

**Light hadron spectrum  
in 2+1 flavor full QCD  
by CP-PACS and JLQCD  
collaboration**

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**YITP Workshop  
“Progress in Particle Physics”  
Yukawa Institute for Theoretical Physics at Kyoto University  
June 29 - July 2, 2004**

# Introduction

## ■ Lattice QCD

- One of the regularization of QCD
- The only systematic way to calculate non-perturbative property of QCD from its first principles.
- Numerical simulation can be applied.
- Various application :
  - ◆ Hadron spectrum, Weak matrix elements, Finite-temperature and finite-density system, Confinement, Topology ...

## ■ Light hadron spectrum

- Direct test of QCD at low energy scale
- Determination of fundamental parameters  
quark masses, QCD coupling, ....

# Systematic studies by CP-PACS and JLQCD collab.

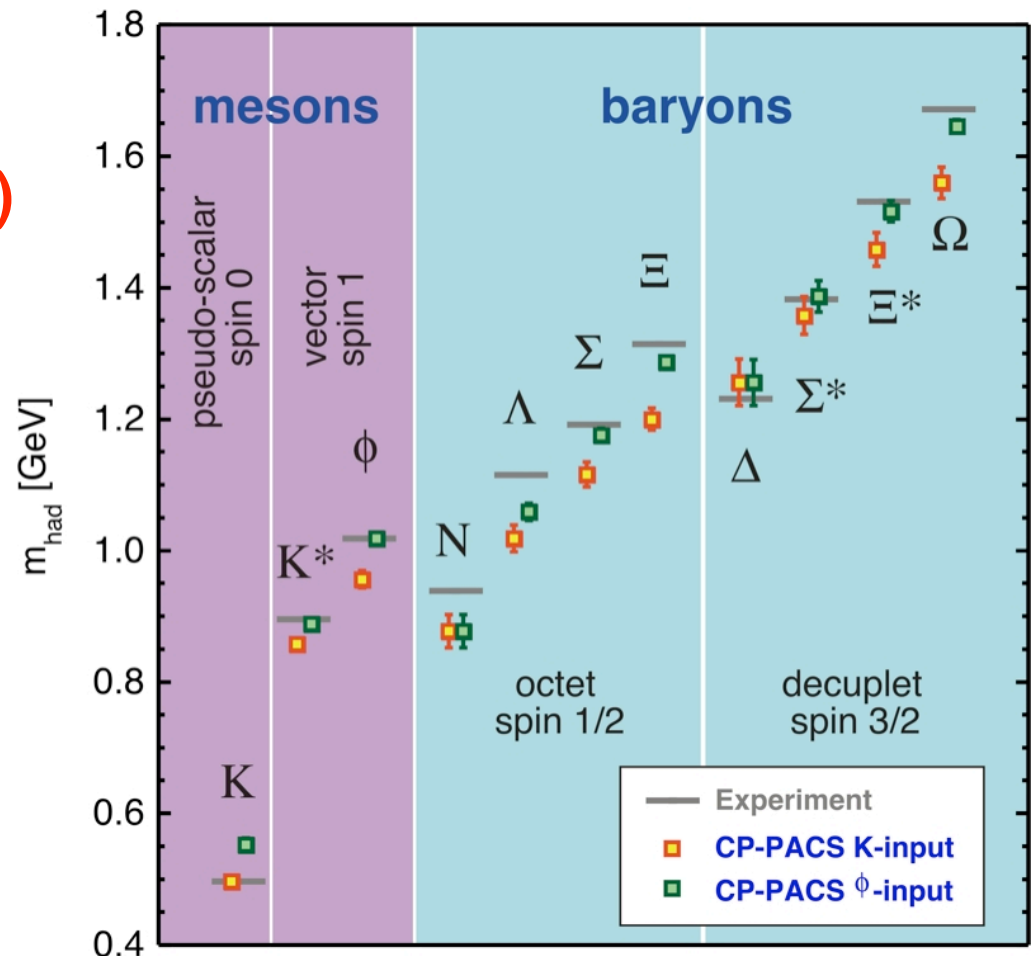
## quenched QCD (continuum limit)

- ◆ RG-improved gauge + clover quark (tad.imp.  $c_{SW}$ ) (CP-PACS, 2001)
- ◆ plaquette + Wilson (CP-PACS, 2001)

Systematic deviation  
from experiment (5-10%)



artifact due to the  
quenched approx.



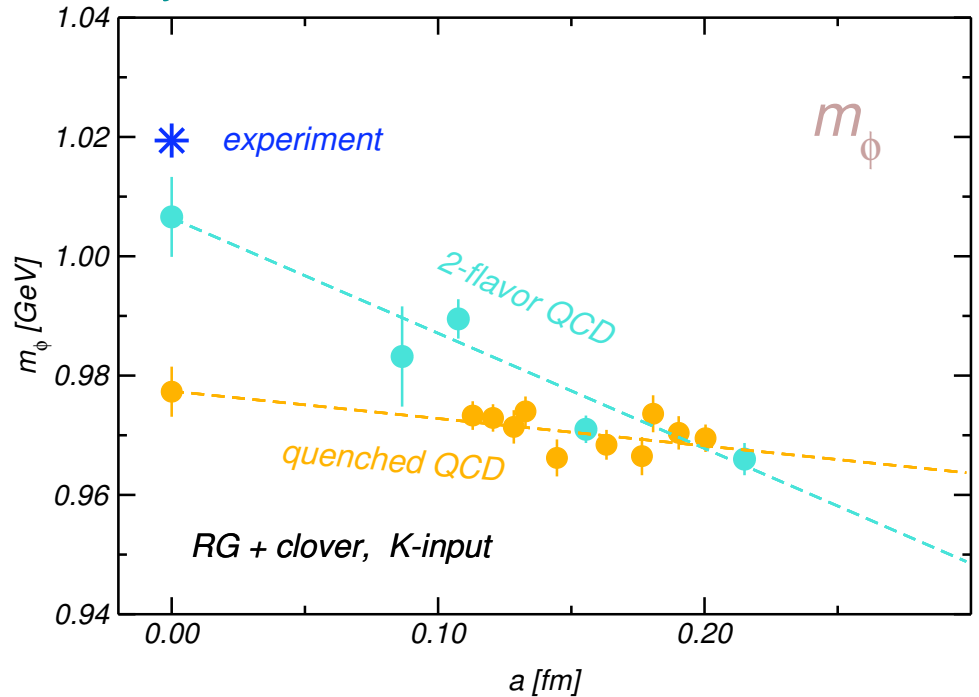
## □ 2-flavor QCD (continuum limit)

ud : dynamical

s : quenched

- ◆ RG-improved gauge  
+ clover quark (tad.imp.  $C_{SW}$ )  
(CP-PACS, 2001)

deviation is reduced

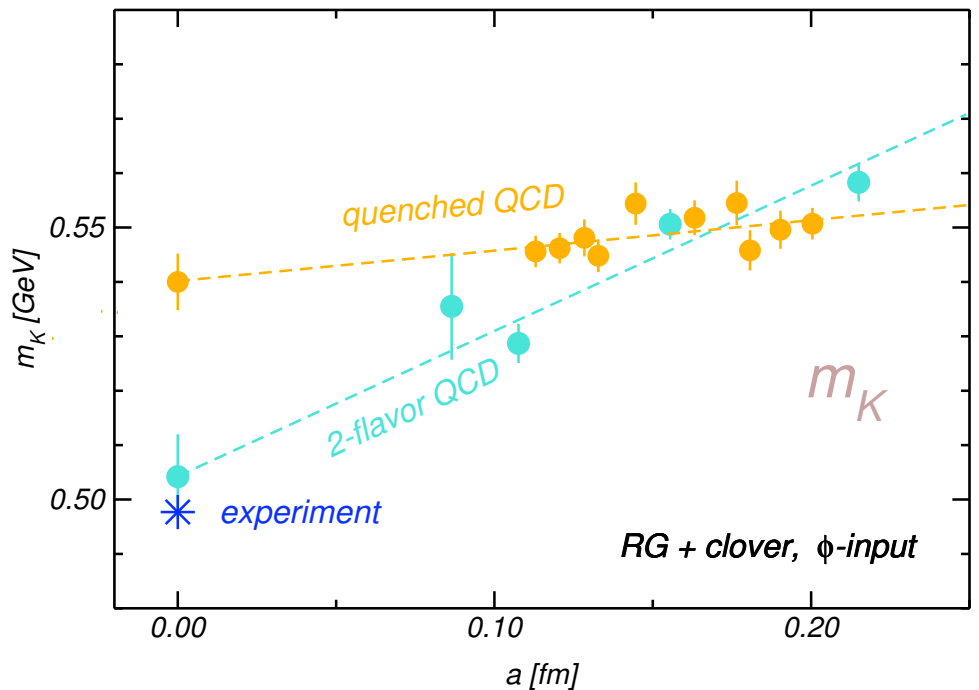


Next Step :

## □ 3-flavor full QCD

ud : dynamical

s : dynamical



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# Algorithm

- with degenerate up and down quarks and strange quark
- Algorithm (odd flavor algorithm)

- Pseudo-fermion method

$$Z = \int \mathcal{D}U \det [D]^{N_f} \exp [-S_g] = \int \mathcal{D}U \mathcal{D}\phi^\dagger \mathcal{D}\phi \exp [-S_g - \phi^\dagger D^{-N_f} \phi]$$

$U$  : link variable,  $D$  : Dirac matrix, ;  $\phi$  : complex scalar

- ◆  $N_f = \text{even}$  :  $\phi^\dagger D^{-N_f} \phi = |D^{-N_f/2} \phi|^2 \leftarrow \text{real positive}$

- ◆  $N_f = \text{odd}$  :  $\phi^\dagger D^{-N_f} \phi \leftarrow \text{complex } (?)$

- Polynomial HMC (Forcrand and Takaishi, 1997, K-I.Ishikawa et.al., 2002)

- ◆ Polynomial approx. of  $D^{-1}$  :  $D^{-1} \sim P_{2N_{poly}} [D] = \bar{T}[D]T[D]$

- ◆ Metropolis test for correction factor  $\det [P_{2N_{poly}} [D]D]$

$\Rightarrow$  exact algorithm

We employ PHMC algorithm for strange quark.

# Simulation parameters

## ■ Lattice action

□ gauge : RG improved action

□ quark : **non-perturbatively  $\mathcal{O}(a)$  improved Wilson action**

■  $\beta = 1.9$  ,  $c_{SW} = 1.715$  , ( lattice spacing  $a \sim 0.1$  fm )

■ **Lattice size** :  $20^3 \times 40$  ( $La \sim 2.0$ fm)  
 $16^3 \times 32$  ( $La \sim 1.6$ fm)

small for baryons  $\implies$  concentrate on meson sector

## ■ Computing facilities

Earth Simulator@JAMSTEC, SR8K/F1@KEK,



CP-PACS,



VPP-5000,



SR8K/G1@Univ. of Tsukuba



## ■ $K$ parameters

- 5 ud for  $20^3 \times 40$  and  
6 ud for  $16^3 \times 32$

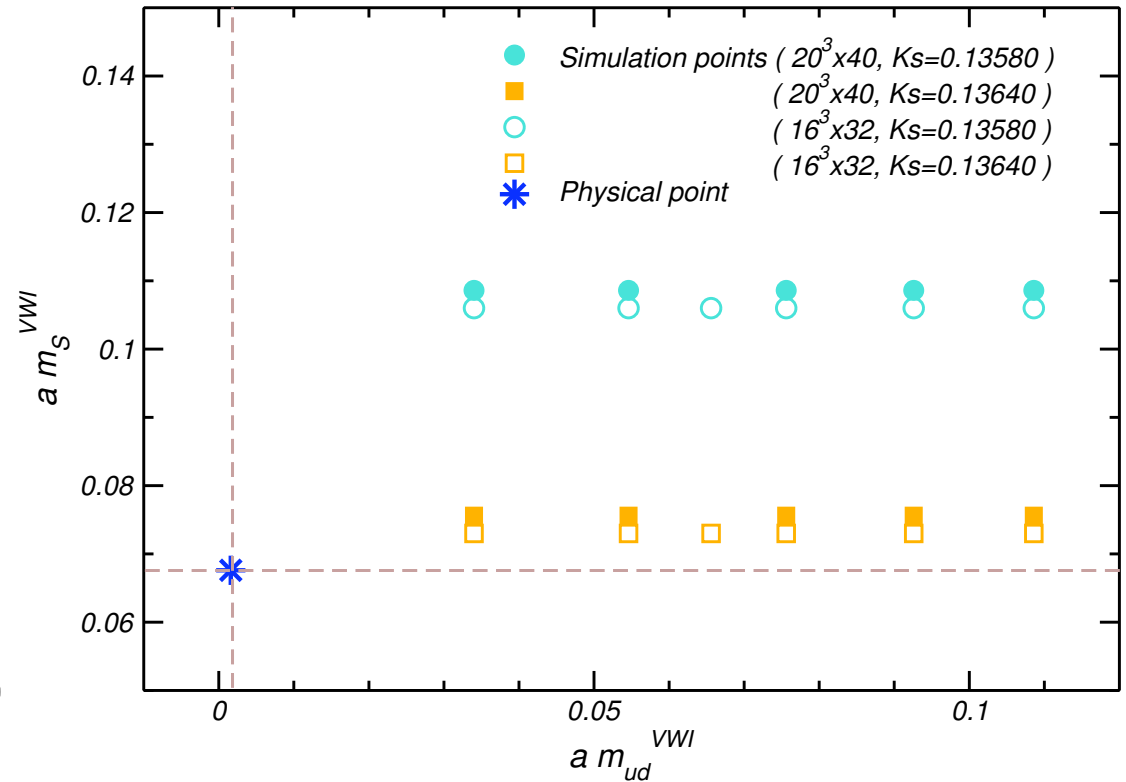
$$m_\pi/m_\rho \sim 0.62 - 0.77$$

(0.18 : experiment)

- 2 strange

$$m_{\eta_s}/m_\phi \sim 0.71, 0.77$$

(0.68 : 1-loop ChPT)



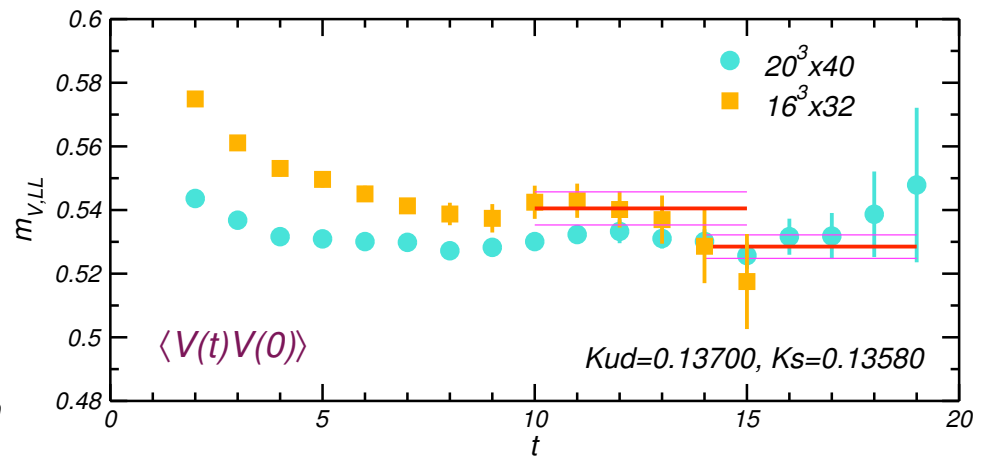
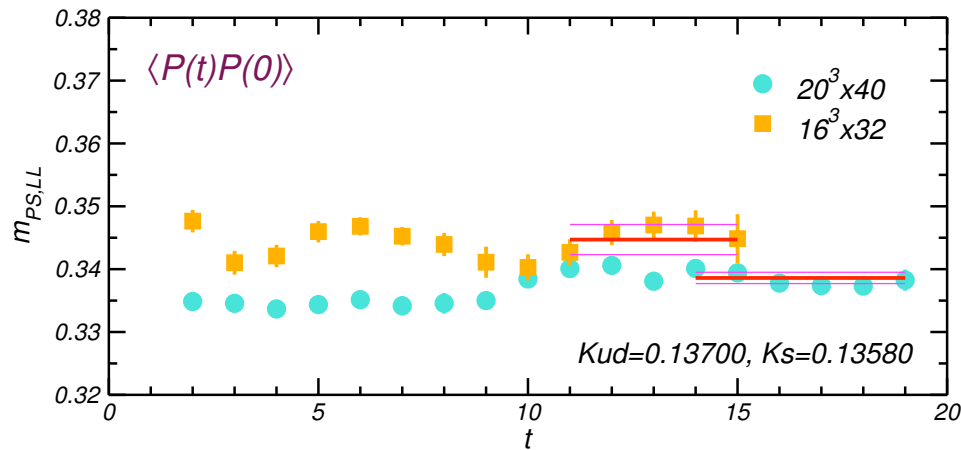
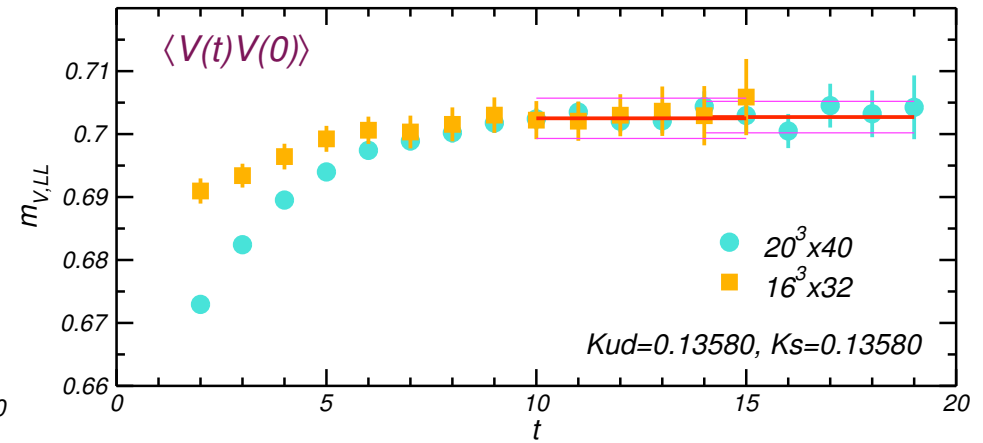
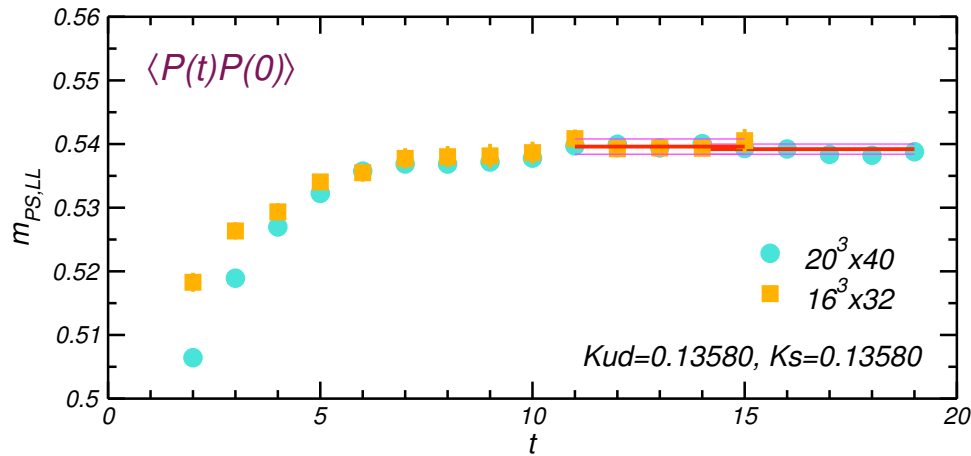
## ■ Statistics

- **5000~8000** traj at each simulation point for  $20^3 \times 40$   
3000 traj at each simulation point for  $16^3 \times 32$
- measure meson masses every 10 trajectories
- statistical error ← jack-knife with bin size of 100 traj



# Finite size effect

## ■ effective mass plot



- FSE is observed at the simulation point where quark masses are small.

## measured meson mass

diff. between L=16 and 20  
 $\sim 2\%$

assumption

$$\frac{m(L) - m(\infty)}{m(\infty)} \sim \frac{c}{L^3}$$

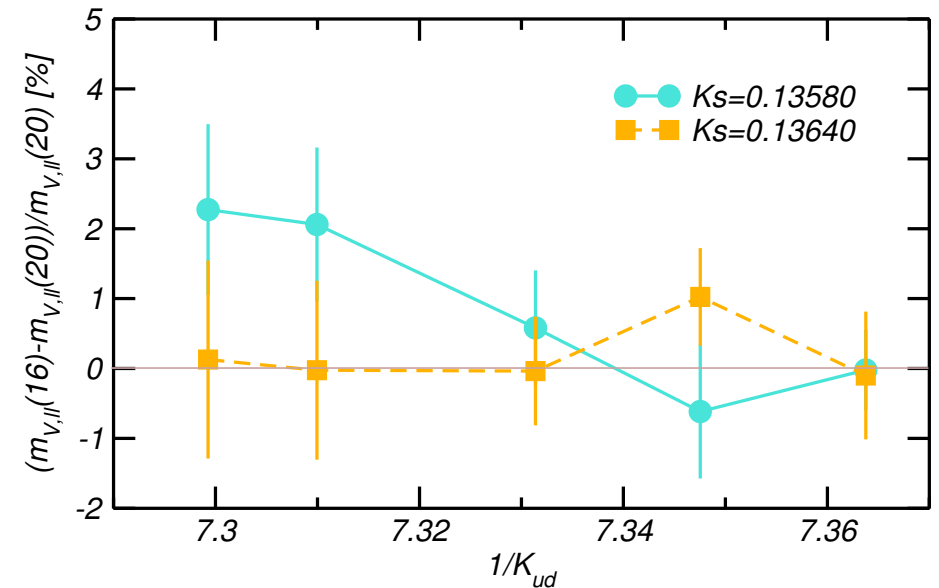
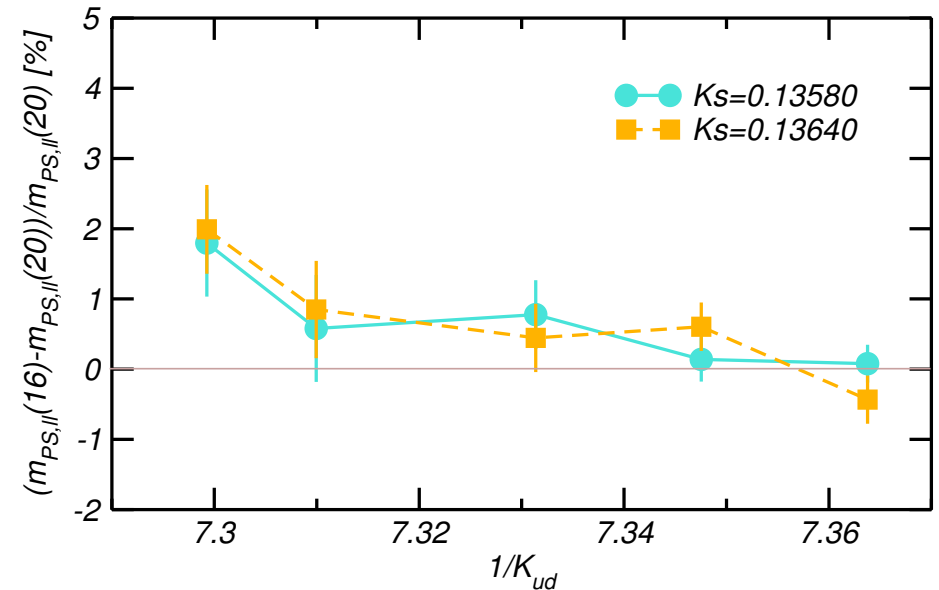
(Fukugita et al., 1992)



diff. between L=20 and  $\infty$   
 $\sim 2\%$

FSE is comparable to or slightly larger than statistical error of spectrum at physical points.

It does not change conclusions below.



# Chiral extrapolation

■ **fit function** ——— polynomial in quark masses

■ **Ambiguity of fit forms**

□ **linear in**  $m_{\text{sea}}$  and  $m_{\text{valence}}$

$$f(m_q) = A + B_S m_{\text{sea}} + B_V m_{\text{val}} + D_{SV} m_{\text{sea}} m_{\text{val}}$$

$$m_{\text{sea}} = 2m_{ud} + m_s, \quad m_{\text{val}} = m_{\text{val}1} + m_{\text{val}2}$$

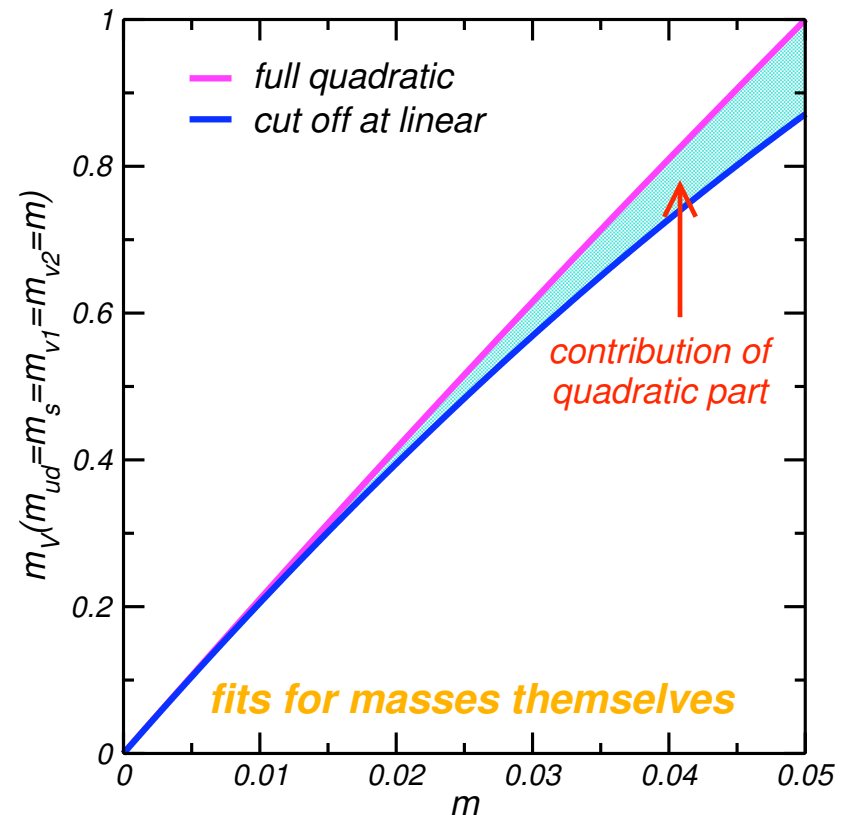
□ **general quadratic polynomial**

$$f(m_q) = A + B_S m_{\text{sea}} + B_V m_{\text{val}} + D_{SV} m_{\text{sea}} m_{\text{val}} + C_S m_{\text{sea}}^2 + C_V m_{\text{val}}^2$$

**Naive fitting yields large contribution of quadratic part.**



**Convergency is not well.**



## ■ Fits for masses normalized by $r_0$

- Sommer scale  $r_0(m_{\text{sea}})$

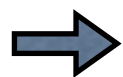
$$r^2 \frac{dV(r)}{dr} \Big|_{r=r_0} = 1.65$$

- Masses normalized by  $r_0$



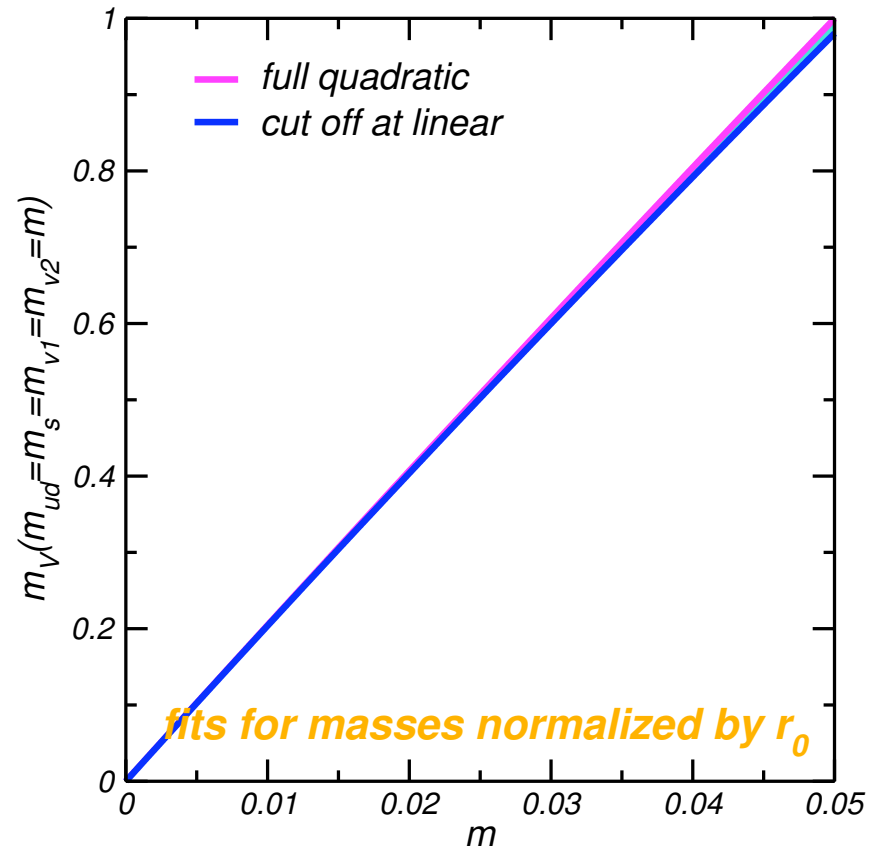
absorption of  $m_{\text{sea}}$  dependences  
of the effective lattice spacing

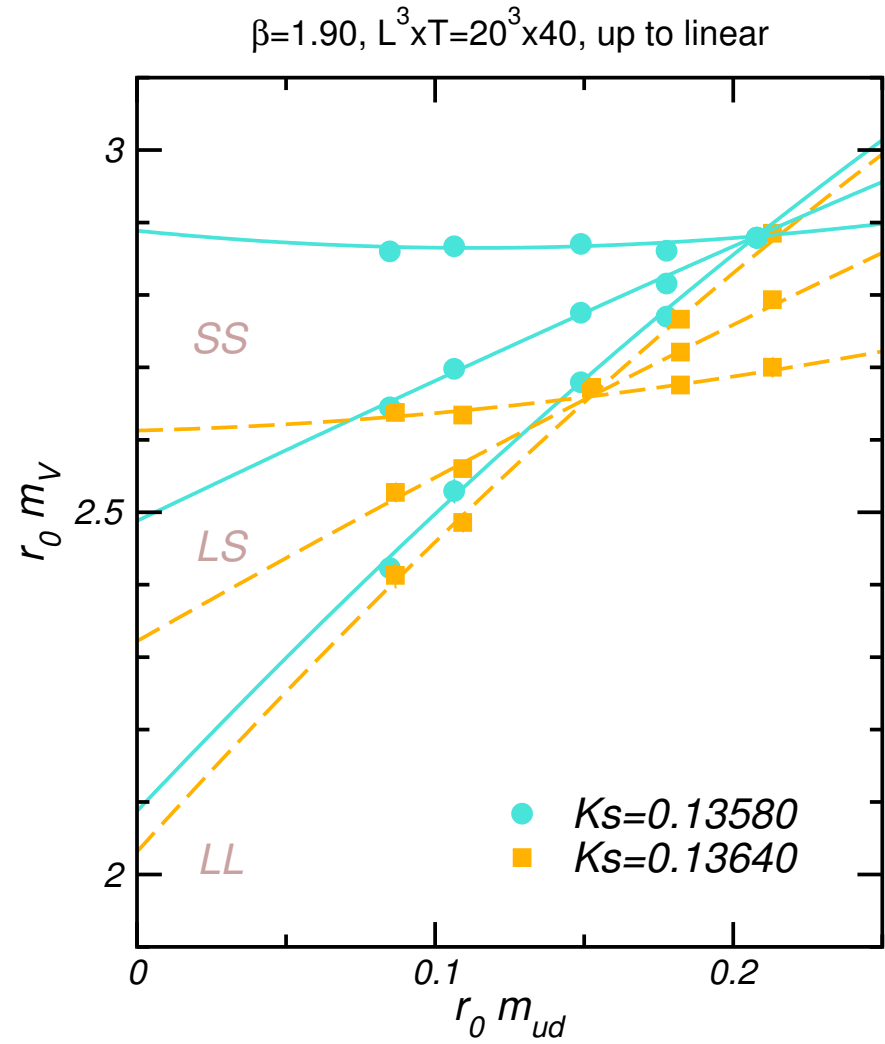
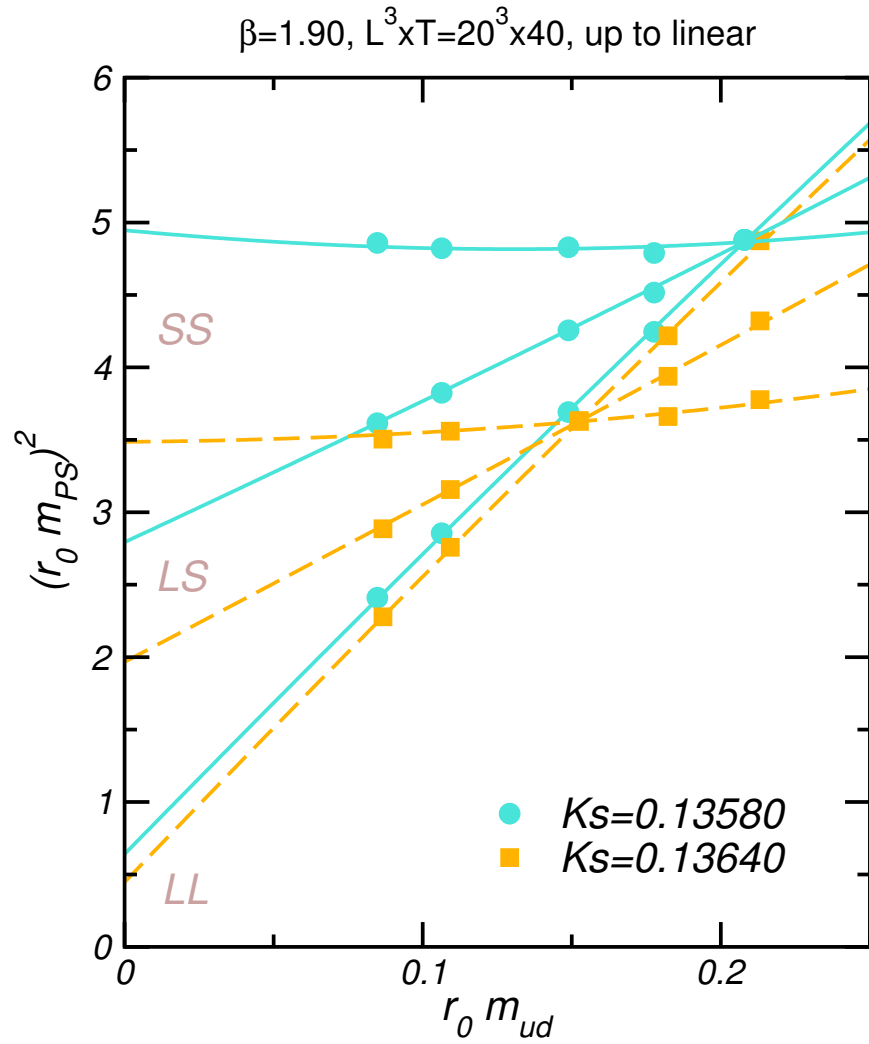
This fits are well reproduced  
by linear function.



our best estimation of  
central value and statistical error

- ◆ Others: used to estimate the systematic uncertainty of chiral fits





**L : light quarks (up, down), S : strange quark**

# Light meson spectrum (preliminary)

## Inputs to fix the quark masses

$$\begin{aligned} m_{ud} &\longleftarrow \frac{m_{PS}(m_{ud}, m_{ud})}{m_V(m_{ud}, m_{ud})} = \frac{m_\pi}{m_\rho} \\ m_s(K\text{-input}) &\longleftarrow \frac{m_{PS}(m_{ud}, m_s)}{m_V(m_{ud}, m_{ud})} = \frac{m_K}{m_\rho} \\ m_s(\phi\text{-input}) &\longleftarrow \frac{m_V(m_s, m_s)}{m_V(m_{ud}, m_{ud})} = \frac{m_\phi}{m_\rho} \end{aligned}$$

## Input to fix the lattice spacing

$$a \longleftarrow m_\rho$$
$$a = \begin{cases} 0.0948(34) \text{ fm} & (K\text{-input}) \\ 0.0954(35) \text{ fm} & (\phi\text{-input}) \end{cases}$$

■ At a  $\sim 0.1$  fm

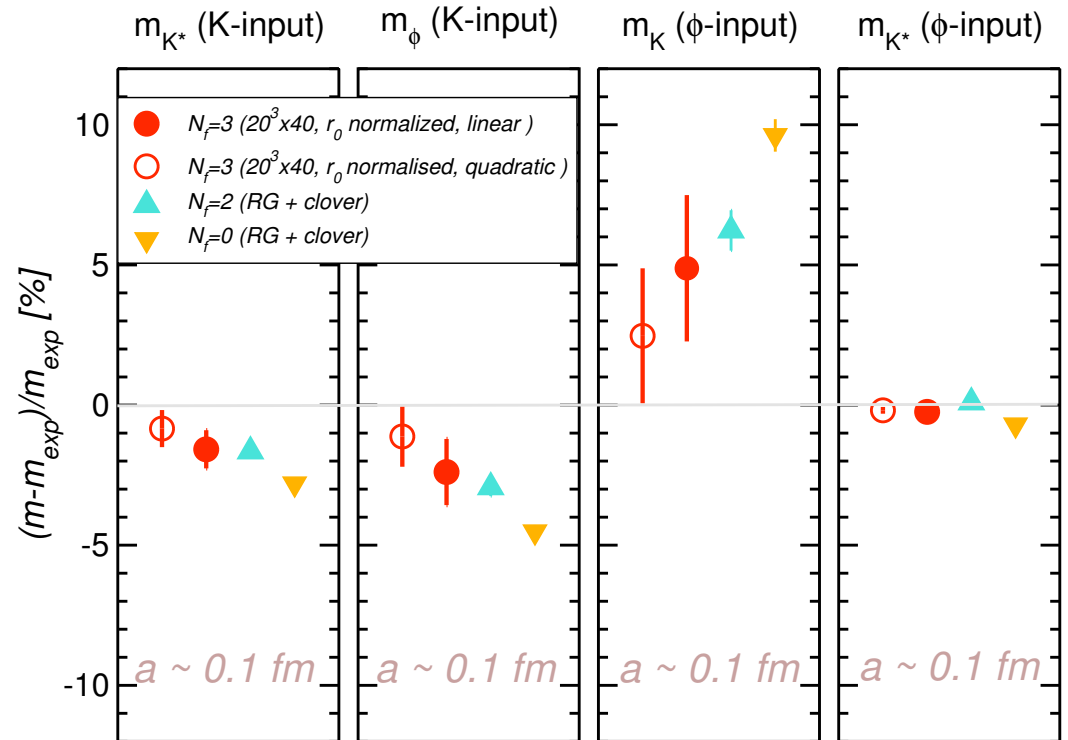
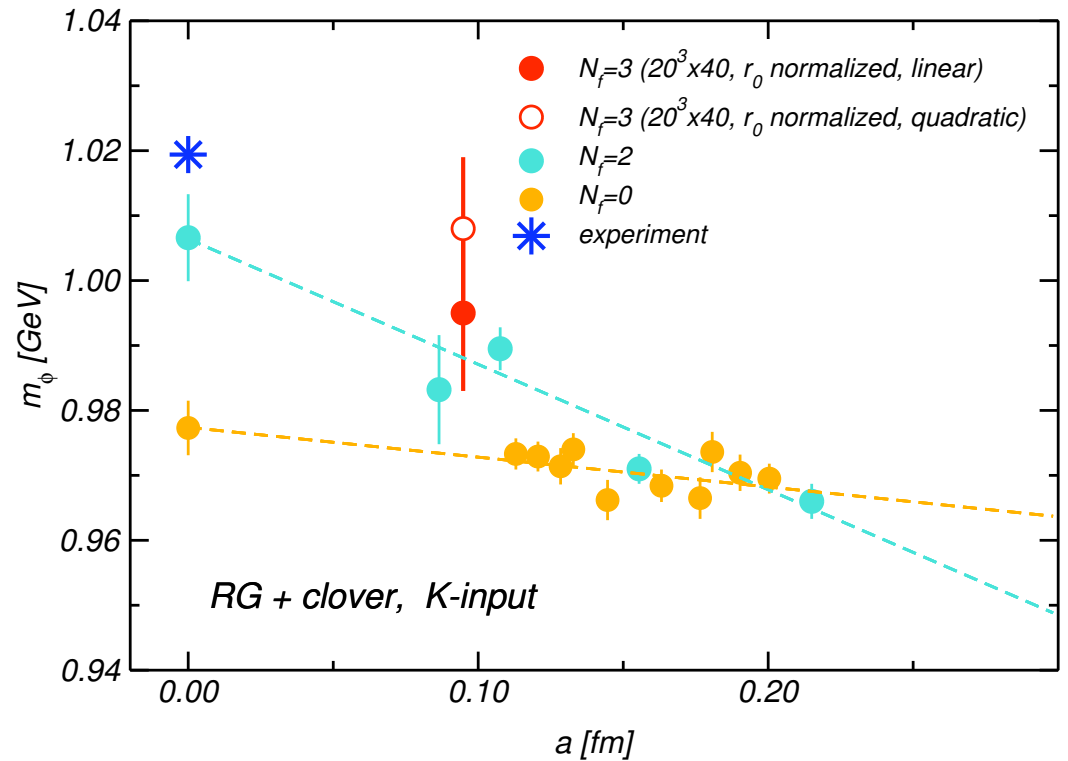
We observe that masses in  $N_f=3$  are closer to experiment than in  $N_f=2$  and  $N_f=0$  at a  $\sim 0.1$  fm.

■ We may expect that

our observation is unchanged in the continuum limit.



This point should be checked in the future study.



# Quark masses (preliminary)

## ■ VWI quark mass

$$m_q = \frac{1}{2} \left( \frac{1}{K} - \frac{1}{K_c} \right)$$

- VWI ud quark mass has **negative** value.

← due to the chiral symmetry breaking

## ■ AWI quark mass ( We use. )

$$m_q = \frac{\langle \Delta_4 A_4(t) P(0) \rangle}{2 \langle P(t) P(0) \rangle}$$

- no such problem as in the VWI quark mass
- The scaling violation is small in  $N_f = 2$  case.

## ■ renormalization

- MF-improved 1-loop matching with  $\overline{\text{MS}}$  at  $\mu = a^{-1}$
- 4-loop running to  $\mu = 2 \text{ GeV}$

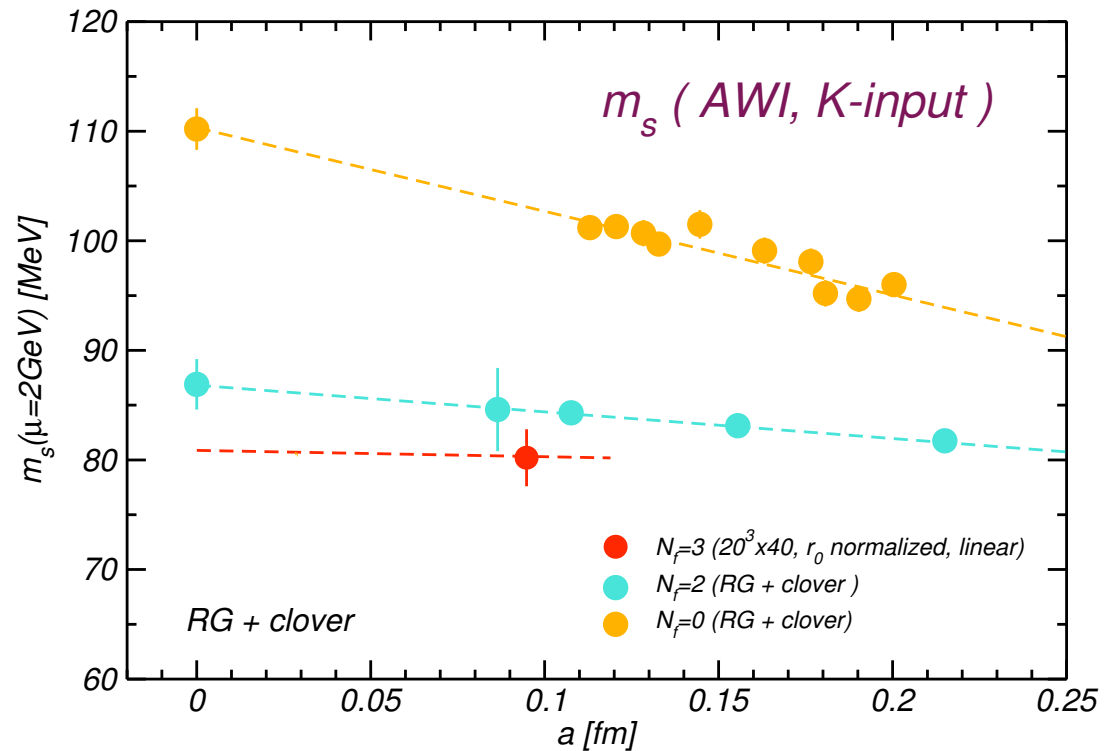
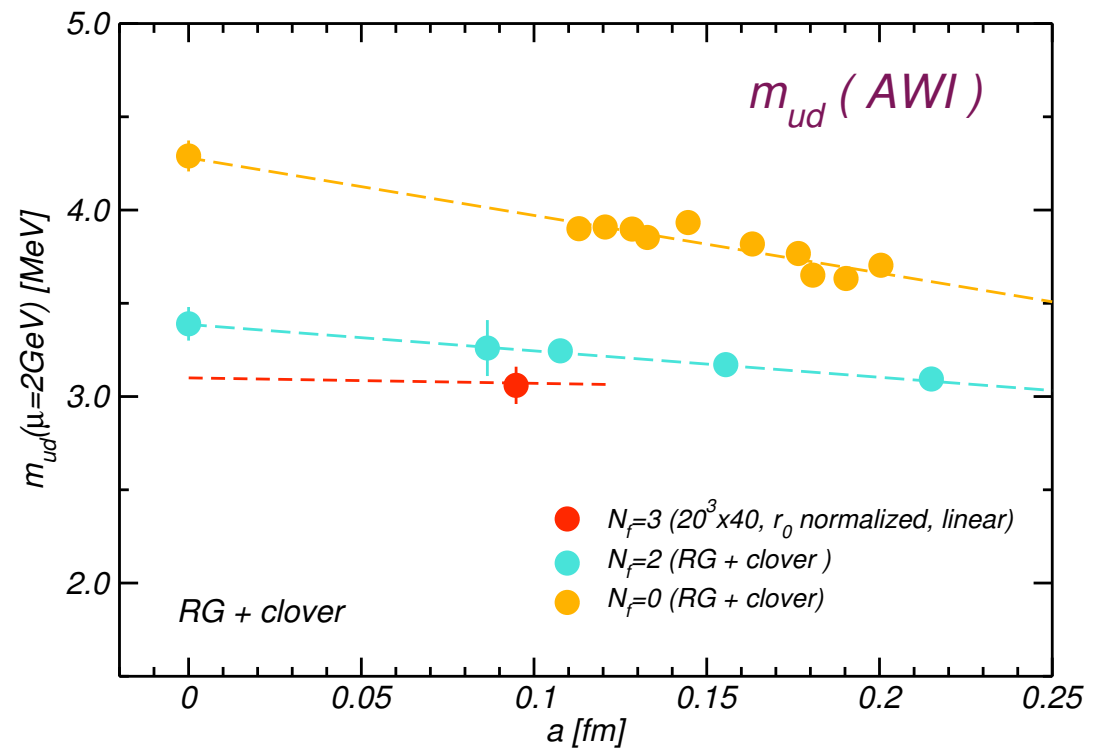


■ Non-perturbatively  
 $\mathcal{O}(a)$  improved  
Wilson quark action



small scaling violation

Assuming that  
the results at  $a \sim 0.1$  fm  
are the same as  
in the continuum limit,  
...



□ Dynamical quarks reduce quark masses.

□  $m_{ud}, m_s$ :

**10 % smaller than in  $N_f = 2$**

**Note: Finite size effect is not observed in quark masses.**

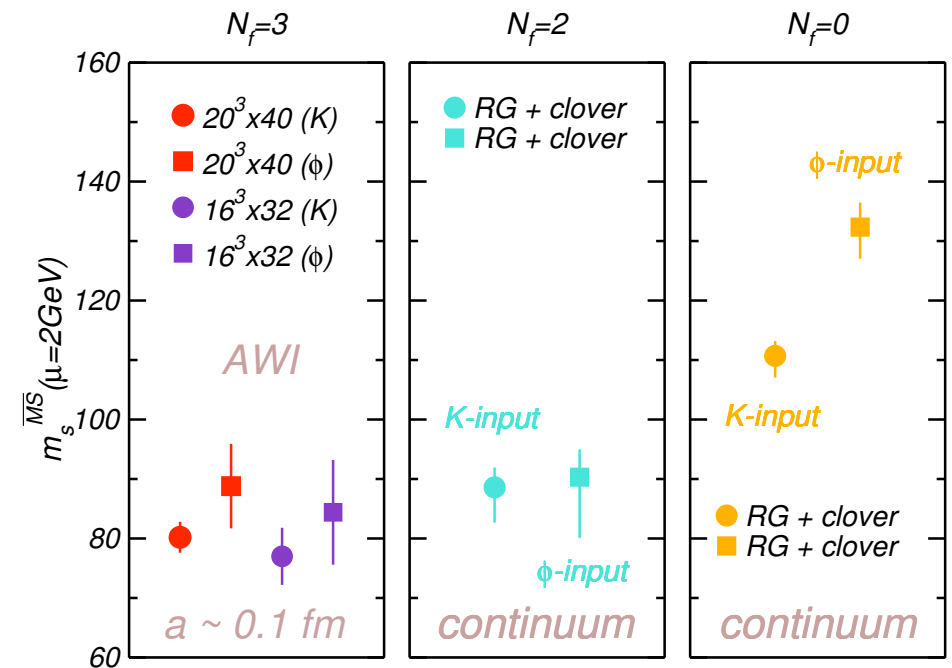
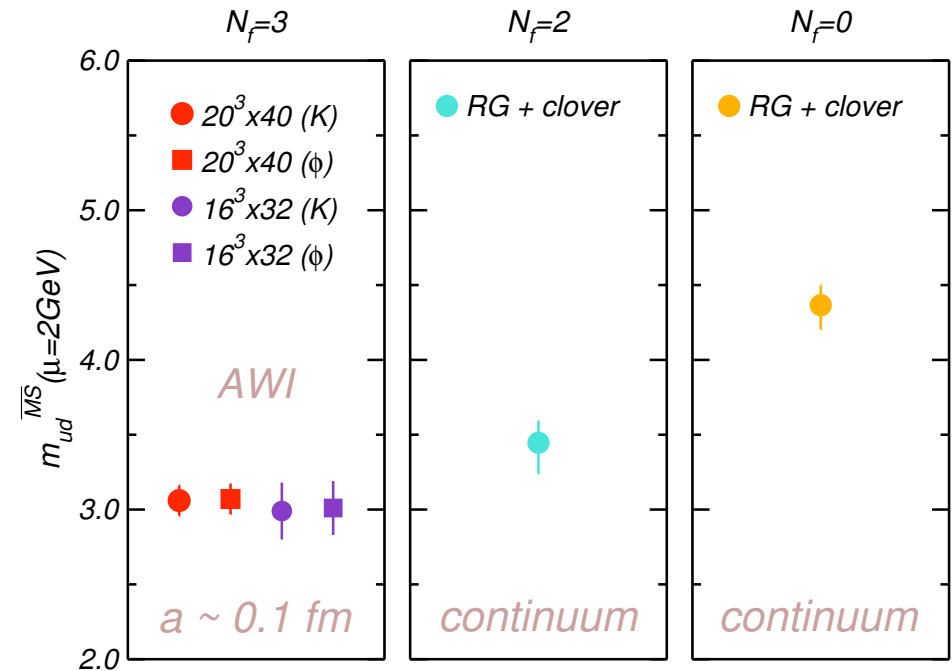
■  **$\overline{\text{MS}}$  scheme at  $\mu = 2 \text{ GeV}$**

$$m_{ud} = 3.06(10)^{+0.03}_{-0.53} \text{ [MeV]}$$

$$m_s = 80.2(2.6)^{+8.6}_{-0.5} \text{ [MeV]}$$

$$m_s/m_{ud} = 26.2(1.2)$$

**(central value: K-input)**



# Summary and future work

- Although our simulation is performed only at one lattice spacing, our result is consistent with the following picture :
  - Light meson spectrum
    - ◆ The result of meson spectrum in  $N_f = 3$  is closer to experiment than in  $N_f = 2$ .
  - Quark mass
    - ◆ Dynamical quarks ( u,d,s) reduce the quark masses.
    - ◆ Quark masses in  $N_f = 3$  is about 10% smaller than in  $N_f = 2$ .
- Simulations are on-going at two other lattice spacings (  $\Rightarrow$  continuum extrapolation )

$$a \sim 0.0707[\text{fm}], \quad L^3 \times T = 28^3 \times 56, \quad (\text{finer lattice})$$

$$a \sim 0.1225[\text{fm}], \quad L^3 \times T = 16^3 \times 32, \quad (\text{coarser lattice})$$