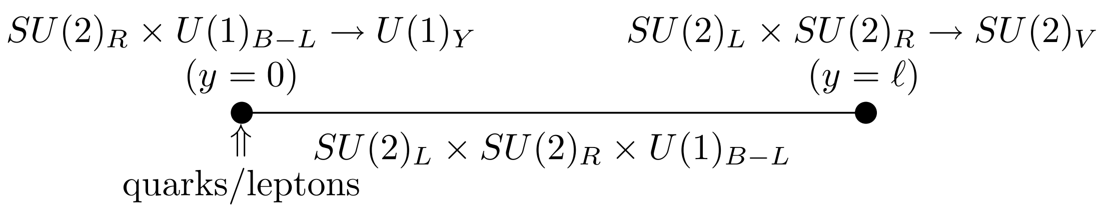
$\#(U_j) = \#(A_\mu^j).$

[ND] case

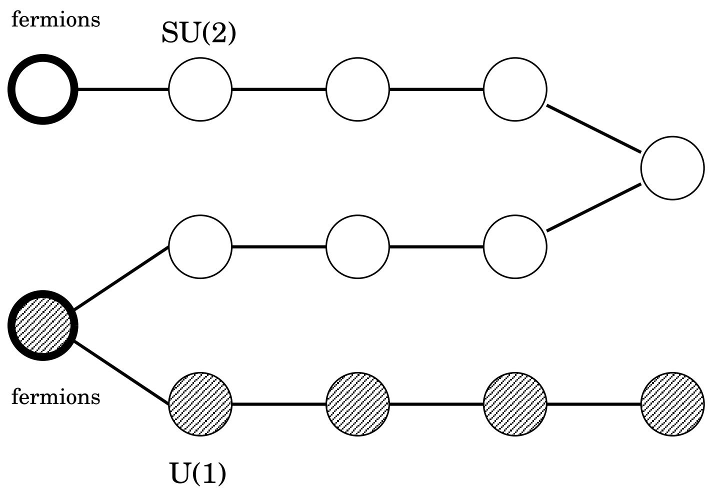


$\#(U_j) = \#(A_\mu^j).$

Deconstruction of 5D Higgsless models



and its deconstruction



5D Higgsless models are described by linear moose.

§.2. Technicolor & BESS model

We note a similarity between technicolor and 5D Higgsless models.

Existence of massive spin-1 resonance:

- Techni-ρ in technicolor ($\bar{Q}Q$ bound state)
- KK-mode of W and Z

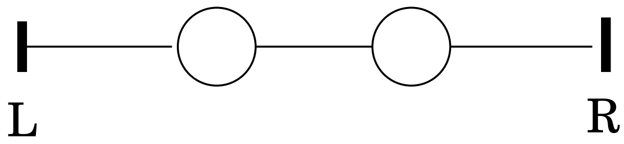
The QCD ρ meson is described as a dynamical gauge boson in the Hidden Local Symmetry (HLS) formalism.

- M.Bando, T.Kugo, S.Uehara, K.Yamawaki, and T.Yanagida, Phys.Rev.Lett. **54** (1985) 1215.
- M.Bando, T.Kugo, K.Yamawaki, Phys.Rept. **164** (1988) 217.

Application of HLS formalism in the electroweak symmetry breaking (a low energy effective theory of technicolor): BESS model

- R.Casalbuoni, S.De Curtis, D.Dominici, and R. Gatto, Phys.Lett.**B155** (1985) 95.

Properties of QCD ρ and a_1 mesons are described by a linear moose:



- ρ meson (vector)
- a_1 meson (axial-vector)

We have two gauge groups between L and R.

We thus obtain

$$\mathcal{FT}\langle T[J_L^\mu(x)J_R^\nu(0)]\rangle \propto \frac{1}{q^4} \qquad \text{for } |q^2| \gg 1\ \text{GeV}^2,$$

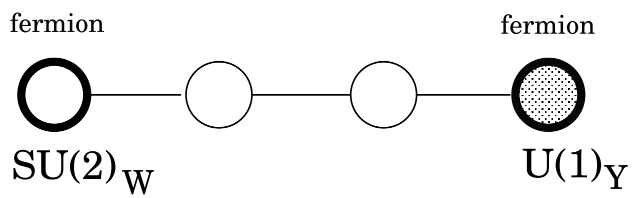
which is consistent with the behavior derived from the Weinberg sum rules,

$$\mathcal{FT}\langle T[J_L^\mu(x)J_R^\nu(0)]\rangle \propto \frac{\langle(\bar{\psi}\psi)^2\rangle}{q^4}.$$

Here we used dimensional analysis with

$$\dim[\mathcal{FT}] = 4, \quad \dim[J^\mu] = 3, \quad \dim[(\bar{\psi}\psi)^2] = 6.$$

The effective theory of one-doublet QCD-like technicolor (BESS model) is described in terms of a linear moose



$SU(2)_W$ and $U(1)_Y$ are weakly gauged.

Techni-ρ and techni-a_1 are described also by a linear moose!

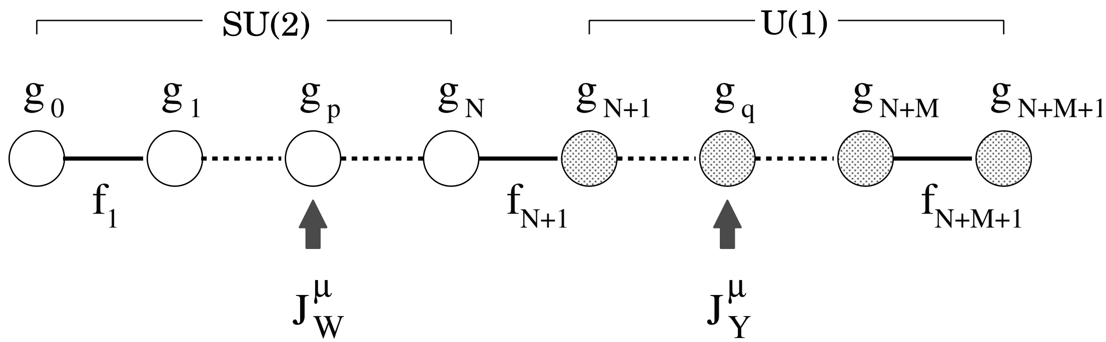
Linear moose provides a general framework to describe Higgsless models with and without an extra dimesion.

The precision tests of the electroweak interactions severely restrict technicolor models.

General structure of corrections to electroweak interactions in linear moose models

§.3. Linear moose models

General linear moose with localized J^{μ}_{W} and J^{μ}_{Y}:



$$\mathcal{L} = \frac{1}{4}\sum_{j=1}^{N+M+1} f_j^2 \mathrm{tr}((D_\mu U_j)^\dagger (D^\mu U_j)) - \sum_{j=0}^{N+M+1} \frac{1}{4g_j^2} F^j_{\mu\nu} F^{j\mu\nu}$$

with
$$D_\mu U_j = \partial_\mu U_j - iA^{j-1}_\mu U_j + iU_j A^j_\mu .$$

Low energy Fermi-constant: (EW symmetry is broken collectively)

$$\sqrt{2}G_F = \frac{1}{v^2_{\mathrm{CC}}} = \sum_{j=p+1}^{N+1} \frac{1}{f_j^2}, \qquad \frac{1}{v^2_{\mathrm{NC}}} = \sum_{j=p+1}^{q} \frac{1}{f_j^2}.$$

Low energy QED coupling:
$$\frac{1}{e^2} = \sum_{j=0}^{N+M+1} \frac{1}{g_j^2}.$$

- Delay of unitarity violation

  For large $N-p$, we are able to achieve the delay of unitarity violation

$$\min(f_j) \gg v \simeq 250 \text{ GeV}, \qquad \sqrt{8\pi}\min(f_j) \gg \sqrt{8\pi}v \simeq 1.2\text{TeV}.$$

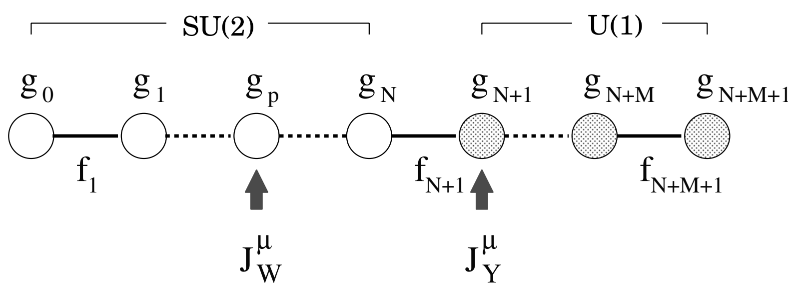
  Note, however,
$$N \lesssim 100$$

  if we assume perturbative gauge coupling : $g_j^2/(4\pi) \lesssim 1$.

  The perturbative unitarity is eventually violated at or below 10 TeV.

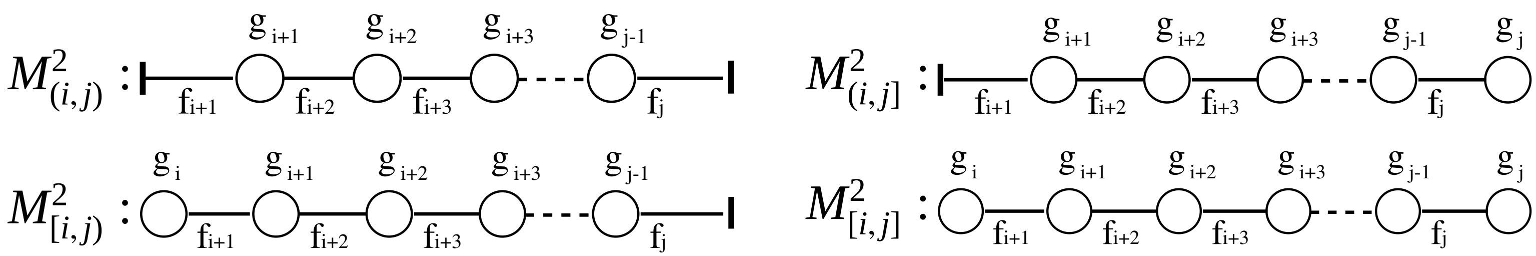
- ρ parameter $$\rho = 1 + \Delta\rho \equiv \frac{v_{CC}^2}{v_{NC}^2} = 1 + v^2 \sum_{j=N+2}^{q} \frac{1}{f_j^2} \geq 1.$$

  In this talk, we concentrate our attention on "Case I" models ($q = N + 1$):



  in which $\Delta\rho = 0$ is satisfied automatically.

“open” and “closed” interval notation:



Gauge boson mass matrices and propagators:

- Neutral current (NC):

$$M^2_{\rm NC} = M^2_{[0,N+M+1]}, \quad [G_{\rm NC}(Q^2)]_{i,j} = g_i g_j([Q^2 + M^2_{\rm NC}]^{-1})_{i,j}$$

- Charged current (CC):

$$M^2_{\rm CC} = M^2_{[0,N+1)}, \quad [G_{\rm CC}(Q^2)]_{i,j} = g_i g_j([Q^2 + M^2_{\rm CC}]^{-1})_{i,j}$$

We also define

$$[G_{\rm CC}(Q^2)]_{WW} = [G_{\rm CC}(Q^2)]_{p,p}, \quad [G_{\rm NC}(Q^2)]_{WW} = [G_{\rm NC}(Q^2)]_{p,p},$$
$$[G_{\rm NC}(Q^2)]_{WY} = [G_{\rm NC}(Q^2)]_{p,q}, \quad [G_{\rm NC}(Q^2)]_{YY} = [G_{\rm NC}(Q^2)]_{q,q}.$$

Four-fermion amplitudes ($f\bar{f} \to f'\bar{f}'$) in the linear moose model

- Neutral current process:

$$
\begin{aligned}
-\mathcal{A}_{\rm NC} &= [G_{\rm NC}(Q^2)]_{WW} I_3 I_3' + [G_{\rm NC}(Q^2)]_{WY} I_3 (Q' - I_3') \\
&\quad + [G_{\rm NC}(Q^2)]_{WY} (Q - I_3) I_3' + [G_{\rm NC}(Q^2)]_{YY} (Q - I_3)(Q' - I_3') \\
&= A(Q^2) I_3 I_3' + B(Q^2)(I_3 Q' + Q I_3') + C(Q^2) Q Q',
\end{aligned}
$$

with

$$
\begin{aligned}
A(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{WW} - 2[G_{\rm NC}(Q^2)]_{WY} + [G_{\rm NC}(Q^2)]_{YY}, \\
B(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{WY} - [G_{\rm NC}(Q^2)]_{YY}, \\
C(Q^2) &\equiv [G_{\rm NC}(Q^2)]_{YY}.
\end{aligned}
$$

- Charged current process:

$$
-\mathcal{A}_{\rm CC} = [G_{\rm CC}(Q^2)]_{WW} (I_+ I_-' + I_- I_+')/2
$$

Deviations of $[G_{\rm NC}(Q^2)]$ and $[G_{\rm CC}(Q^2)]$ from their SM values

$\Downarrow$

Corrections to the electroweak interactions in linear moose models

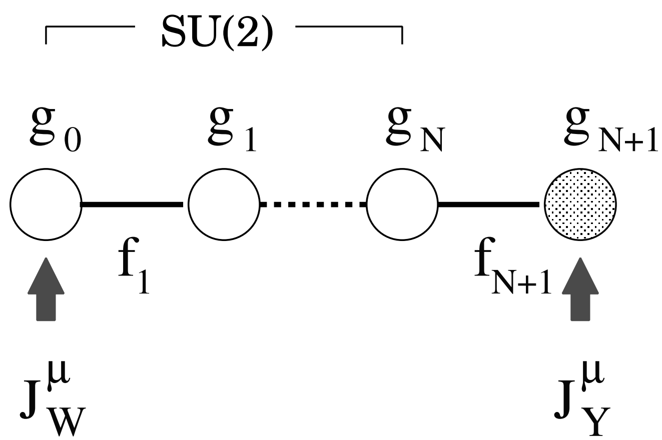
Electroweak correction parameters S, T, $\alpha\delta$: ($\Delta\rho = 0$ is assumed)

$$
\begin{aligned}
-\mathcal{A}_{\rm NC} &= e^2\frac{QQ'}{Q^2} + \frac{(I_3 - s^2Q)(I_3' - s^2Q')}{\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2} - \alpha T\right)} \\
&\quad +4\sqrt{2}G_F\frac{\alpha\delta}{4s^2c^2}I_3I_3' - 4\sqrt{2}G_F\alpha T(Q - I_3)(Q' - I_3'), \\
-\mathcal{A}_{\rm CC} &= \frac{(I_+I_-' + I_-I_+')/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right)Q^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} \\
&\quad +4\sqrt{2}G_F\frac{\alpha\delta}{4s^2c^2}\frac{(I_+I_-' + I_-I_+')}{2}.
\end{aligned}
$$

c.f. R.Barbieri, A.Pomarol, R.Rattazzi, and A. Strumia, hep-ph/0405040
zero-momentum formalism ($\hat{S}$, $\hat{T}$, W, Y)

§.4. Corrections to electroweak interactions

We first consider a special class of models $p = 0$, $M = 0$ in “Case I”:



The BESS model (effective theory of technicolor) is included in this class of models.

Number of free parameters: $2N + 3 = (N + 2) + (N + 1)$.
The model is specified completely once $e^2, G_F, M_Z, M_{Z\hat{k}}, M_{W\hat{n}}$ $(\hat{k}, \hat{n} = 1, 2, \cdots, N)$ are fixed.

Properties of $[G_{\rm NC}(Q^2)]_{WY}$:

- Poles at $Q^2 = 0, -M_Z^2, -M_{Z_1}^2, \cdots, -M_{Z_N}^2$.
- Charge universality of QED determines the pole residue of photon:

$$\lim_{Q^2 \to 0} Q^2 [G_{\rm NC}(Q^2)]_{WY} = e^2.$$

- Generalized Weinberg sum rules:

$$[G_{\rm NC}(Q^2)]_{WY} \propto \frac{1}{Q^{2(N+2)}}, \qquad \text{for } Q^2 \gg M_{Z_N}^2.$$

These conditions are enough to determine the form of $[G_{\rm NC}(Q^2)]_{WY}$ completely:

$$[G_{\rm NC}(Q^2)]_{WY} = \frac{e^2}{Q^2} \frac{M_Z^2}{Q^2 + M_Z^2} \left[\prod_{\hat{k}=1}^{N} \frac{M_{Z_{\hat{k}}}^2}{Q^2 + M_{Z_{\hat{k}}}^2} \right].$$

We rewrite

$$[G_{\rm NC}(Q^2)]_{WY} = \frac{[\xi_\gamma]_{WY}}{Q^2} + \frac{[\xi_Z]_{WY}}{Q^2+M_Z^2} + \sum_{\hat{k}=1}^{N} \frac{[\xi_{Z_{\hat{k}}}]_{WY}}{Q^2+M_{Z_{\hat{k}}}^2},$$

with

$$[\xi_\gamma]_{WY} = e^2, \qquad [\xi_Z]_{WY} = -e^2\left[\prod_{\hat{k}=1}^{N} \frac{M_{Z_{\hat{k}}}^2}{M_{Z_{\hat{k}}}^2 - M_Z^2}\right], \qquad \cdots.$$

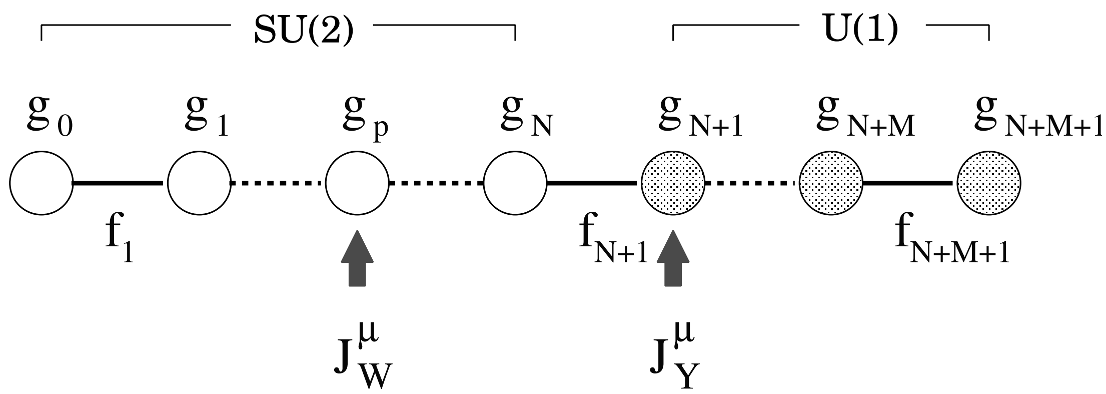
The S parameter can be defined as $[\xi_Z]_{WY} = -e^2\left[1+\frac{\alpha}{4s_Z^2c_Z^2}S\right]$.

We thus obtain manifestly positive expression of S

$$\alpha S = 4s_Z^2c_Z^2 \sum_{\hat{k}=1}^{N} \frac{M_Z^2}{M_{Z_{\hat{k}}}^2} > 0. \qquad (M_{Z_{\hat{k}}}^2 \gg M_Z^2 \text{ is assumed.})$$

We need Z' lighter than a TeV in order to achieve tree-level unitarity at 1TeV. Such a light Z' gives significant effect to the S parameter: $S \gtrsim 1$, and ruled out by the existing precision tests.

We finally consider general "Case I" models:



We find

$$\alpha S - 4c_Z^2 \alpha T + \frac{\alpha\delta}{c_Z^2} = 4s_Z^2 M_W^2 \Sigma_{(p,N+1)} > 0.$$

Tree level unitarity says $\Sigma_{(p,N+1)} > \dfrac{1}{8\pi v^2}$. We thus obtain

$$S - 4c_Z^2 T + \frac{\delta}{c_Z^2} \gtrsim 1,$$

in any unitary theory.

§.5. Which models are viable?

No linear moose model with localized $J^{\mu}_{W,Y}$ can satisfy the precision electroweak tests and the tree level unitarity at 1 TeV simultaneously.

This conclusion is independent of

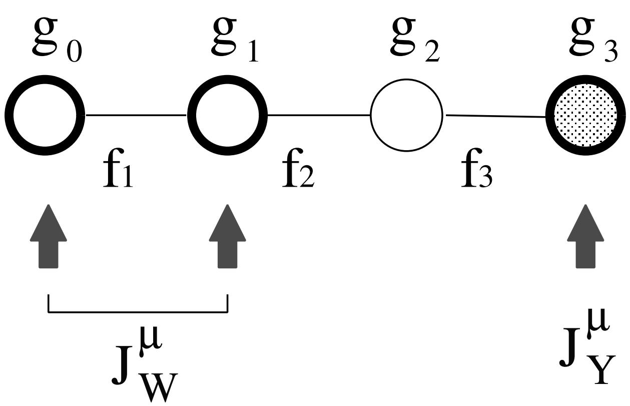
- **background 5D geometry**
- **spatially dependent gauge couplings**
- **brane kinetic terms**
- **etc.**

We thus need "delocalization" of $J_{W,Y}$ to construct a viable model.

Continuum 5D theories with delocalized fermion:

- G.Cacciapaglia, C.Csaki, C.Grojean, and J.Terning, hep-ph/0409126.
- R.Foadi, S.Gopalakrishna, and C.Schmidt, hep-ph/0409266.

Example: a linear moose with delocalized fermion



$$x_0 J_W^\mu A_\mu^0 + x_1 J_W^\mu A_\mu^1,$$

$$x_0 + x_1 = 1$$

- 8 free parameters (g_j, f_j, x_1)
- 4 inputs: $e^2, M_Z, G_F, S = 0$
- 3 ansatzes: $f_2 = f_3, g_1 = g_2 = 4.$

Amount of required delocalization:

f_1	2000GeV	1000GeV	300GeV
x_1	0.48	0.14	0.015

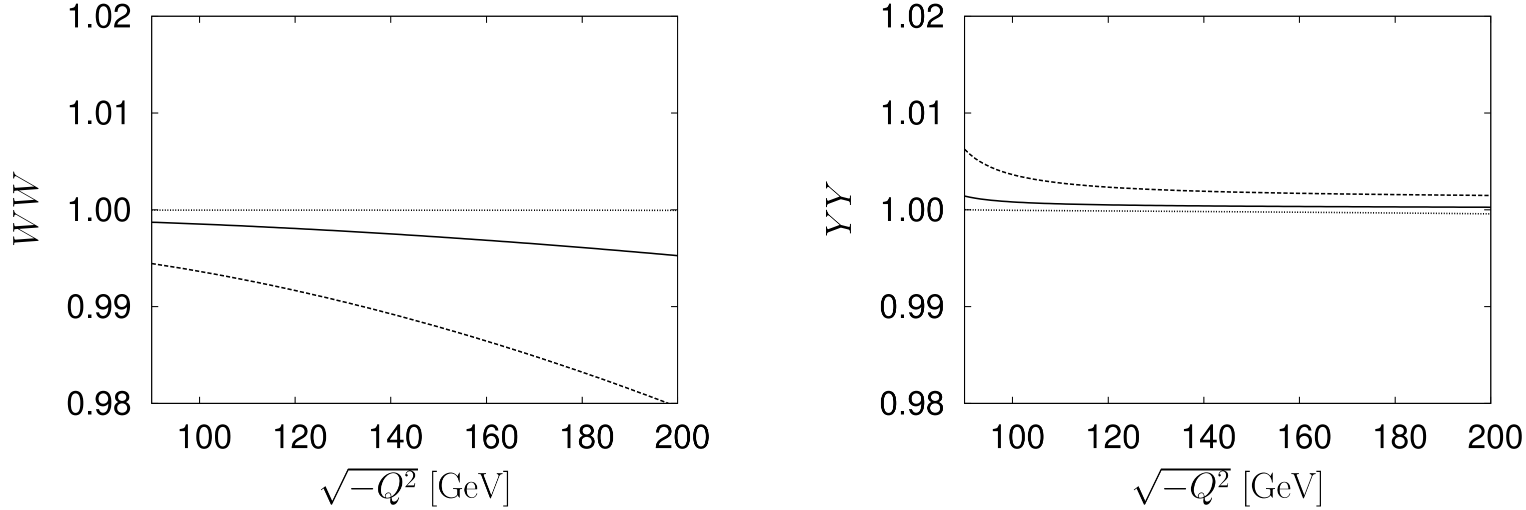
We find

$$T \simeq 0, \qquad U \simeq 0, \qquad \Delta\rho = 0, \qquad \delta \propto x_1.$$

We are able to make the size of electroweak correction small enough consistent with the precision measurements.

- R.S.Chivukula, E.H.Simmons, H.-J.He, M.Kurachi, M.T., in preparation.

$[G_{\rm NC}(Q^2)]/[G_{\rm NC}(Q^2)]^{\rm SM}$ for the LEP energy range



for f_1 = 2000, 1000, 300 GeV.

The 4-site model with delocalized fermion is consistent with precision measurements at LEP for $f_1 \lesssim 1$TeV, still achieving the delay of unitarity violation.

§.6. Summary and Outlooks

- **Higgsless theory is an interesting alternative to the standard model Higgs, achieving tree level unitarity at 1TeV.**
- **Both 4D and 5D Higgsless models are described in terms of linear moose.**
- **We need "delocalization" of fermions in order to construct a viable Higgsless model satisfying various requirements of the precision electroweak tests. (flavor universal extended technicolor in technicolor model.)**
- **KK-modes of W and Z (W', Z' or techni-ρ) exist below a TeV in these models. Collider phenomenology of these particles should be investigated.**