# Non-Perturbative Aspects of Kaluza-Klein Modes in 5-dim. Supersymmetric QCD on $S^1$

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# §. Introduction

## ✡ model

5-dim. $N=1$ supersymmetric gauge theory

with gauge group $SU(2)$

$N_f$ fundamental matters

compactify on $S^1$ (radius $R$)

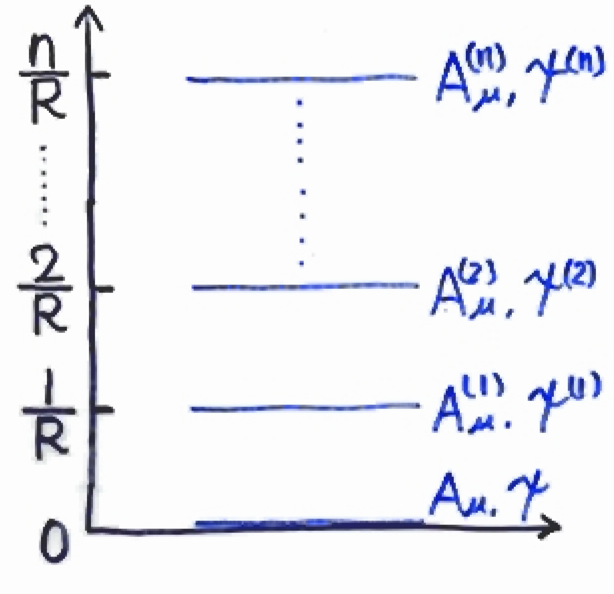
$x^5 \sim x^5 + 2\pi R$

$\phi(x^\mu, x^5) = \sum_{n\in\mathbb{Z}} \phi_n(x^\mu)\, e^{\frac{inx^5}{R}}$

$\int d^5x\, \phi^\dagger \Box \phi \to \sum_{n\in\mathbb{Z}} \int d^4x\, \phi_n^\dagger \left(\Box + \frac{n^2}{R^2}\right) \phi_n$

4-dim. $N=2$ supersymmetric gauge theory

classical spectrum




$\Longrightarrow$ How is it modified by quantum effects?

# §. Seiberg-Witten Theory

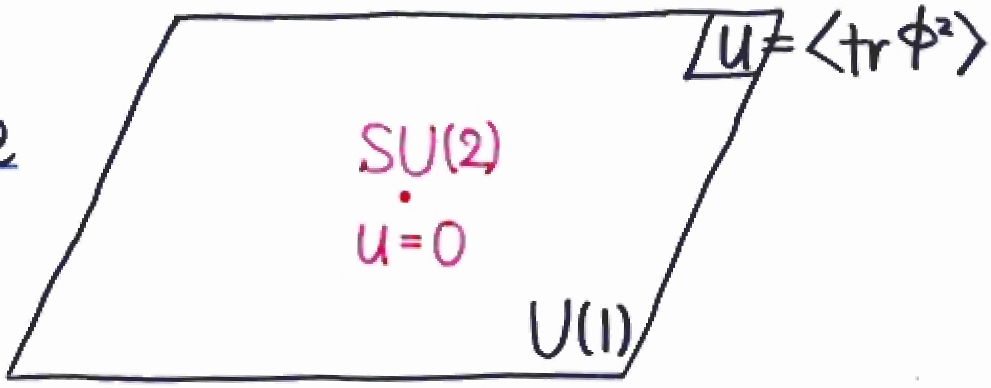
Seiberg-Witten, hep-th/9407087

## ☆ 4-dim. $\mathcal{N}=2$ $SU(2)$ Yang-Mills theory

gauge multiplet $\quad \begin{matrix} & A_\mu & \\ \lambda & & \chi \\ & \phi & \end{matrix}$ $\qquad V(\phi) = [\phi, \phi^\dagger]^2$

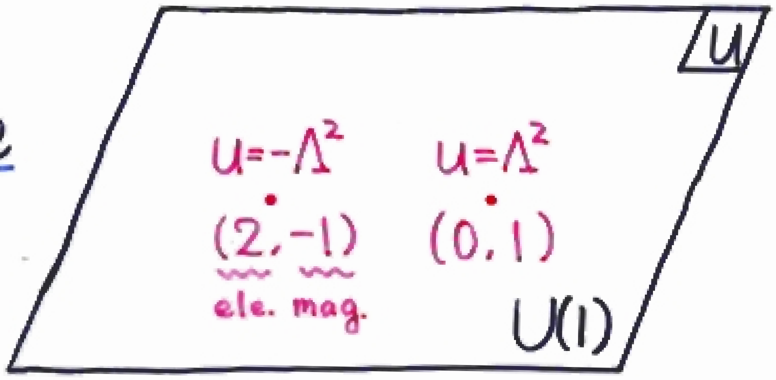
$\Rightarrow \phi = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}$

classical moduli space




$\Downarrow$

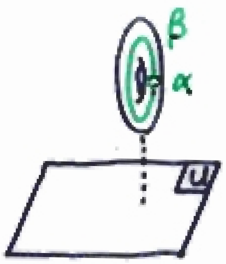
quantum moduli space



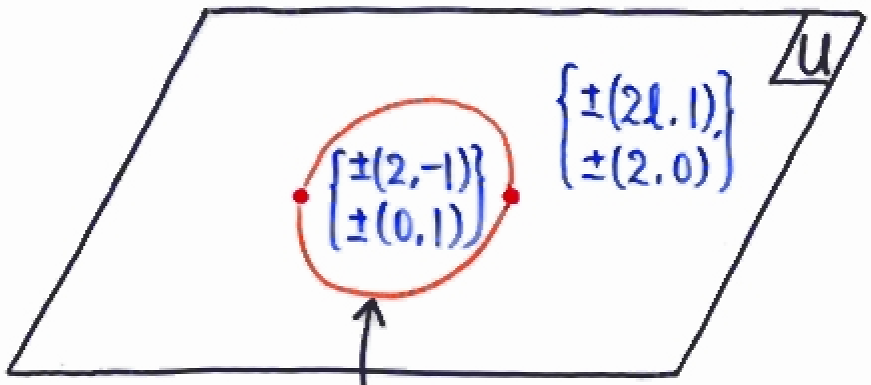

$y^2 = x^3 + f(u)x + g(u)$

$\Rightarrow a(u) = \oint_\alpha \frac{dx}{y} du, \quad a_D(u) = \oint_\beta \frac{dx}{y} du$

$\Rightarrow M_{(p,q)}(u) = |p\, a(u) + q\, a_D(u)|$

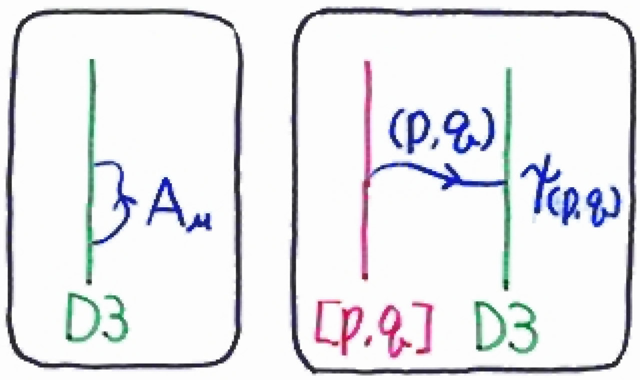


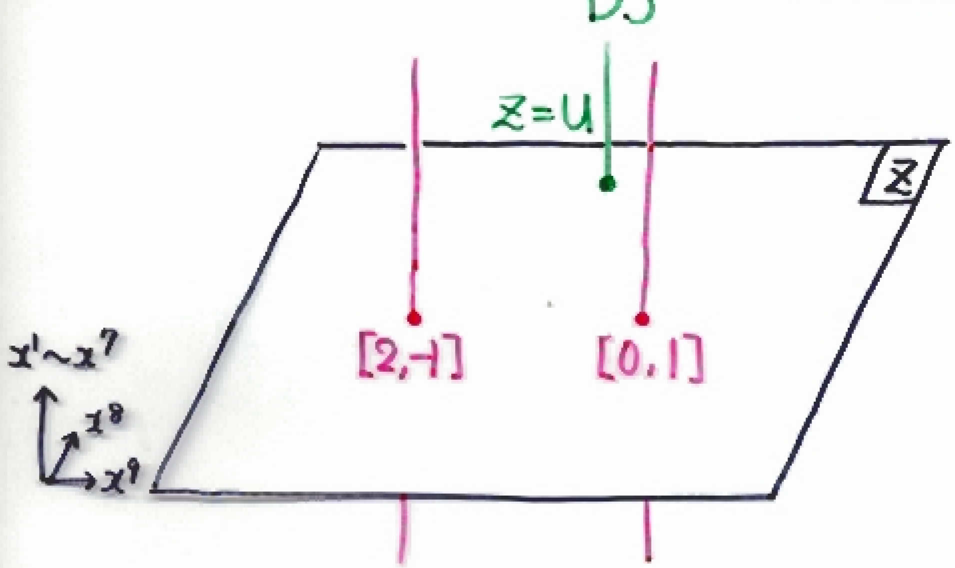
## ☆ Spectrum



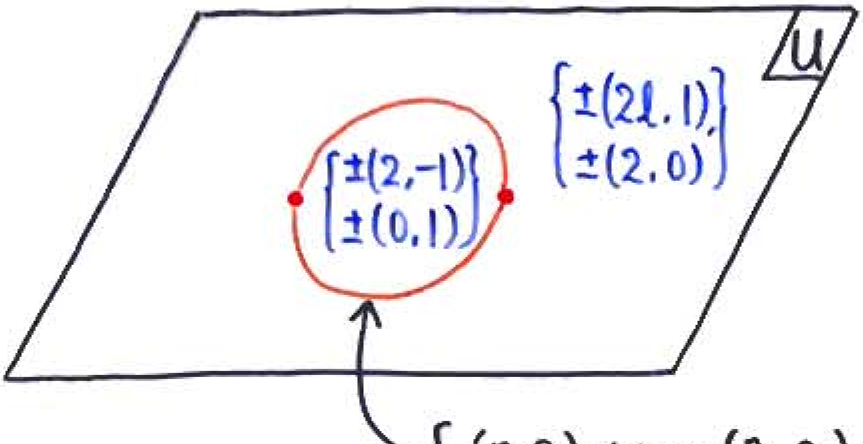

$$\begin{cases} (p,q) \leftrightarrow n_1(p_1,q_1) + n_2(p_2,q_2) \\ M_{(p,q)}(u) = n_1 M_{(p_1,q_1)}(u) + n_2 M_{(p_2,q_2)}(u) \end{cases}$$

## cf. IIB string theory



# ✡ Spectrum

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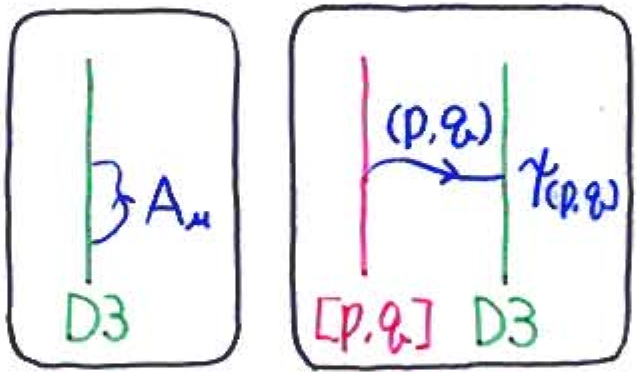
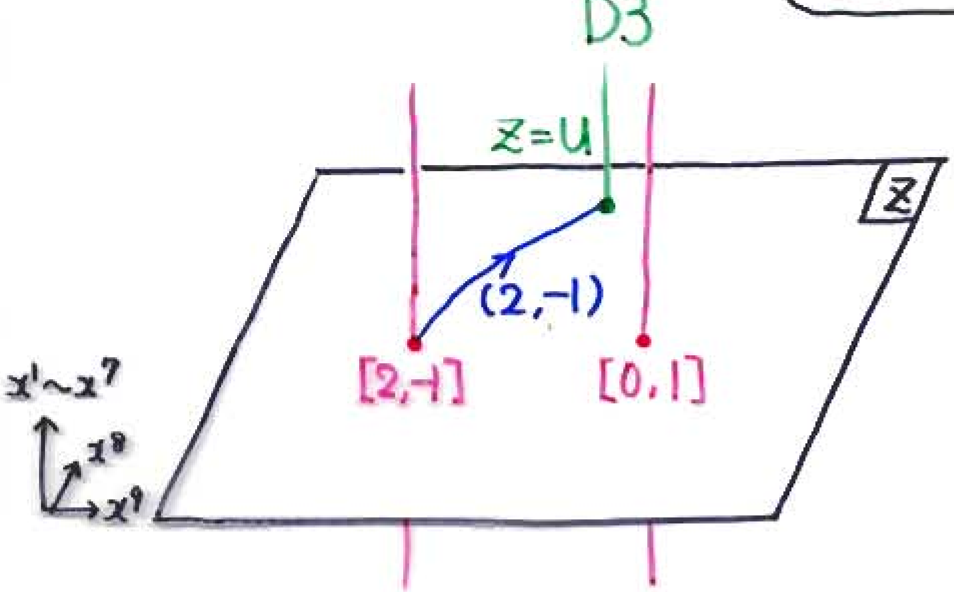
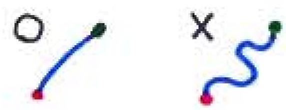
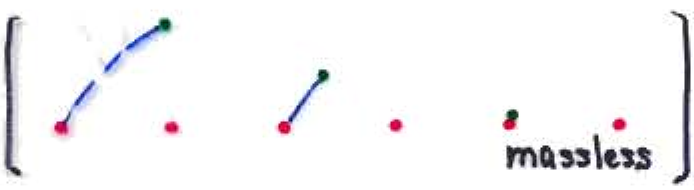



$$\int_C T_{p,q}(z)\, ds = M_{(p,q)}(u)$$

c :



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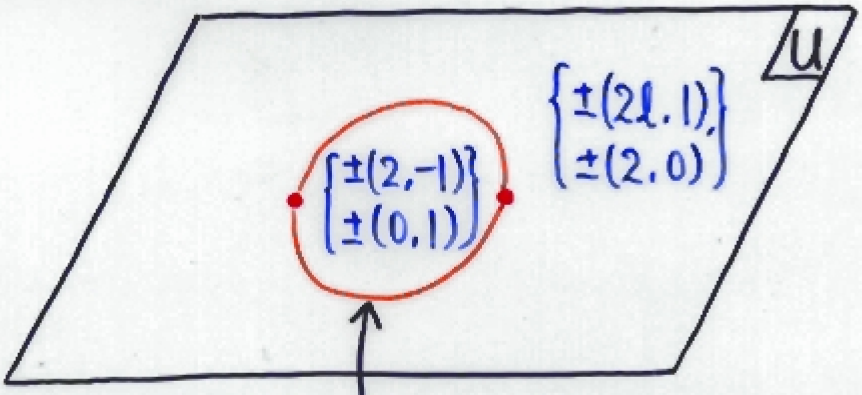


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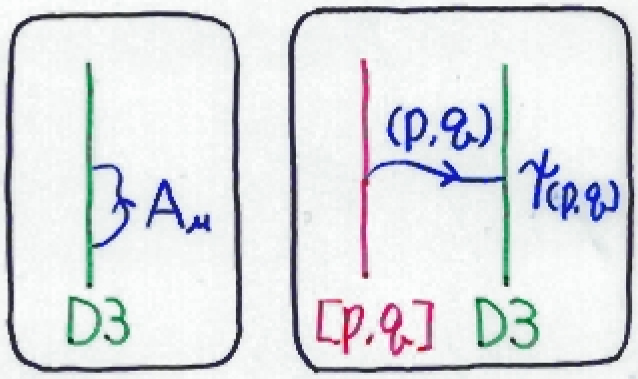
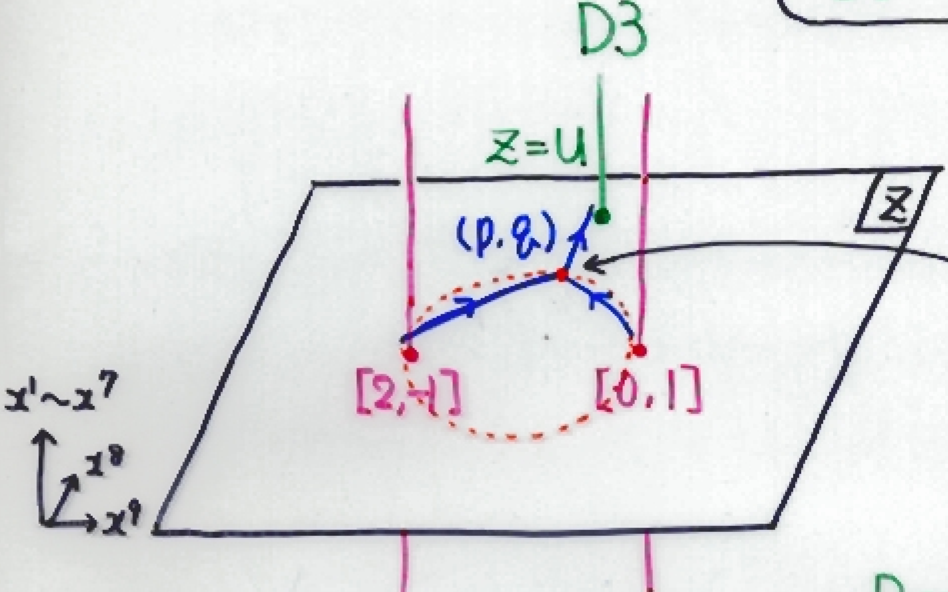


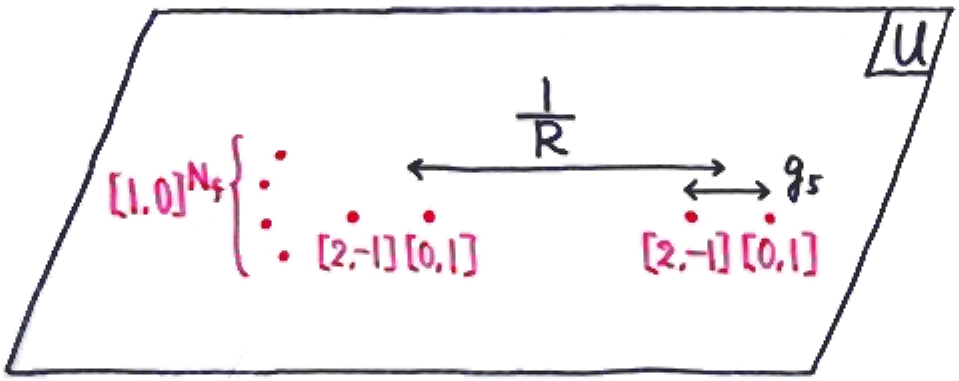




string tensions must be balanced at the junction.

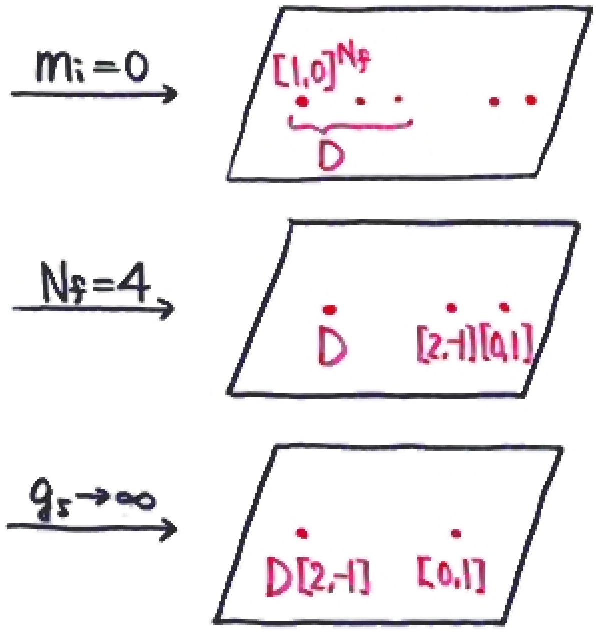

Bergman-Fayyazuddin, hep-th/9802033

# § Spectrum of 5-dim. SQCD on $S^1$

## ☆ Seiberg-Witten solution




simplify




- $y^2 = x^3 + f(u)x + g(u)$

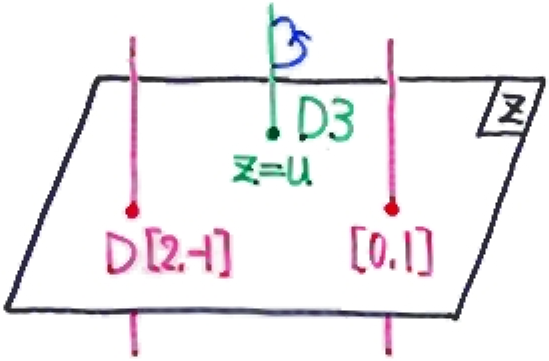
  $f(u) = u^2 - bu\,,\quad g(u) = c^2u^2$

$\Rightarrow\ a(u) = \oint_\alpha \frac{dx}{y}\,du\,,\quad a_D(u) = \oint_\beta \frac{dx}{y}\,du$

$\Rightarrow$ brane b.g. $T_{p,q}(z)\,dz = p\,da(z) + q\,da_D(z)$

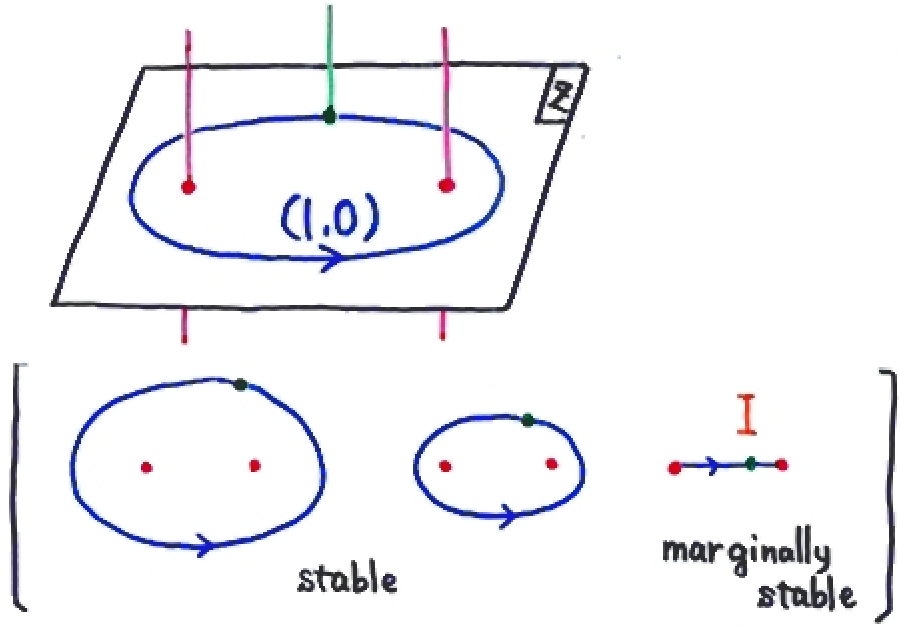
★ $g_s \to \infty$ Ohtake, hep-th/0409004

∘ $A_\mu^{(0)}$



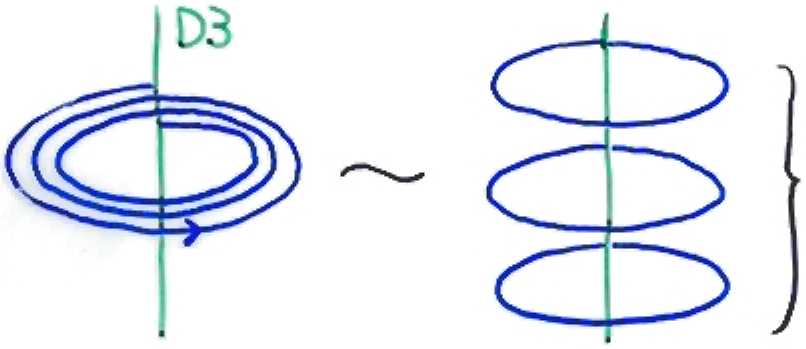

⇨ stable for $^\forall u$

∘ $A_\mu^{(1)}$




⇨ stable for $u \notin I$, marginally stable for $u \in I$

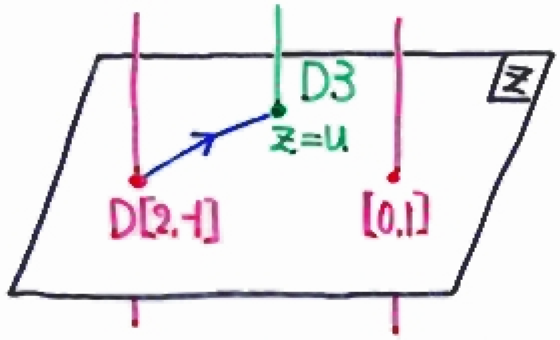
∘ $A_\mu^{(n)}$, $n \geq 2$




⇨ marginally stable for $^\forall u$

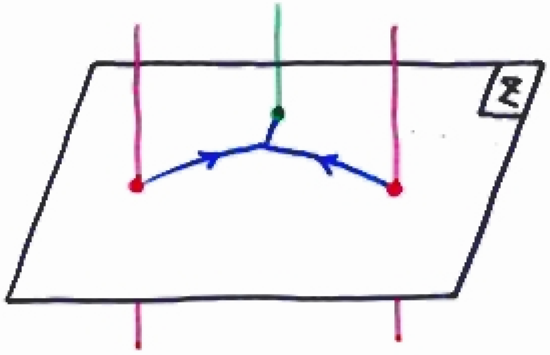
☆ $g_s \to \infty$

- $\chi^{(0)}$




⇨ stable for ∀u

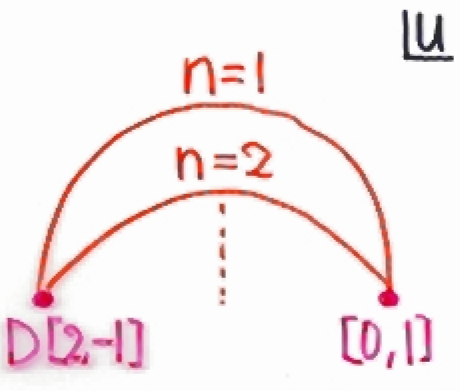
- $\chi^{(1)}$




⇨ a marginal stabily curve appears

- $\chi^{(n)}$ ⇨ " "

⇨

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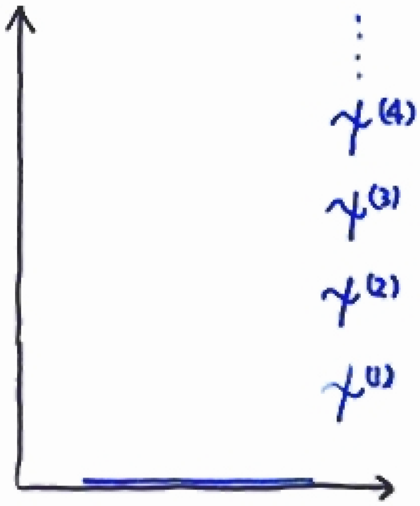
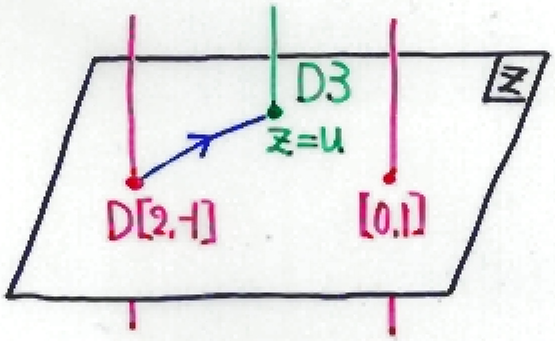


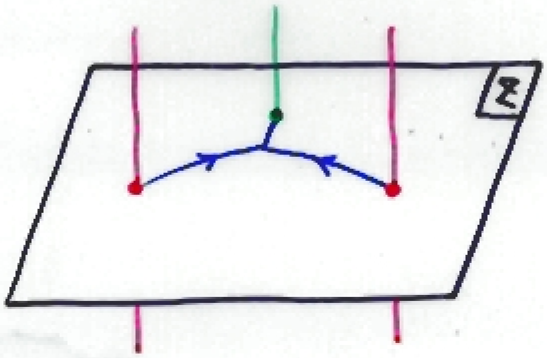
☆ $g_s \longrightarrow \infty$

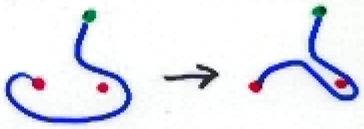
∘ $\chi^{(0)}$




⇨ stable for $\forall u$

∘ $\chi^{(1)}$



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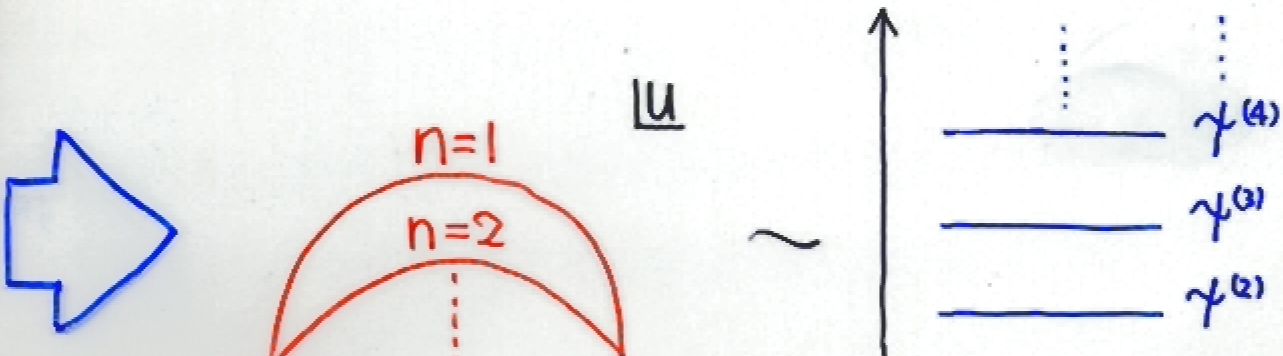
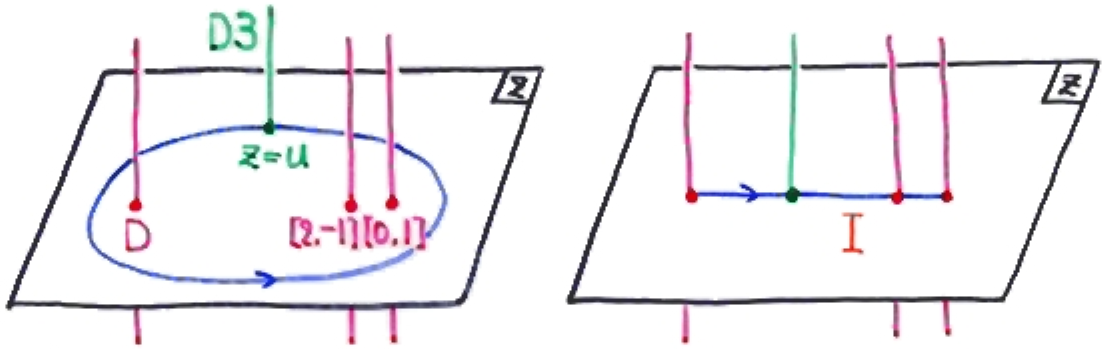
∘ $\chi^{(n)}$ ⇨ 〃 〃



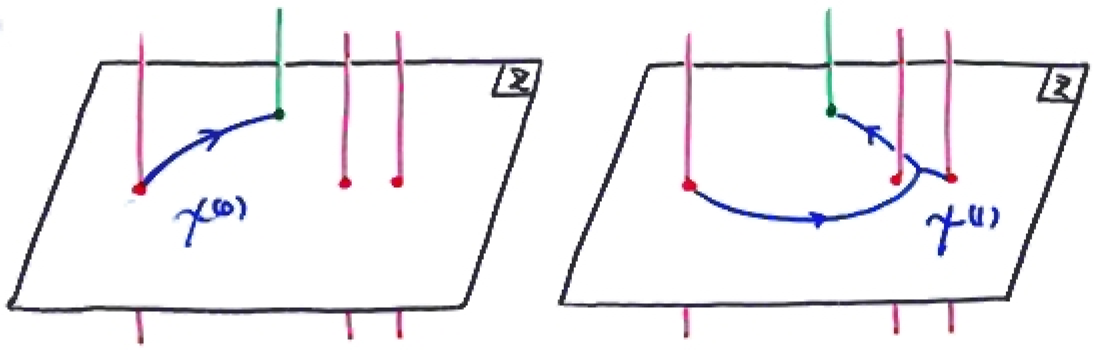
☆ $g_s < \infty$ Ohtake-Wakabayashi, to appear

∘ $A_\mu^{(n)}$

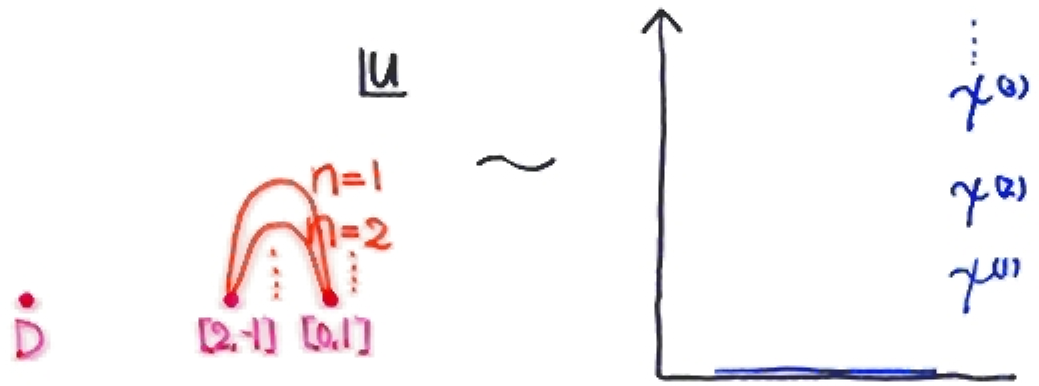



⇨ $A_\mu^{(0)}$ is stable for $\forall u$.
$A_\mu^{(1)}$ is stable for $u \notin I$, marginally stable for $u \in I$
$A_\mu^{(n)}$ $(n \geq 2)$ are marginally stable for $\forall u$

∘ $\gamma^{(n)}$

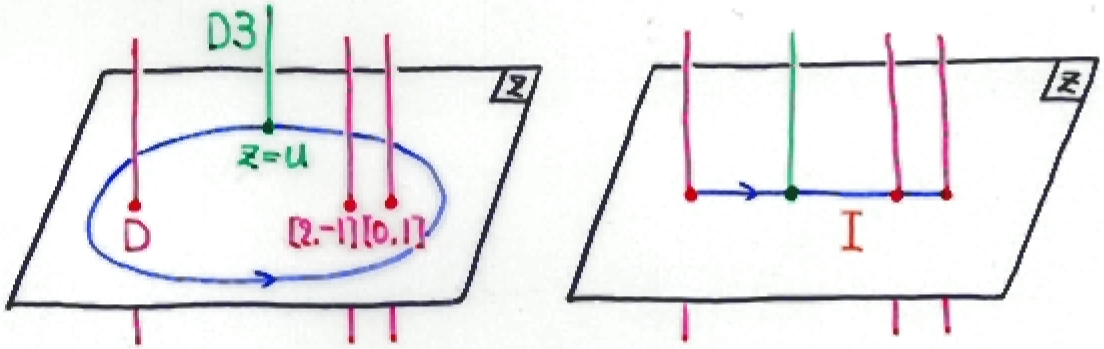



⇨ $\gamma^{(0)}$ is stable for $\forall u$
$\gamma^{(n)}$ $(n \geq 1)$ has a marginal stability curve



☆ $g_5 < \infty$ Ohtake-Wakabayashi, to appear

∘ $A_\mu^{(n)}$

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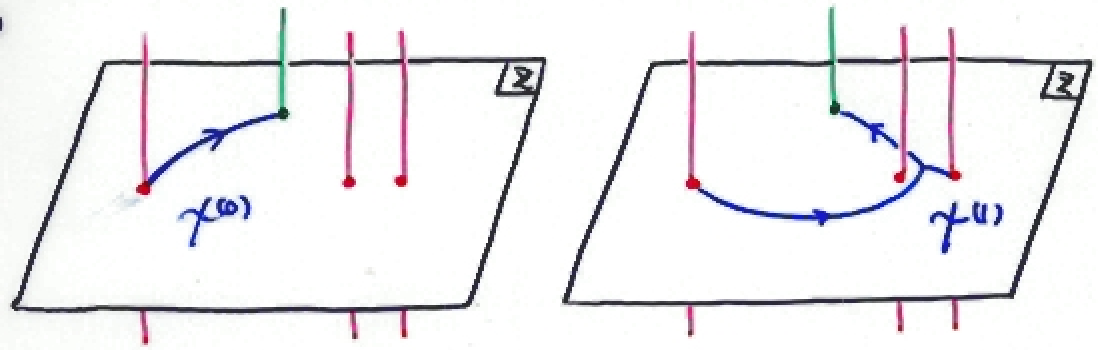


⇨ $\gamma^{(0)}$ is stable for $\forall u$
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