# Soft gluon resummation in transversly polarized Drell-Yan process

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- Introduction
- Soft gluon resummation
- 1-loop results
- Summary

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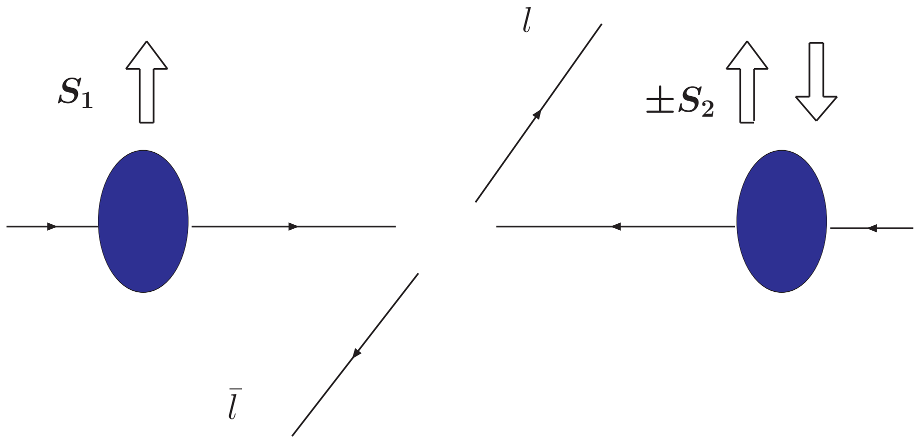
# 1. Introduction

Transversely polarized Drell-Yan

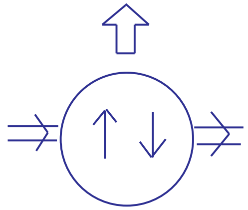
(at RHIC 2001 ∼)

$$P^{\uparrow}P^{\uparrow\downarrow} \to l^{+}l^{-}X$$

$\Rightarrow$ Spin struture of proton




$$A_{TT} = \frac{d\sigma^{p^{\uparrow}p^{\uparrow}} - d\sigma^{p^{\uparrow}p^{\downarrow}}}{d\sigma^{p^{\uparrow}p^{\uparrow}} + d\sigma^{p^{\uparrow}p^{\downarrow}}} \quad \Rightarrow \delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$$

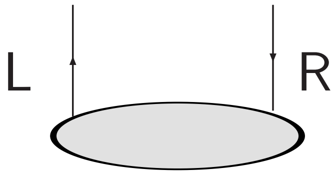


— New parton distribution function (not appear in DIS)

Chiral-Odd distribution

— $\delta q(x) \neq \Delta q(x) = q^{+}(x) - q^{-}(x)$ (longitudinal pol.)

$\langle P,S|\bar{q}i\sigma^{\mu\nu}q|P,S\rangle$ :



## $\boxed{\text{1-loop QCD corrections}}$ Former calculations

### Total cross section

- Massive Gluon Regularization — Vogelsang, Weber ('93)
- Dimentional Reduction — Contogouris et. al. ('94)
- Relation between Schemes — Kamal ('96)

$\longrightarrow$ Dimensional Regularization (indirect) — Vogelsang ('98)

### $q_T$ distributions

- Massive Gluon Regularization — Vogelsang, Weber ('93)

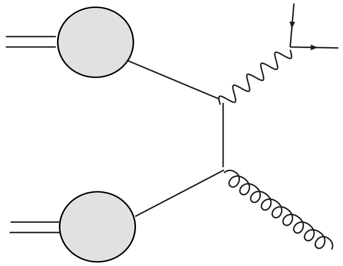
♣ D-dimensional Calculation

Transvers pol. : $d\sigma \propto \cos(2\phi)$

— Phase Space integral keeping azimuthal dependence

No direct calculation so far

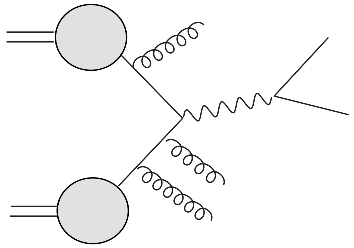
## $q_T$ distribution



- $q_T \sim Q$ : Single scale

  $\Rightarrow$ Perturbation works well.

  (good convergence)



- $q_T \ll Q$ : Double scales

  $\Rightarrow$ $\alpha_s^n \ln^{2n}(Q^2/q_T^2)$ spoil perturbation.

  $\rightarrow$ all order resummation

$*$ phase space boundary $\Rightarrow$ incomplete cancellation of soft contributions (real vs. virtual)

♣ We calculated 1-loop corrections in dim. reg.

$\rightarrow$ matched to resummation formula (formulated in $\overline{MS}$)

$\rightarrow$ obtained $d\sigma/dq_T$ reliable in both large $q_T^2 \sim Q^2$ and small $q_T^2 \ll Q^2$ region.

# 2. Soft gluon resummation

## What is resummation ?

$$\frac{d\sigma}{dq_T^2 dQ^2} \sim \frac{1}{q_T^2}\left[\alpha_s(L+1) + \alpha_s^2(L^3+L^2+L+1) + \alpha_s^3(L^5+L^4+\cdots+1) + \cdots\right]$$

- when $\alpha_s, \quad \alpha_s L^2 \ll 1 \quad (q_T \sim Q) \longrightarrow$ Truncate at a Fixed Order
- when $\alpha_s \ll 1, \alpha_s L^2 \sim 1 \quad (q_T \ll Q) \longrightarrow$ Resum $(\alpha_s L^2)^n$ terms

$$\begin{aligned}
\frac{d\sigma}{dq_T^2 dQ^2} \quad &\sim \quad \frac{1}{q_T^2}\left[\, \alpha_s L + \alpha_s^2 L^3 + \alpha_s^3 L^5 + \cdots\right] && \text{Leading Log (LL)} \\
&+ \quad \frac{1}{q_T^2}\left[\, \alpha_s + \alpha_s^2 L^2 + \alpha_s^3 L^4 + \cdots\right] && \text{Next to Leading Log (NLL)} \\
&+ \quad \cdots \\
&+ \quad \frac{1}{Q^2}\left[\, \alpha_s + \alpha_s^2 + \alpha_s^3 + \cdots\right] && \text{Finite Terms}
\end{aligned}$$

1-loop explicit calculation $\longleftrightarrow$ NLL resummation

## Resummation formula *à la* CSS Collins.Soper.Sterman ('85)

- Momentum conservation $\longrightarrow$ Impact Parameter space : b

$$\delta(\vec{q}_T - \vec{k}_{1,T} - \vec{k}_{2,T} - \cdots) = \int dbe^{i\vec{b}\cdot(\vec{q}_T - \vec{k}_{1,T} - \vec{k}_{2,T} - \cdots)}$$

- Factorization & RG eq.

$$\frac{d\sigma}{dQ^2 dq_T^2 dy d\phi} = N \left\{ \frac{1}{4\pi} \int d^2b \; e^{i\,\vec{b}\cdot\vec{q}_T} \; e^{S(b,Q)} R(b,Q,y) + Y(q,Q,y) \right\}$$

Log terms in $S(b,Q)$ Finete terms

$$S(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{d\kappa^2}{\kappa^2} \left\{ \ln\frac{Q^2}{\kappa^2} A_c(\alpha_s(\kappa^2)) + B_c(\alpha_s(\kappa^2)) \right\}$$

$$R(b,Q) = \sum_{a,b,c} C_{ca} \otimes f_a \left( x_1^0, \frac{b_0^2}{b^2} \right) \cdot C_{\bar{c}b} \otimes f_b \left( x_2^0, \frac{b_0^2}{b^2} \right)$$

$C_{ca}$ : Coeff. function $f_a$ : Parton Distribution Function

# Matching at NLL

(1) Calculate 1-loop corrections.

(2) Expand resummation formula at NLL level.

$$S(b,Q) \sim -\int_{b_0^2/b^2}^{Q^2} \left\{ \ln\left(\frac{Q^2}{\kappa^2}\right) \left[A^{(1)}\alpha_s(\kappa^2) + A^{(2)}\alpha_s^2(\kappa^2)\right] + B^{(1)}\alpha_s(\kappa^2) \right\}$$

Coefficients : $A^{(1)}, A^{(2)}, B^{(1)}$ Universal $\leftrightarrow$ Splitting function

LL : $A^{(1)} = C_F$ Dokshitzer et. al.('78)

NLL : $A^{(2)} = \frac{1}{2} C_F \left[ \left( \frac{67}{18} - \frac{\pi^2}{6} \right) C_G - \frac{5}{9} N_f \right]$ Kodaira, Trentadue ('82)

$B^{(1)} = -\frac{3}{2} C_F$ Davies, Stirling, Webber ('85)

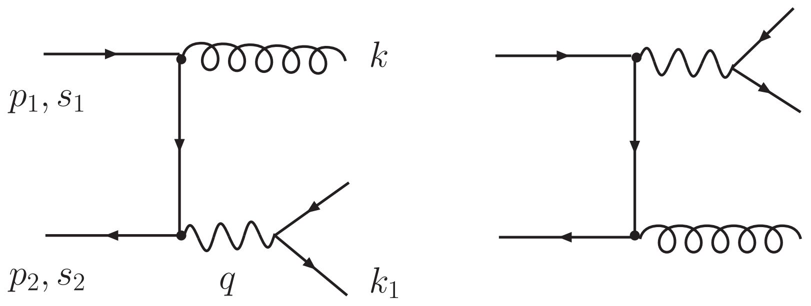
$$C_{ab}(z,\alpha) = \delta_{ab}\delta(1-z) + \frac{\alpha_s}{2\pi} C_{ab}^{(1)}(z)$$

(3) Compare with explicit 1-loop results and extract $C_{ab}(z)$ and $Y(q_T, Q, y)$. $\longrightarrow$ Final result

# 3. 1-loop result

$$\Delta_T d\sigma \equiv 1/2[d\sigma(s_1, s_2) - d\sigma(s_1, -s_2)]$$

## Real emission



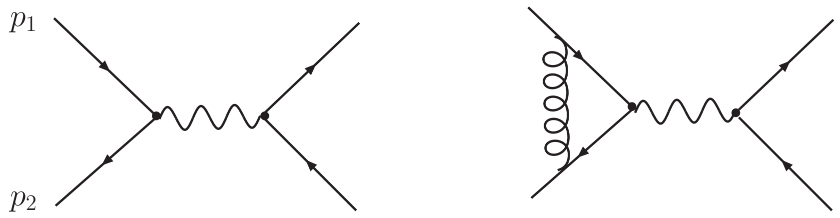

$$q(p_1, s_1) + \bar{q}(p_2, s_2) \rightarrow l(k_1) + \bar{l}(k_2) + g(k)$$
$$(q = k_1 + k_2)$$

- Integration with $\phi_{k_1}$ being kept

$$\int (\text{amplitude})\, d\Omega_q^{D-2} d\cos\theta_{k_1}^1 \longrightarrow \text{Difficult in general}$$

— only for $(D \neq 4, q_T^2 = 0)$, $(D = 4, q_T^2 \neq 0)$ necessary!!

## Virtual corrections




- soft div. cancel with real emission
- collinear div. factorized into PDF

$\boxed{\text{Results}}$ $X$ : singular at $q_T^2 = 0$, $Y$: finite at $q_T^2 = 0$

$$\frac{\Delta_T d\sigma^H}{dQ^2 dq_T^2 dy d\phi_{k_1}} = N \cos(2\phi_{k_1} - \phi_1 - \phi_2) \left[ X(q_T^2, Q^2, y) + Y(q_T^2, Q^2, y) \right]$$

$$\begin{aligned}
X^{(0)} &= H(x_1^0, x_2^0)\, \delta(q_T^2) \\
X^{(1)} &= \frac{\alpha_s}{2\pi} C_F \left\{ H(x_1^0, x_2^0) \left[ 2 \left( \frac{\ln Q^2/q_T^2}{q_T^2} \right)_+ - \frac{3}{(q_T^2)_+} + \left( -8 + \pi^2 \right) \delta(q_T^2) \right] \right. \\
&\quad + \left( \frac{1}{(q_T^2)_+} + \delta(q_T^2) \ln \frac{Q^2}{\mu_F^2} \right) \\
&\quad \left. \times \left[ \int_{x_1^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) H\left( \frac{x_1^0}{z}, x_2^0 \right) + \int_{x_2^0}^1 \frac{dz}{z} \Delta_T P_{qq}(z) H\left( x_1^0, \frac{x_2^0}{z} \right) \right] \right\}
\end{aligned}$$

Splitting Function : $\Delta_T P_{qq}(z) = \frac{2z}{(1-x)_+} + \frac{3}{2}\delta(1-z)$ Artru & Mekhi ('90)

$$H(x_1, x_2) = \sum_i e_i^2 [\delta q_i(x_1) \delta \bar{q}_i(x_2) + (1 \leftrightarrow 2)]$$

$x_1^0,\ x_2^0$ : Functions of $(Q^2,\ q_T^2,\ y)$

# Finite terms

$$Y\left(q_T^2\,,\,Q^2\,,\,y\right)=\frac{\alpha_s}{\pi}C_F$$

$$\times\left\{\frac{1}{q_T^2}\left[\int dx_1 H_1+\int dx_2 H_2+H(x_1^0\,,\,x_2^0)\,\ln\frac{(1-x_1^+)(1-x_2^+)}{(1-x_1^0)(1-x_2^0)}\right]\right.$$

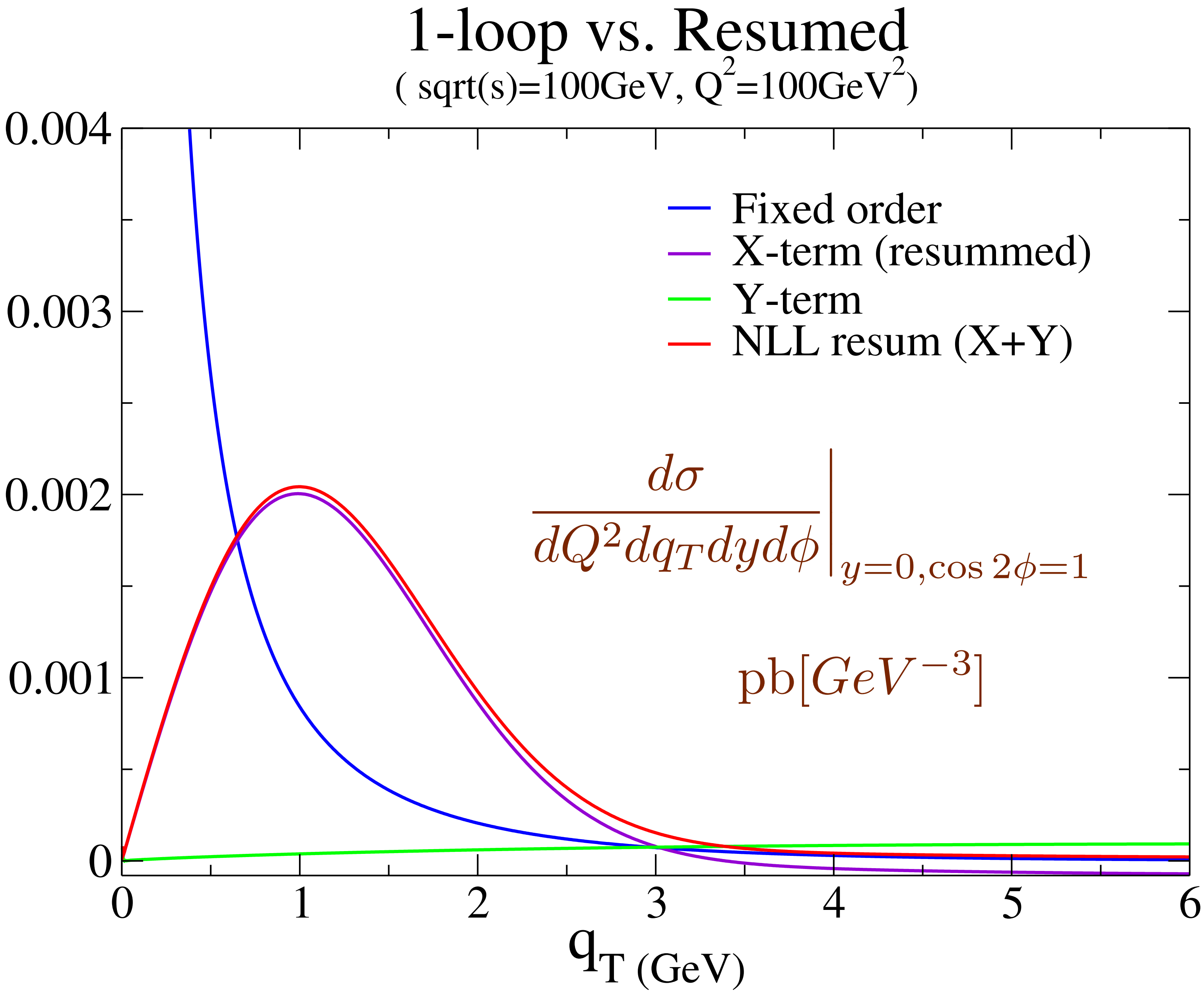
$$+\left[\frac{2}{Q^2}-\frac{3}{q_T^2}\ln\left(1+\frac{q_T^2}{Q^2}\right)\right]$$

$$\left.\times\left[\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{x_1-x_1^+}H(x_1\,,\,x_2^*)\,\frac{\tau}{x_1x_2^*}+\int_{\sqrt{\tau_+}\,e^{-y}}^{1}\frac{dx_2}{x_1-x_2^+}H(x_1^*\,,\,x_2)\,\frac{\tau}{x_1^*x_2}\right]\right\}$$

$$\int dx_1 H_1=\int_{\sqrt{\tau_+}\,e^{y}}^{1}\frac{dx_1}{x_1-x_1^+}\left[H(x_1,x_2^*)\frac{\tau}{x_1x_2^*}-H(x_1^0,x_2^0)\right]-(q_T^2\to 0)$$

All terms are finite at $q_T^2=0$ $(x_{1,2}^+=x_{1,2}^*=x_{1,2}^0)$

Numerical Results $\sqrt{S} = 100$ GeV, $Q$=10$GeV$



# 4. Summary

(1) Transversly polarized DY process

$\longrightarrow$ Chiral-odd distribution $\delta q(x)$

(2) We calculated 1-loop corrections to total cross section and $q_T$ distribution in dimensional regularization.

— more complicated than unpolarized case.

— reproduced the known result for total cross section.

(3) By matching to resummation formula, we obtained results reliable in both $q_T \sim Q$ and $q_T \ll Q$ region.

$$\sim N \cos\left(2\phi_{k_1} - \phi_1 - \phi_2\right) \left\{ \frac{1}{4\pi} \int d^2 b \; e^{i\,\vec{b}\cdot\vec{q_T}} \; e^{S(b,Q)} R(b,Q) + Y\left(q_T^2, Q^2, y\right) \right\}$$