Optimal-Observable Analysis of Top Production/Decay at Photon-Photon Colliders

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1. Introduction
2. Basic Framework
3. Optimal-Observable Analyses
4. Summary

1. Introduction

Discovery of Top-quark

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We have now all the SM-fermions,

but · · · · · ·

- Is the 3rd generation a copy of the 1st & 2nd.?
- Isn't there any New-Physics in top-quark couplings?

*Based on collaboration with B. Grządkowski (Warsaw U), K.Ohkuma (Fukui U. Thechnology) and J. Wudka (UC Riverside).

So far, we have studied

$$e^+e \;\;\rightarrow\;\; t\bar{t}$$

to explore possible non-SM top-quark couplings.

Here we perform a similar analysis in

$$\gamma\gamma \rightarrow \;\; t\bar{t}$$

2. Basic Framework

What we aim to do is "A Model-Independent Analysis".

For this purpose, what terms must be taken into account?

• In the case of $e\bar{e} \rightarrow t\bar{t}$:

We are able to write down

"the most general covariant $t\bar{t}\gamma/Z$ amplitude."

$$\Gamma_v^\mu = \frac{g}{2}\bar{u}(p_t)\Big[\,\gamma^\mu(A_v - B_v\gamma_5) + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t}(C_v - D_v\gamma_5)\Big]v(p_t). \quad (1)$$

$$(v = \gamma \text{ or } Z)$$

However in $\gamma\gamma \rightarrow t\bar{t}$, t or $\bar{t}$ in $t\bar{t}\gamma$ coupling is virtual.

$$\Downarrow$$

Those which were dropped in $e\bar{e} \rightarrow t\bar{t}$ thanks to the on-shell condition can contribute

Take

$$\bar{\psi}\psi$$

as an example. If ψ and $\bar{\psi}$ are both on-shell,

$$\bar{\psi}F(\Box)\psi \qquad \Longrightarrow \qquad \bar{\psi}F(m^2)\psi$$

So they are all equivalent to $\bar{\psi}\psi$. However if ψ is virtual, we have infinite numbers of

$$\bar{u}F(q^2)S_F(q)\cdots$$

in an amplitude since F can be arbitrary.

We decided to perform an analysis in the framework of

Effective Operator Approach à la Buchmüller & Wyler

Basic assumption

- New Physics with Energy scale Λ
- Below Λ , we have only SM particles

The leading non-SM interactions are dim.-6 operators:

$$\begin{array}{ll}
\mathcal{O}_{uB} = i\bar{u}\gamma_\mu D_\nu u B^{\mu\nu} & \mathcal{O}_{qB} = i\bar{q}\gamma_\mu D_\nu q B^{\mu\nu} \\
\mathcal{O}_{qW} = i\bar{q}\tau^i\gamma_\mu D_\nu q W^{i\;\mu\nu} & \mathcal{O}'_{uB} = (\bar{q}\sigma^{\mu\nu}u)\tilde{\varphi}B_{\mu\nu} \\
\mathcal{O}_{uW} = (\bar{q}\sigma^{\mu\nu}\tau^i u)\tilde{\varphi}W_{i\;\mu\nu} & \mathcal{O}_{\varphi\tilde{W}} = (\varphi^\dagger\varphi)\tilde{W}^i_{\mu\nu}W^{i\;\mu\nu} \\
\mathcal{O}_{\varphi\tilde{B}} = (\varphi^\dagger\varphi)\tilde{B}_{\mu\nu}B^{\mu\nu} & \mathcal{O}_{\tilde{W}B} = (\varphi^\dagger\tau^i\varphi)\tilde{W}^i_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{\varphi W} = (\varphi^\dagger\varphi)W^i_{\mu\nu}W^{i\;\mu\nu} & \mathcal{O}_{\varphi B} = (\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} \\
\mathcal{O}_{WB} = (\varphi^\dagger\tau^i\varphi)W^i_{\mu\nu}B^{\mu\nu} &
\end{array} \tag{2}$$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \Big[\,\frac{1}{\Lambda^2}\sum_i \alpha_i\mathcal{O}_i + (h.c.)\,\Big]$$

One new discovery:

$$\mathcal{O}_{uB}, \quad \mathcal{O}_{qB}, \quad \mathcal{O}_{qW}$$

are not independent of the others

$$\mathcal{O}_{uB} = -ig_u\mathcal{O}'_{uB} + [\ -ig_u\mathcal{O}'_{uB}\ ]^\dagger + \cdots$$
$$\mathcal{O}_{qB} = ig_u\mathcal{O}'_{uB} + [\ ig_u\mathcal{O}'_{uB}\ ]^\dagger + \cdots$$
$$\mathcal{O}_{qW} = ig_u\mathcal{O}_{uB} + [\ ig_u\mathcal{O}'_{uB}\ ]^\dagger + \cdots$$

via some equations of motion

Independent operators lead to the following **Feynman rules**.

(1) CP-conserving $t\bar{t}\gamma$ vertex

$$\frac{\sqrt{2}}{\Lambda^2} v\alpha_{\gamma 1}\, \not{k}\gamma_\mu, \tag{3}$$

(2) CP-violating $t\bar{t}\gamma$ vertex

$$i\frac{\sqrt{2}}{\Lambda^2} v\alpha_{\gamma 2}\, \not{k}\gamma_\mu\gamma_5, \tag{4}$$

(3) CP-conserving $\gamma\gamma H$ vertex

$$-\frac{4}{\Lambda^2} v\alpha_{h1}\, [\ (k_1 k_2) g_{\mu\nu} - k_{1\nu} k_{2\mu}\ ], \tag{5}$$

(4) CP-violating $\gamma\gamma H$ vertex

$$\frac{8}{\Lambda^2} v\alpha_{h2}\, k_1^\rho k_2^\sigma \epsilon_{\rho\sigma\mu\nu}, \tag{6}$$

Here, k & $k_{1,2}$ are incoming photon momenta, $\alpha_{\gamma 1,\gamma 2,h1,h2}$ are defined as

$$\alpha_{\gamma 1} \equiv \sin\theta_W \mathrm{Re}(\alpha_{uW}) + \cos\theta_W \mathrm{Re}(\alpha'_{uB}), \tag{7}$$

$$\alpha_{\gamma 2} \equiv \sin\theta_W \mathrm{Im}(\alpha_{uW}) + \cos\theta_W \mathrm{Im}(\alpha'_{uB}), \tag{8}$$

$$\alpha_{h1} \equiv \sin^2\theta_W \mathrm{Re}(\alpha_{\varphi W}) + \cos^2\theta_W \mathrm{Re}(\alpha_{\varphi B}) - 2\sin\theta_W\cos\theta_W \mathrm{Re}(\alpha_{WB}), \tag{9}$$

$$\alpha_{h1} \equiv \sin^2\theta_W \mathrm{Re}(\alpha_{\varphi\tilde{W}}) + \cos^2\theta_W \mathrm{Re}(\alpha_{\varphi\tilde{B}}) - \sin\theta_W\cos\theta_W \mathrm{Re}(\alpha_{\tilde{W}B}). \tag{10}$$

On the other hand, the general amplitude for $t \to bW$ can be written as

$$\Gamma^{\mu}_{Wtb} = -\frac{g}{\sqrt{2}}\,\bar{u}(p_b)\Big[\gamma^{\mu}P_L - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}f_2^R P_R\Big]u(p_t), \tag{11}$$

where $P_{L,R} \equiv (1 \pm \gamma_5)/2$ and f_2^R is

$$f_2^R = \frac{1}{\Lambda^2}\Big[-\frac{4M_W v}{g}\alpha_{uW} - \frac{M_W v}{2}\alpha_{Du}\Big]. \tag{12}$$

for $m_b = 0$ and on-shell W approximation.

Using them, we calculated the cross section of $\gamma\gamma \to \ell^{\pm}X$ via **FORM**.

HOWEVER, the result is too long to show here. Sorry!

3. Optimal-Observable Analysis

How can we determine several unknown parameters simultaneously?

$\Longrightarrow$ **Optimal-observable method**

Brief summary of this method: Suppose we have a distribution

$$\frac{d\sigma}{d\phi}(\equiv \mathit{\Sigma}(\phi)) = \sum_i c_i f_i(\phi)$$

where $f_i(\phi)$ are calculable functions, and c_i are the parameters we try to determine.

Determining c_i

$\Longrightarrow$ We make weighting functions $w_i(\phi)$ which satisfies :

$$\int w_i(\phi)\Sigma(\phi)d\phi = c_i$$

The one which minimizes the statistical uncertainty of c_i is

$$w_i(\phi) = \sum_j X_{ij} f_j(\phi)/\mathit{\Sigma}(\phi)\,,$$

where X is the inverse matrix of

$$M_{ij} \equiv \int \frac{f_i(\phi) f_j(\phi)}{\mathit{\Sigma}(\phi)} d\phi$$

This X gives

$$\mathit{\Delta} c_i = \sqrt{X_{ii}\,\sigma_T/N}\,,$$

where $\sigma_T \equiv \int (d\sigma/d\phi) d\phi$, $N = L_{\rm eff}\sigma_T$ is the total number of the events, $L_{\rm eff}$ is the product of the **integrated luminosity** and **detection efficiency**.

We applied this procedure to the angular & energy distribution of $\gamma\gamma \to t\bar{t} \to \ell^+ X$:

$$\frac{d\sigma}{dE_\ell d\cos\theta_\ell} = f_{\rm SM}(E_\ell, \cos\theta_\ell) + \alpha_{\gamma 1} f_{\gamma 1}(E_\ell, \cos\theta_\ell) + \alpha_{\gamma 2} f_{\gamma 2}(E_\ell, \cos\theta_\ell) + \alpha_{h1} f_{h1}(E_\ell, \cos\theta_\ell) + \alpha_{h2} f_{h2}(E_\ell, \cos\theta_\ell) + \alpha_d f_d(E_\ell, \cos\theta_\ell) \tag{13}$$

in $e\bar{e}$-CM frame, where

- f_{SM} is the SM contribution,
- $f_{\gamma 1,\gamma 2}$ are CP-conserving- & CP-violating-$t\bar{t}\gamma$-vertices contribution,
- $f_{h1,h2}$ are CP-conserving- & CP-violating-$\gamma\gamma H$-vertices contribution,

and

- f_d is from the anomalous tbW-vertex

$$\alpha_d = {\rm Re}(f_2^R).$$

Parameters

Higgs mass : $m_H = 100, 300, 500$ GeV,

Polarizations of e & $\bar{e}$: $P_e = P_{\bar{e}} = 1$,

Polarization of the Laser:

(1) Linear Polarization

$P_e = P_{\bar{e}} = 1$, $P_t = P_{\tilde{t}} = P_\gamma = P_{\tilde{\gamma}} = 1/\sqrt{2}$ and $\chi(\equiv \varphi_1 - \varphi_2) = \pi/4$

(2) Circular Polarization

$P_e = P_{\bar{e}} = P_\gamma = P_{\tilde{\gamma}} = 1.$

Problems

It turned out that our results for X_{ij} are very unstable:

even a tiny fluctuation of M_{ij} changes X_{ij} significantly.

$$\Downarrow$$

Some of f_i have similar shapes?

The only option in such a case is to refrain from determining all the couplings at once through this process alone.

$$\Downarrow$$

We assumed some parameters can be determined in other processes:

Result :

We found several sets of solution in two-parameter analysis

1) Linear polarization

- Independent of m_H

$$\Delta\alpha_{\gamma 2} = 73/\sqrt{N_\ell}, \quad \Delta\alpha_d = 1.9/\sqrt{N_\ell}, \tag{14}$$

- $m_H = 100$ GeV

$$\Delta\alpha_{h2} = 107/\sqrt{N_\ell}, \quad \Delta\alpha_d = 1.6/\sqrt{N_\ell}, \tag{15}$$

- $m_H = 300$ GeV

$$\Delta\alpha_{h1} = 3.4/\sqrt{N_\ell}, \quad \Delta\alpha_d = 3.2/\sqrt{N_\ell}, \tag{16}$$

Here $\sqrt{N_\ell} \simeq 63$ for $L_{e\bar{e}}^{\rm eff} = 500$ fb^{-1}.

2) Circular polarization

- $m_H = 100$ GeV

$$\Delta\alpha_{h1} = 9.0/\sqrt{N_\ell}, \quad \Delta\alpha_d = 3.0/\sqrt{N_\ell}, \tag{17}$$

- $m_H = 300$ GeV

$$\Delta\alpha_{h1} = 3.5/\sqrt{N_\ell}, \quad \Delta\alpha_d = 3.0/\sqrt{N_\ell}, \tag{18}$$

$$\Delta\alpha_{h2} = 35/\sqrt{N_\ell}, \quad \Delta\alpha_d = 3.1/\sqrt{N_\ell}, \tag{19}$$

- $m_H = 500$ GeV

$$\Delta\alpha_{h1} = 7.7/\sqrt{N_\ell}, \quad \Delta\alpha_d = 2.8/\sqrt{N_\ell}, \tag{20}$$

$$\Delta\alpha_{h2} = 10/\sqrt{N_\ell}, \quad \Delta\alpha_d = 2.8/\sqrt{N_\ell}, \tag{21}$$

Here $\sqrt{N_\ell} \simeq 48$ for $L_{e\bar{e}}^{\rm eff} = 500$ fb^{-1}.

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We also performed a similar analysis using

$$\gamma\gamma \to t\bar{t} \to bX$$

Isn't it harder to study b-quark distribution?

$$\Downarrow$$

b-quark tagging must be done to distinguish $t\bar{t}$ events from possible background (WW production).

1) Linear polarization

• Independent of m_H

$$\Delta\alpha_{\gamma2} = 29/\sqrt{N_b}, \quad \Delta\alpha_d = 2.6/\sqrt{N_b}, \tag{22}$$

• $m_H = 100$ GeV

$$\Delta\alpha_{h2} = 38/\sqrt{N_b}, \quad \Delta\alpha_d = 2.4/\sqrt{N_b}, \tag{23}$$

• $m_H = 300$ GeV

$$\Delta\alpha_{\gamma 2} = 24/\sqrt{N_b}, \quad \Delta\alpha_{h1} = 2.4/\sqrt{N_b}, \tag{24}$$

$$\Delta\alpha_{h1} = 5.4/\sqrt{N_b}, \quad \Delta\alpha_{d} = 4.9/\sqrt{N_b}, \tag{25}$$

• $m_H = 500$ GeV

$$\Delta\alpha_{\gamma 2} = 23/\sqrt{N_b}, \quad \Delta\alpha_{h1} = 5.0/\sqrt{N_b}, \tag{26}$$

$$\Delta\alpha_{h1} = 18/\sqrt{N_b}, \quad \Delta\alpha_{h2} = 22/\sqrt{N_b}, \tag{27}$$

$$\Delta\alpha_{h1} = 8.0/\sqrt{N_b}, \quad \Delta\alpha_{d} = 3.3/\sqrt{N_b}, \tag{28}$$

where $\sqrt{N_b} \simeq 140$ for $L_{e\bar{e}}^{\rm eff} = 500$ fb^{-1}.

2) Circular polarization

• $m_H = 100$ GeV

$$\Delta\alpha_{h1} = 14/\sqrt{N_b}, \quad \Delta\alpha_{d} = 5.2/\sqrt{N_b}, \tag{29}$$

• $m_H = 500$ GeV

$$\Delta\alpha_{h1} = 10/\sqrt{N_b}, \quad \Delta\alpha_{d} = 4.2/\sqrt{N_b}, \tag{30}$$

where $\sqrt{N_b} \simeq 100$ for $L_{e\bar{e}}^{\rm eff} = 500$ fb^{-1}.[♯1]

The above results are for $\Lambda = 1$ TeV. When one takes the new-physics

[♯1] We used the tree-level SM formula for computing N_b, so that we have the same N_b for different m_H.

scale to be $\Lambda' = \lambda\Lambda$, then all the above results ($\Delta\alpha_i$) are replaced with $\Delta\alpha_i/\lambda^2$, which means that the right-hand sides of eqs. (14)–(30) are multiplied by λ^2.

Comparing two results

The following parameter sets are measurable in

(1) Lepton analysis

$(\alpha_{\gamma 2},\ \alpha_d)$, $(\alpha_{h1},\ \alpha_d)$, $(\alpha_{h2},\ \alpha_d)$

(2) b-quark analysis

$(\alpha_{\gamma 2},\ \alpha_{h1})$, $(\alpha_{\gamma 2},\ \alpha_d)$, $(\alpha_{h1},\ \alpha_{h2})$, $(\alpha_{h1},\ \alpha_d)$, $(\alpha_{h2},\ \alpha_d)$

4. Summary

- In order to explore possible anomalous top-quark couplings, we studied $t\bar{t}$ production/decay in $\gamma\gamma$ collisions.

- We assumed a New-Physics with an energy-scale Λ, and we have only the SM particles below Λ.

- All leading non-SM interactions are given in terms of dimension-6 effective operators à la Buchmüller & Wyler.

- We found some new "Equation-of-motion relations" among several operators, which reduced the number of operators necessary in our analysis.

- We found it impossible to determine all the parameters in this process alone, but also found some stable solutions in two-parameter analysis.

- If we encounter phenomena which cannot be described in our framework, it will be an indication of some New-Physics beyond B& W scenario.

References

(1) B.Grzadkowski, Z.Hioki, K.Ohkuma and J.Wudka, *Nucl. Phys.* **B689** (2004) 108 (hep-ph/0310159).

(2) B.Grzadkowski, Z.Hioki, K.Ohkuma and J.Wudka, *Phys. Lett.* **B593** (2004) 189 (hep-ph/0403174).