# Intersecting D-brane Models

-- a brief introduction --

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# 1. Introduction

## Why particle models in string theory?

1. Inclusion of quantum gravity.

   a unified description of all interactions

2. Derivation of coupling constants.

   Many interactions in the Standard Model:

   Gauge interactions,
   Yukawa interactions.

   These couplings are just parameters, and hard to derive within the framework of the quantum field theory.

# Yukawa interactions in field theory models
# -- two typical scenarios --

1. Grand Unified Theories

Unified Yukawa coupling constant(s) at high energies.

↓ many Higgs fields, or
higher-dimensional interactions

Hierarchical Yukawa coupling constants at low energies.

2. Composite models of Higgs and/or quarks and leptons

No Yukawa couplings at high energies.

(ex. technicolor theory: $\Phi \bar{q} q \leftarrow \bar{T} T \bar{q} q / M_{\rm ETC}^2$ )

↓ complicated dynamics
(ex. extended technicolor dynamics)

Hierarchical Yukawa coupling constants among composite particles.

String theory can give understandings beyond field theory models, since

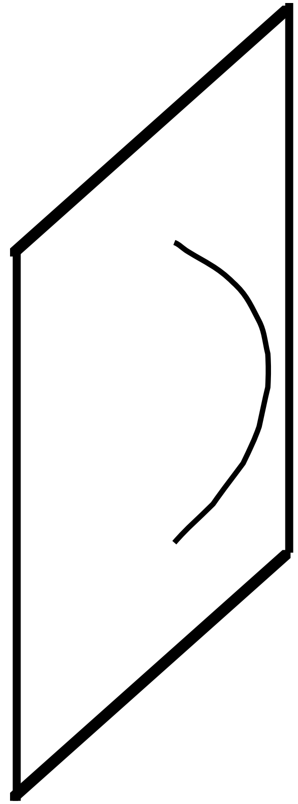
Yukawa interactions,
higher-dimensional interactions,
and gauge interactions

are calculable, in principle.

It also gives a concrete background to other fields theory models with extra dimensions.

# 2. Intersecting D-branes

Dp-brane: a p-dimensional object in string theory
on which the ends of open string are fixed



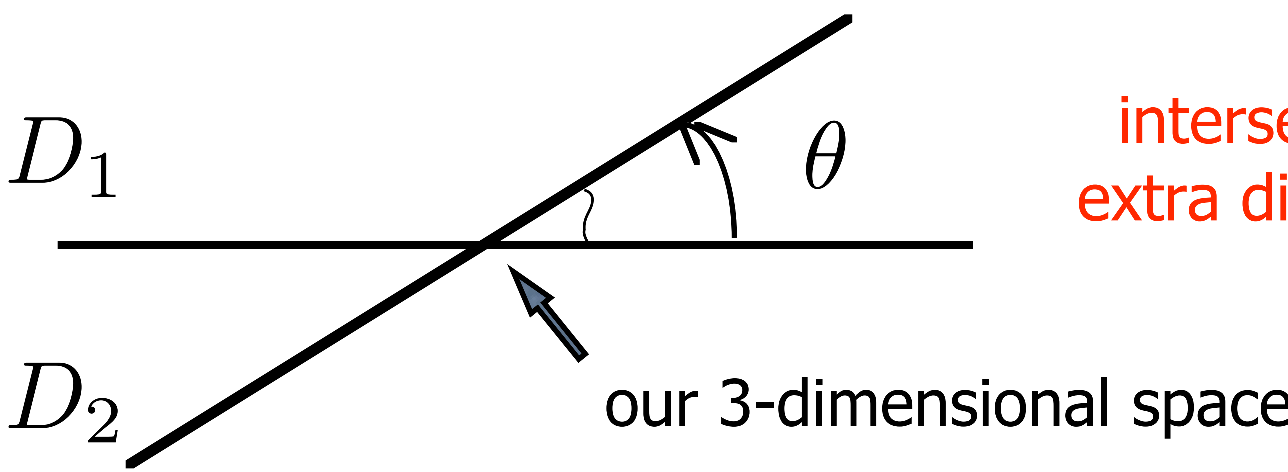
U(1) gauge symmetry
for a single D-brane.

U(N) gauge symmetry
for N D-brane
(multiplicity N)

The gauge field is localized on the
(p+1)-dimensional world-volume.

# Intersecting D6-branes

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 10D: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| D6 : | 0 | 1 | 2 | 3 | 4 | | 5 | | 6 | |
| | | | | | $\theta_1$ | | $\theta_2$ | | $\theta_3$ | |

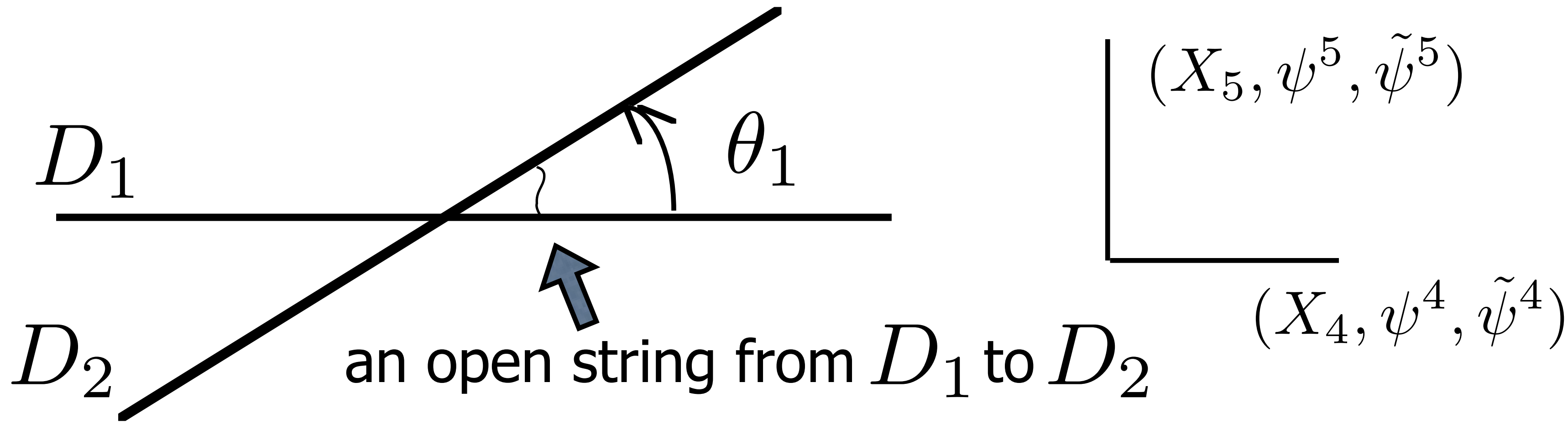



intersecting in extra dimensions

open string localized at the intersection point (3D space)
chiral fermion localized at the intersecting point (3D space)

Berkooz-Douglas-Leigh, hep-th/9606139

# mechanism for chiral fermion (very briefly)




at $\sigma = 0$

$$\begin{cases} \psi^4 = e^{-2\pi i \nu} \tilde{\psi}^4 & \text{Neumann} \\ \psi^5 = -e^{-2\pi i \nu} \tilde{\psi}^5 & \text{Dirichlet} \end{cases}$$

$\nu = 0, 1/2$ (R, NS)

at $\sigma = \pi$

$$\begin{cases} \cos\theta_1 \psi^4 + \sin\theta_1 \psi^5 = \cos\theta_1 \tilde{\psi}^4 + \sin\theta_1 \tilde{\psi}^5 \\ -\sin\theta_1 \psi^4 + \cos\theta_1 \psi^5 = -(-\sin\theta_1 \tilde{\psi}^4 + \cos\theta_1 \tilde{\psi}^5) \end{cases}$$

$$\alpha_i \equiv |\theta_i|/\pi, \quad 0 \le \alpha_i \le 1/2$$

NS-sector states $(\nu = 1/2)$

$$\psi_{\alpha_1 - 1/2 - n}|0\rangle_{\rm NS}, \qquad n \ge 0$$

massive, massless, tachyonic

$$m^2 = -(\alpha_1 - \frac{1}{2} - n) + \frac{1}{2}(-1 + \alpha_1 + \alpha_2 + \alpha_3) = \frac{1}{2}(-\alpha_1 + \alpha_2 + \alpha_3) + n$$

$$\tilde{\psi}_{-\alpha_1 - 1/2 - n}|0\rangle_{\rm NS}, \qquad n \ge 0$$

massive

$$m^2 = -(-\alpha_1 - \frac{1}{2} - n) + \frac{1}{2}(-1 + \alpha_1 + \alpha_2 + \alpha_3) = \frac{1}{2}(2\alpha_1 + \alpha_2 + \alpha_3) + n$$

R-sector states $(\nu = 0)$

$$|{\rm s}^{\rm 4D}_{+}\rangle_{\rm R}$$

massless chiral fermion

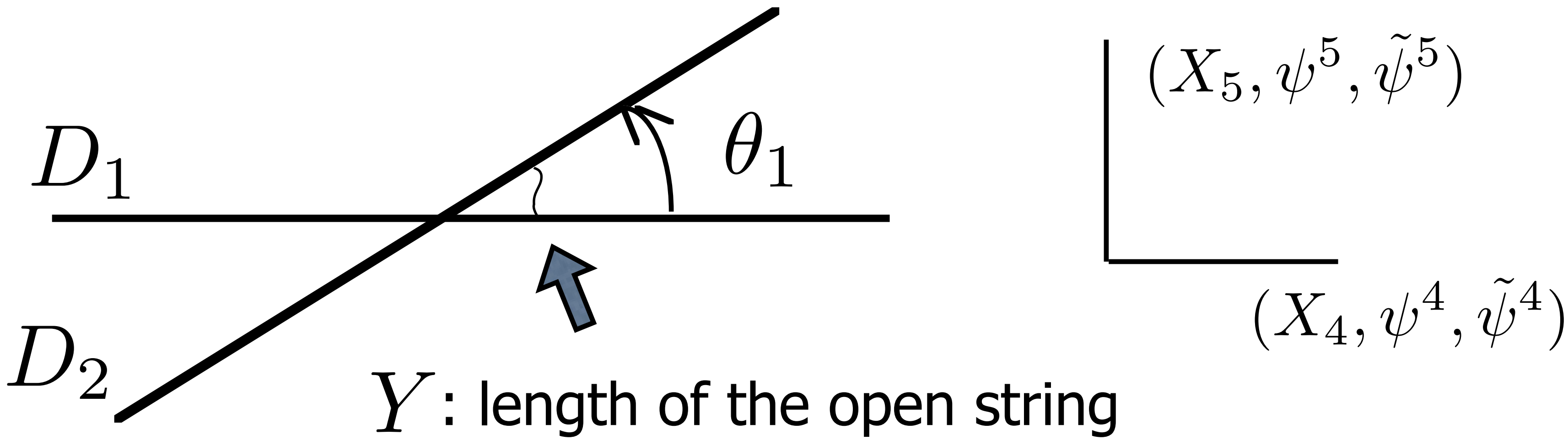
$$\psi_{\alpha_i - 1 - n}|{\rm s}^{\rm 4D}_{-}\rangle_{\rm R}, \qquad n \ge 0 \qquad m^2 = 1 - \alpha_i + n > 0$$

massive

$$\psi_{-\alpha_i - n}|{\rm s}^{\rm 4D}_{-}\rangle_{\rm R}, \qquad n \ge 0 \qquad m^2 = \alpha_i + n > 0$$

massive

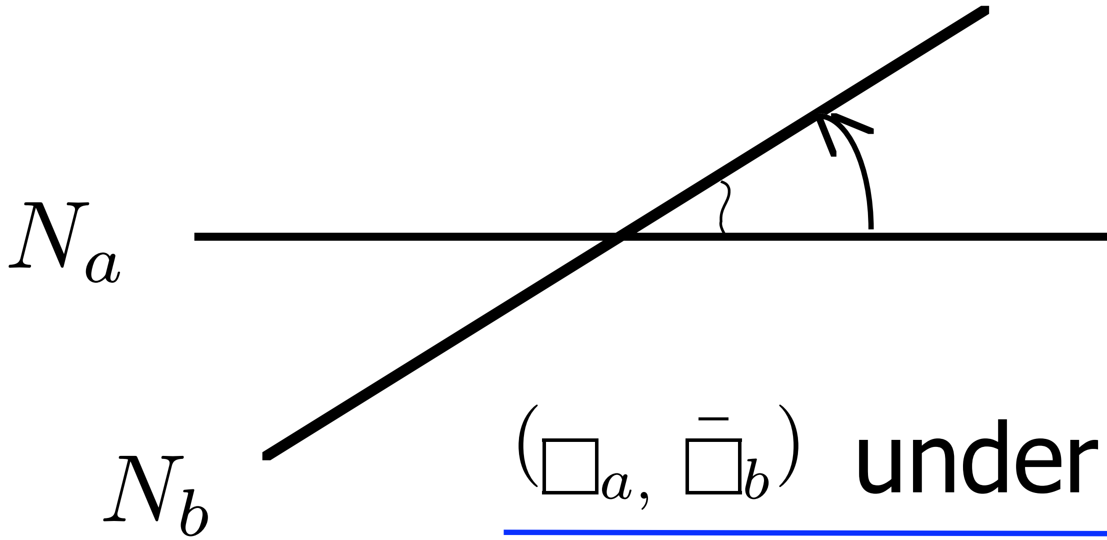
# Localization at the intersecting point




$$\psi_{\alpha_1 - 1/2 - n} |0\rangle_{\text{NS}}, \qquad n \geq 0$$

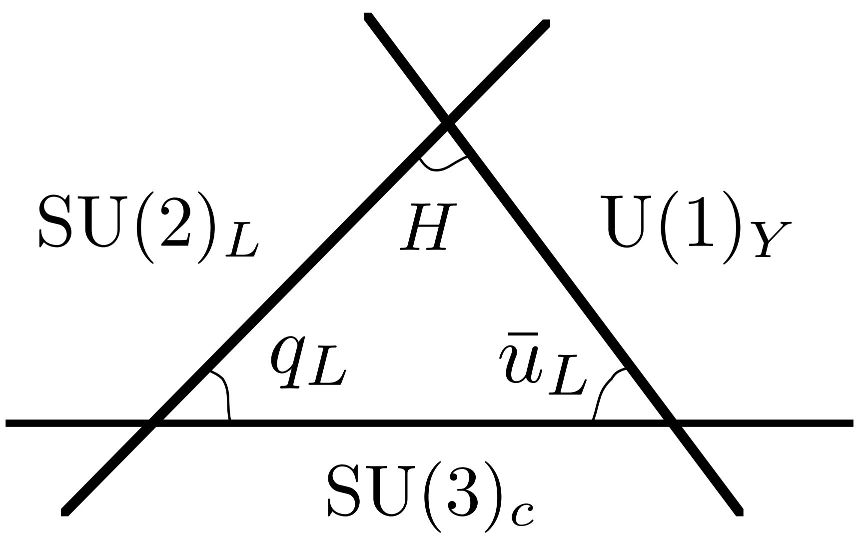
$$\alpha' m^2 = \frac{Y^2}{4\pi^2 \alpha'} + \frac{1}{2}(-\alpha_1 + \alpha_2 + \alpha_3) + n$$

# representation of chiral fermions



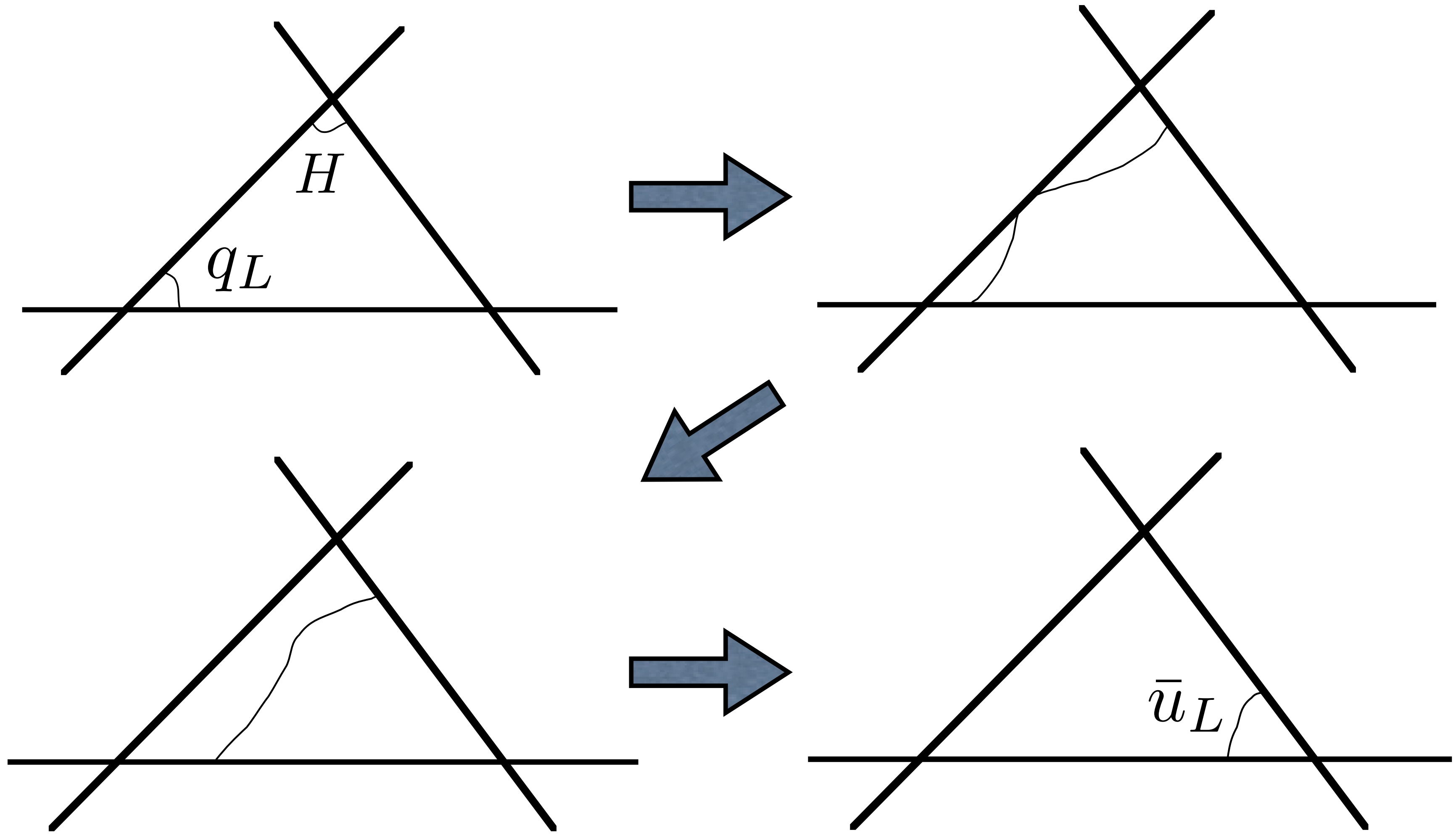

$(\Box_a, \bar{\Box}_b)$ under $\mathrm{U}(N_a) \times \mathrm{U}(N_b)$

# intersecting D-branes




the Standard Model
on intersecting D-branes?

# Yukawa couplings




the strength is geometrically determined: $g_Y \simeq e^{-\frac{A}{2\pi\alpha'}}$

But, not so simple.....

There are constrains on D-brane configurations.

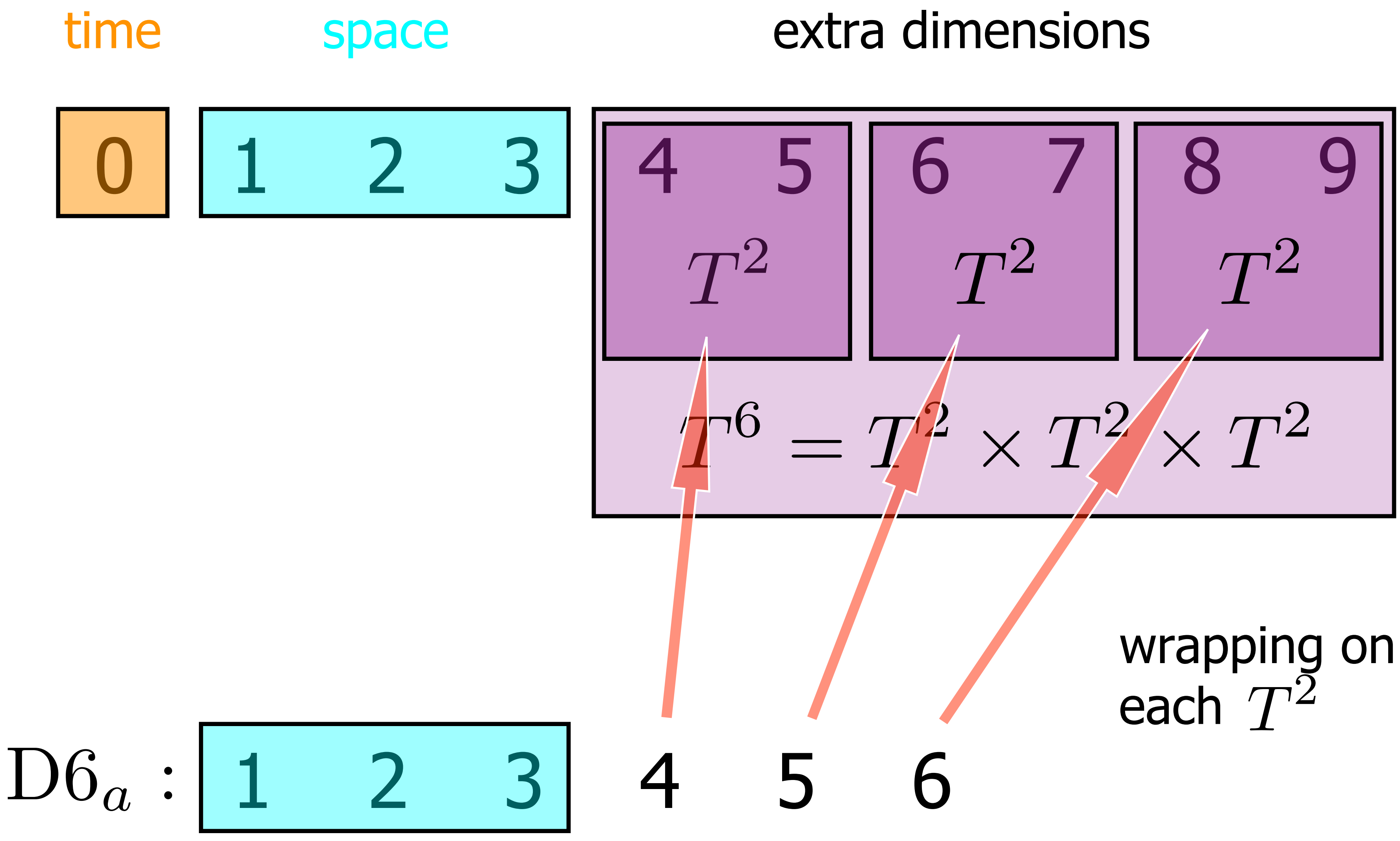
1) supersymmetry conditions

not necessary, but for stable configurations

2) RR tadpole cancellation conditions

for a consistent string theory (Gauss law)
and 4-dimensional Lorentz invariance

# Let's consider a more concrete set up: intersecting in $T^6 = T^2 \times T^2 \times T^2$



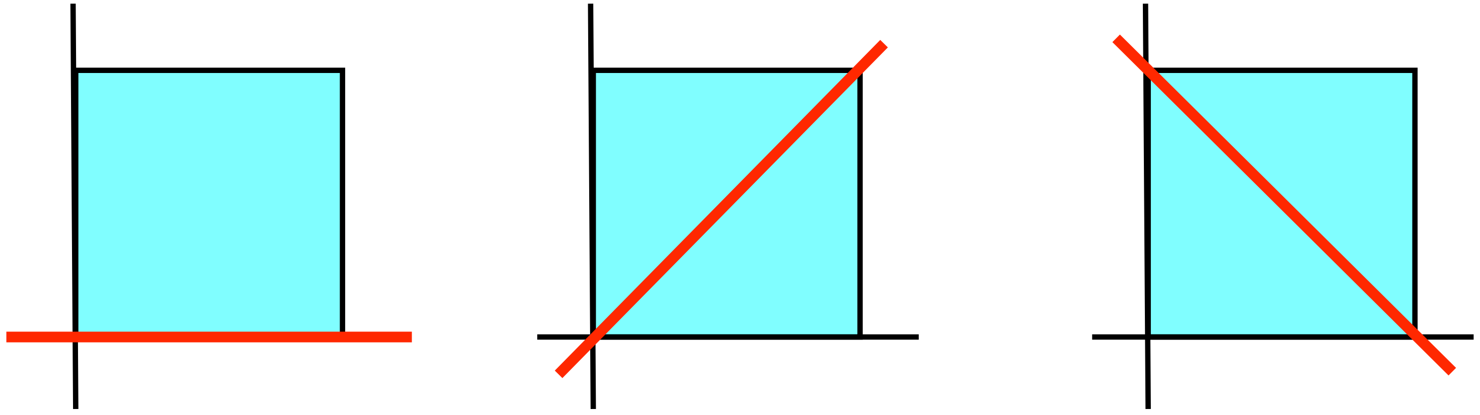
# $\mathrm{D6}_a$-brane wrapping on $T^6 = T^2 \times T^2 \times T^2$

wrapping numbers

$$[(n_a^1, m_a^1), (n_a^2, m_a^2), (n_a^3, m_a^3)]$$

$$T^2 \qquad T^2 \qquad T^2$$

ex.



$(1, 0)$ $(1, 1)$ $(1, -1)$

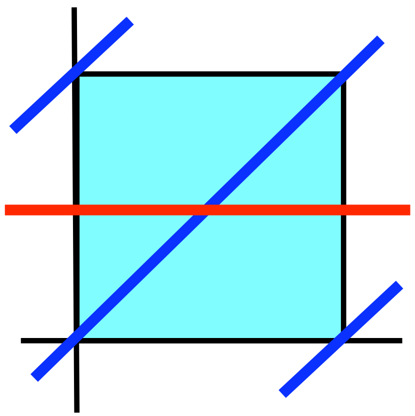
# intersection numbers

number of intersections between $\mathrm{D6}_a$ and $\mathrm{D6}_b$ branes

$$I_{ab} = \prod_{i=1}^{3} \left( n_a^i m_b^i - m_a^i n_b^i \right)$$

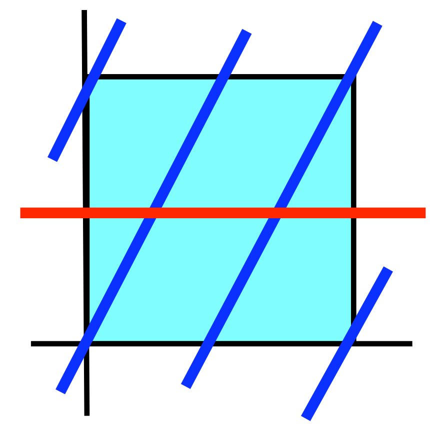
$I = 1$

$(1,0) \times (1,1)$



$I = 2$

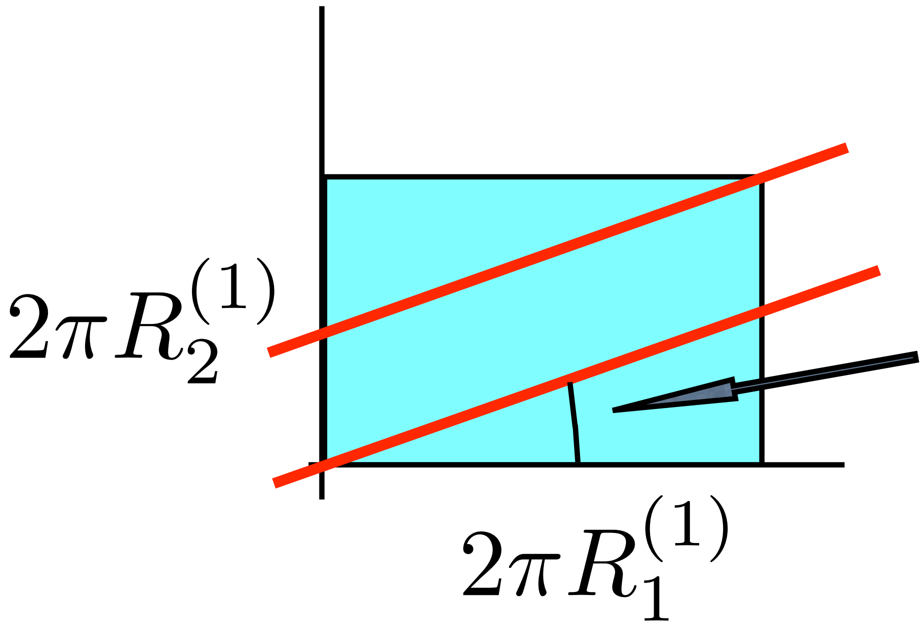
$(1,0) \times (1,2)$



> If two brans are parallel in some of the three torus, the intersecting number becomes zero.

> Multiple intersecting number means multiple fields with same gauge charges.

# supersymmetry conditions




$$\theta_1^a = \tan^{-1}\left(\frac{R_2^{(1)} m_a^1}{R_1^{(1)} n_a^1}\right)$$

$$\theta_1^a + \theta_2^a + \theta_3^a = 0$$

for all $\mathrm{D6}_a$-branes

constraints on winding numbers

Berkooz-Douglas-Leigh, hep-th/9606139

# tadpole cancellation conditions

ex: a construction of type I superstring theory

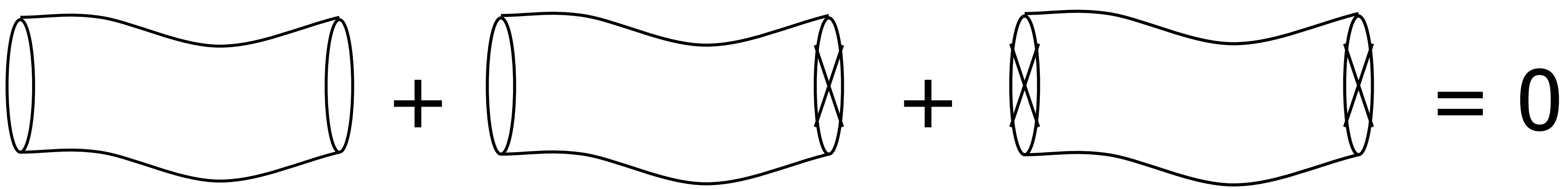
type I = unoriented projection of type IIB
+
32 D9-branes

type I theory: theory of open and closed strings
with N=1 SUSY in 10 dimensions

type IIB theory: theory of closed strings
with N=2 SUSY in 10 dimensions

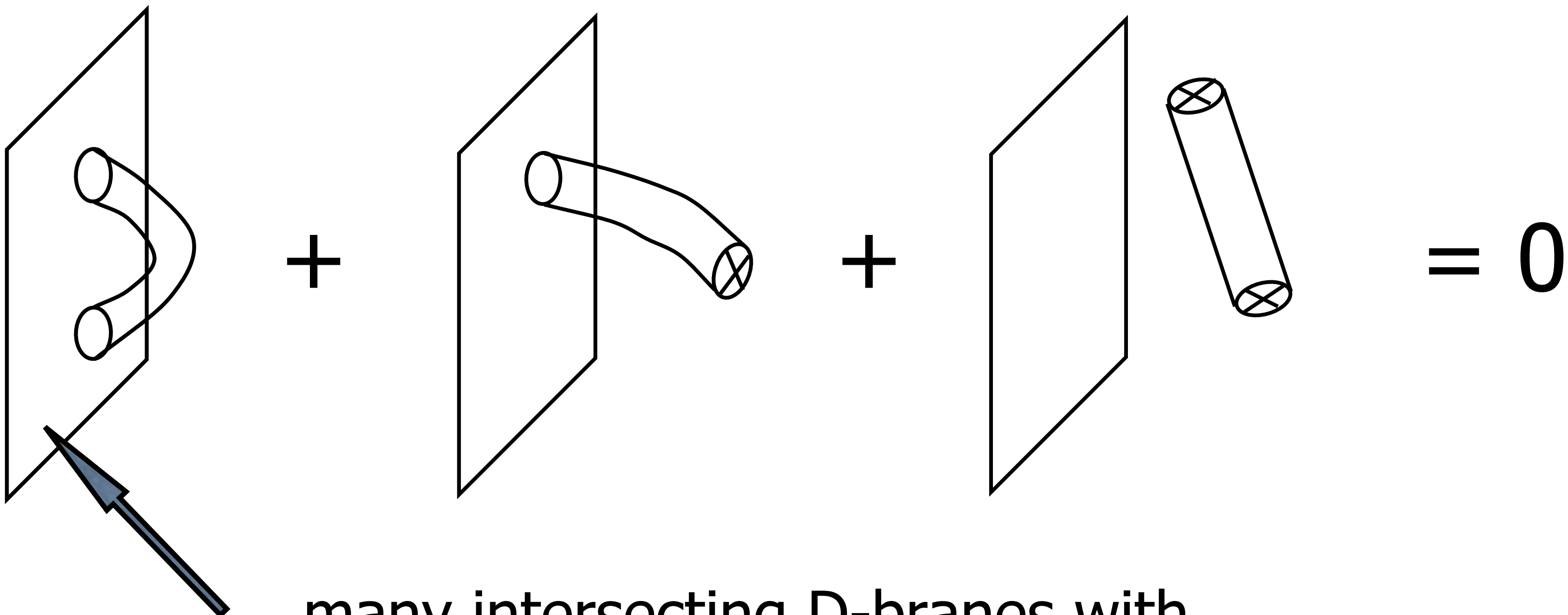
The tadpole cancellation condition determines the number of D9-branes = 32

open string one-loop vacuum diagrams




for no vacuum-to-vacuum amplitudes of closed-string Ramond-Ramond tensor fields (assuming open-closed string duality)

# RR tadpole cancellation conditions in intersecting D-brane systems




many intersecting D-branes with
different multiplicities and winding numbers

constraints on the multiplicities and winding numbers of D-branes

# RR tadpole cancellation conditions

$$\sum_a N_a n_a^1 n_a^2 n_a^3 = 16,$$

$$\sum_a N_a n_a^1 m_a^2 m_a^3 = -16,$$

$$\sum_a N_a m_a^1 n_a^2 m_a^3 = -16,$$

$$\sum_a N_a m_a^1 m_a^2 n_a^3 = -16,$$

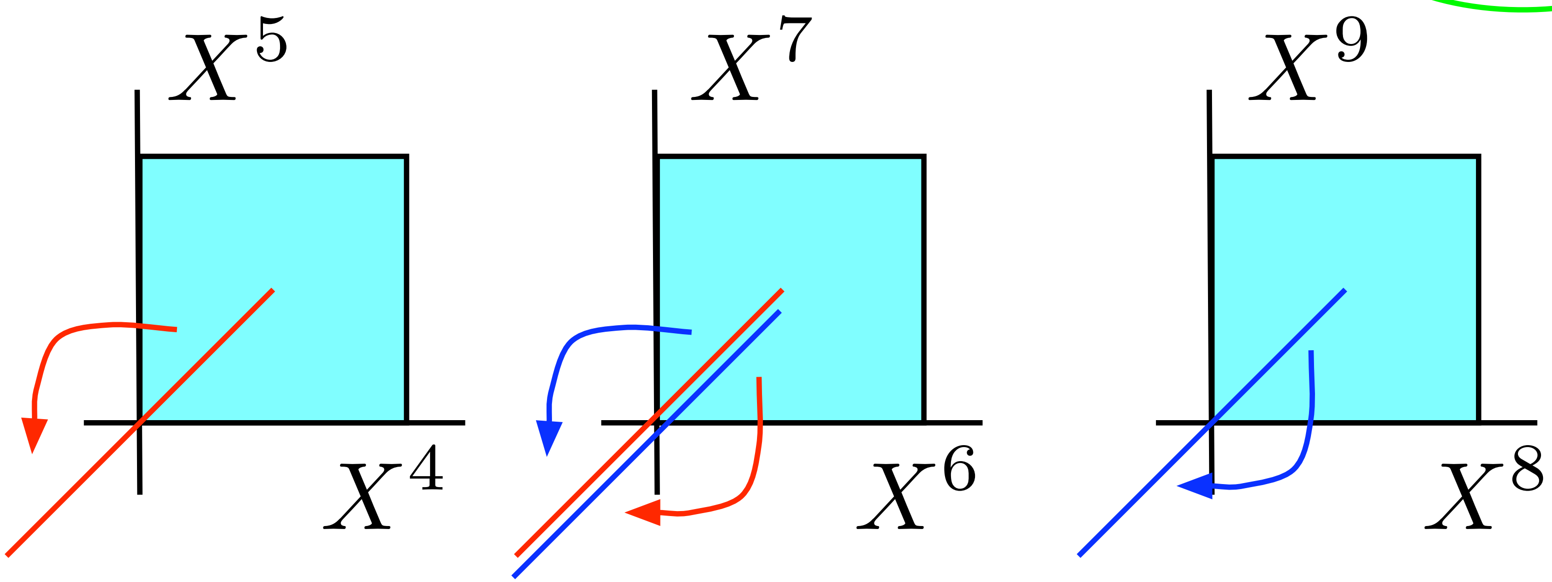
Gauss law for RR charge in compact

$T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ orientifold

Cvetic-Shiu-Uranga, hep-th/0107166

## $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ orientifold in type IIA theory

$T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ orbifold projections $\theta$ and $\omega$



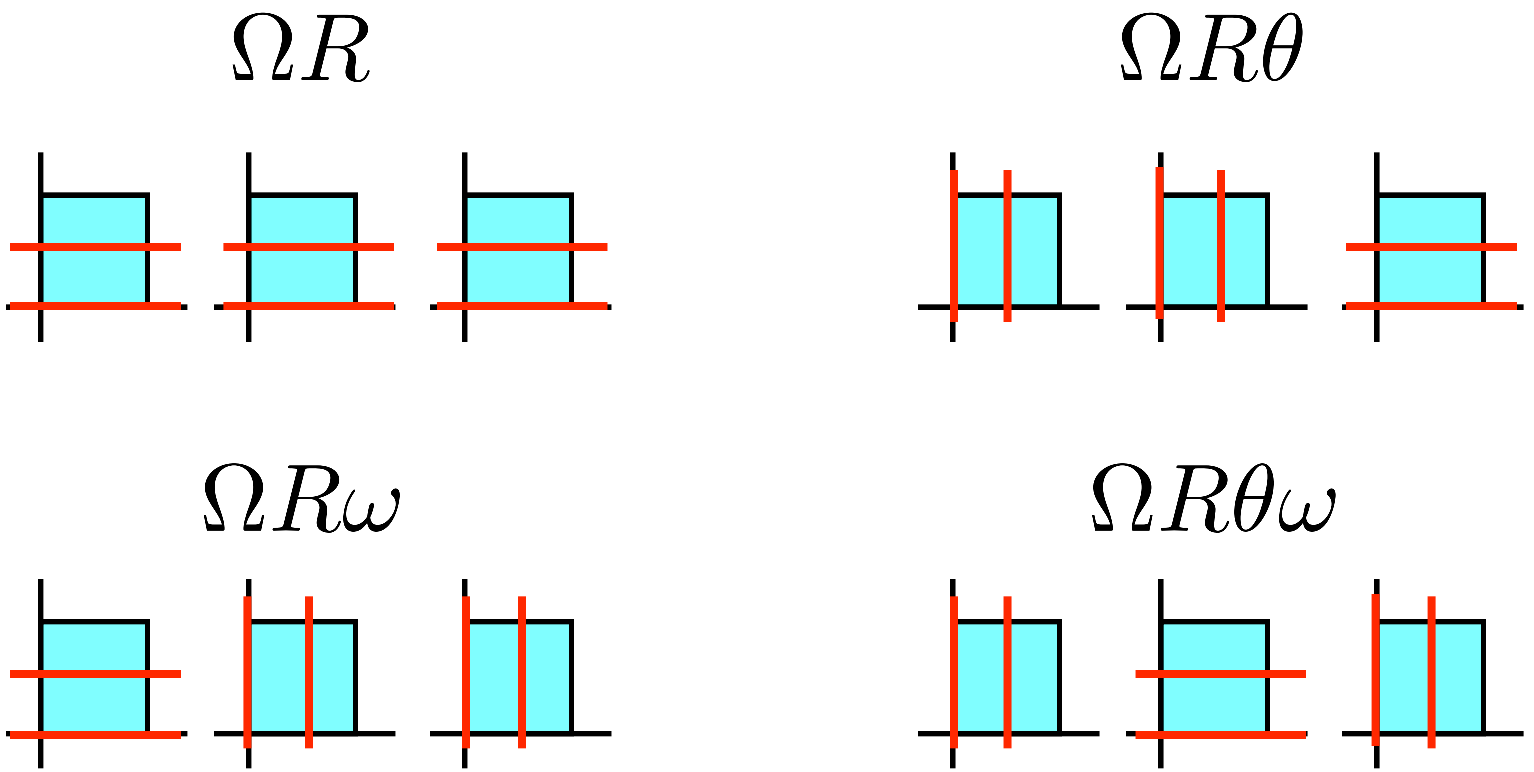

Orientifold projection $\Omega R$

$\Omega$ : world-sheet parity

$R: \; X^{5,7,9} \to -X^{5,7,9}$

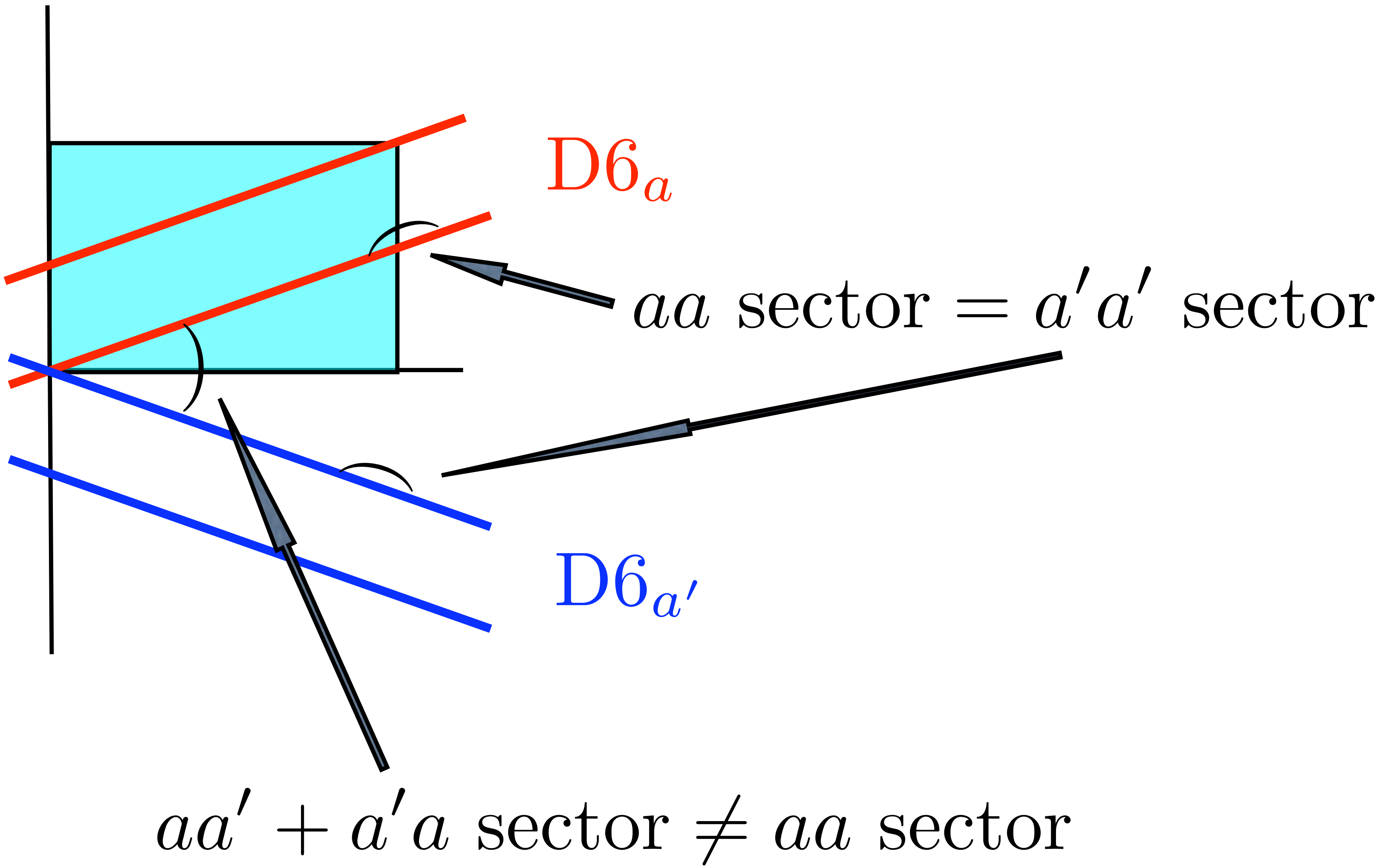
# orientifold fixed planes: $\mathrm{O6}$-planes

eight fixed planes under each four orientifold projection




$\mathrm{USp}(N)$ gauge symmetry is localized on the D-brane parallel to some O6-plane.

# $\mathrm{D6}_a$ -brane and its image: $\mathrm{D6}_{a'}$ -brane



# general massless field contents

| sector | field |
|---|---|
| $aa$ | $\mathrm{U}(N_a/2)$ or $\mathrm{USp}(N_a)$ gauge multiplet.<br>3 $\mathrm{U}(N_a/2)$ adjoint or 3 $\mathrm{USp}(N_a)$ anti-symmetric tensor chiral multiplets. |
| $ab+ba$ | $I_{ab}$ $(\Box_a, \bar{\Box}_b)$ chiral multiplets. |
| $ab'+b'a$ | $I_{ab'}$ $(\Box_a, \Box_b)$ chiral multiplets. |
| $aa'+a'a$ | $\frac{1}{2}\left(I_{aa'} - \frac{4}{2^k} I_{aO6}\right)$ symmetric tensor chiral multiplets.<br>$\frac{1}{2}\left(I_{aa'} + \frac{4}{2^k} I_{aO6}\right)$ anti-symmetric tensor chiral multiplets. |

exotics

moduli fields (massless)
general problem in this construction

# The values of gauge coupling constants at the string scale

$$\alpha_{\mathrm{U}} = \frac{\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{\sqrt{V_6}}{V_3}$$

$$\alpha_{\mathrm{USp}} = \frac{2\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{\sqrt{V_6}}{V_3}$$

$\kappa_4 = \sqrt{8\pi G_N}$

Planck scale

$M_s = 1/\sqrt{\alpha'}$

string scale

$V_6$ : volume of 6D compact space

$V_3$ : 3D volume of D-brane in 6D

Gauge couplings are not unified, but have definite relations at the string scale

# 3. Some examples of models

There are many models constructed.

- SUSY or non-SUSY
- type IIA or type IIB
- different orbifold projections
- different compactified space

in this review:

1) M.Cvetic, G.Shiu and A.M.Uranga, Nucl. Phys. B615 (2001) 3.
   (as a typical model)
2) N.Kitazawa, hep-th/0403278.
   (dynamical generation of Yukawa couplings)

# Three generation model by M.Cvetic, G.Shiu and A.M.Uranga

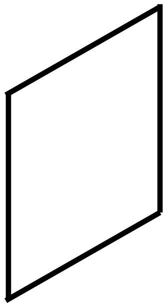
## D6-branes and their winding numbers

| D6-brane | winding number | multiplicity |
|---|---|---|
| $A_1$ | $[(0,1),(0,-1),(2,0)]$ | 8 |
| $A_2$ | $[(1,0),(1,0),(2,0)]$ | 2 |
| $B_1$ | $[(1,0),(1,-1),(1,3/2)]$ | 4 |
| $B_2$ | $[(1,0),(0,1),(0,-1)]$ | 2 |
| $C_1$ | $[(1,-1),(1,0),(1,1/2)]$ | $6+2$ |
| $C_2$ | $[(0,1),(1,0),(0,-1)]$ | 4 |

U(2)

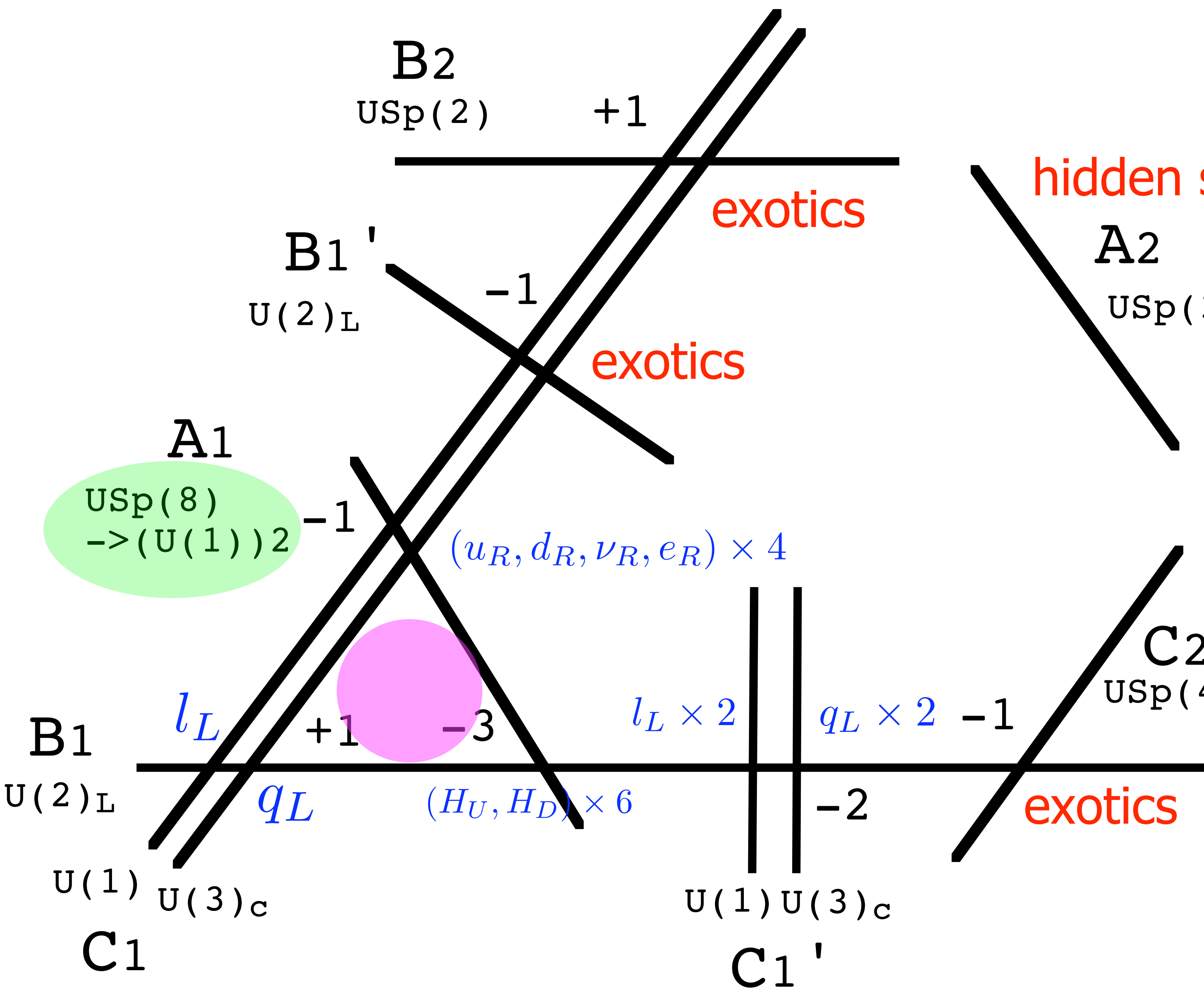
U(3)xU(1)

tilted torus compactification

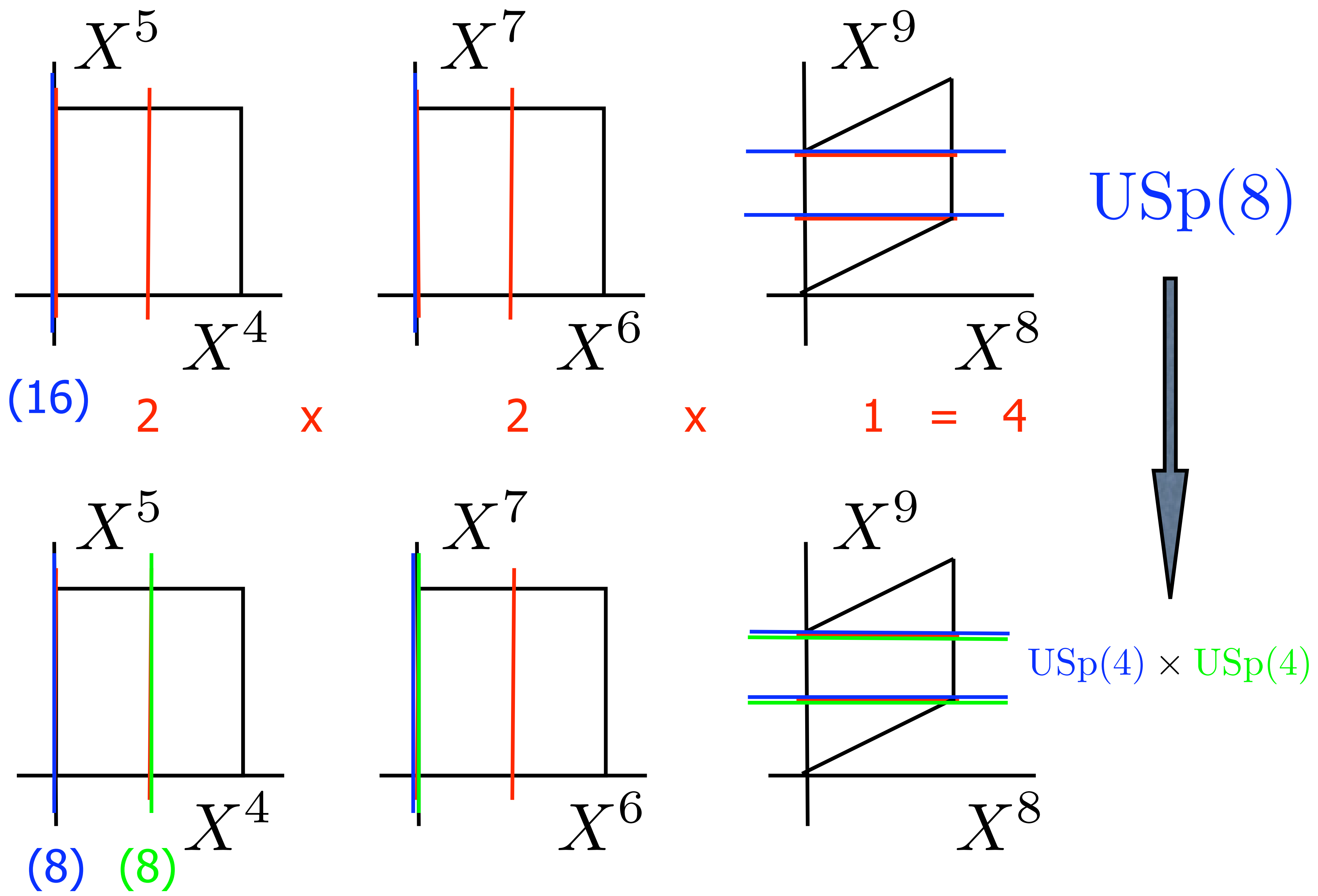


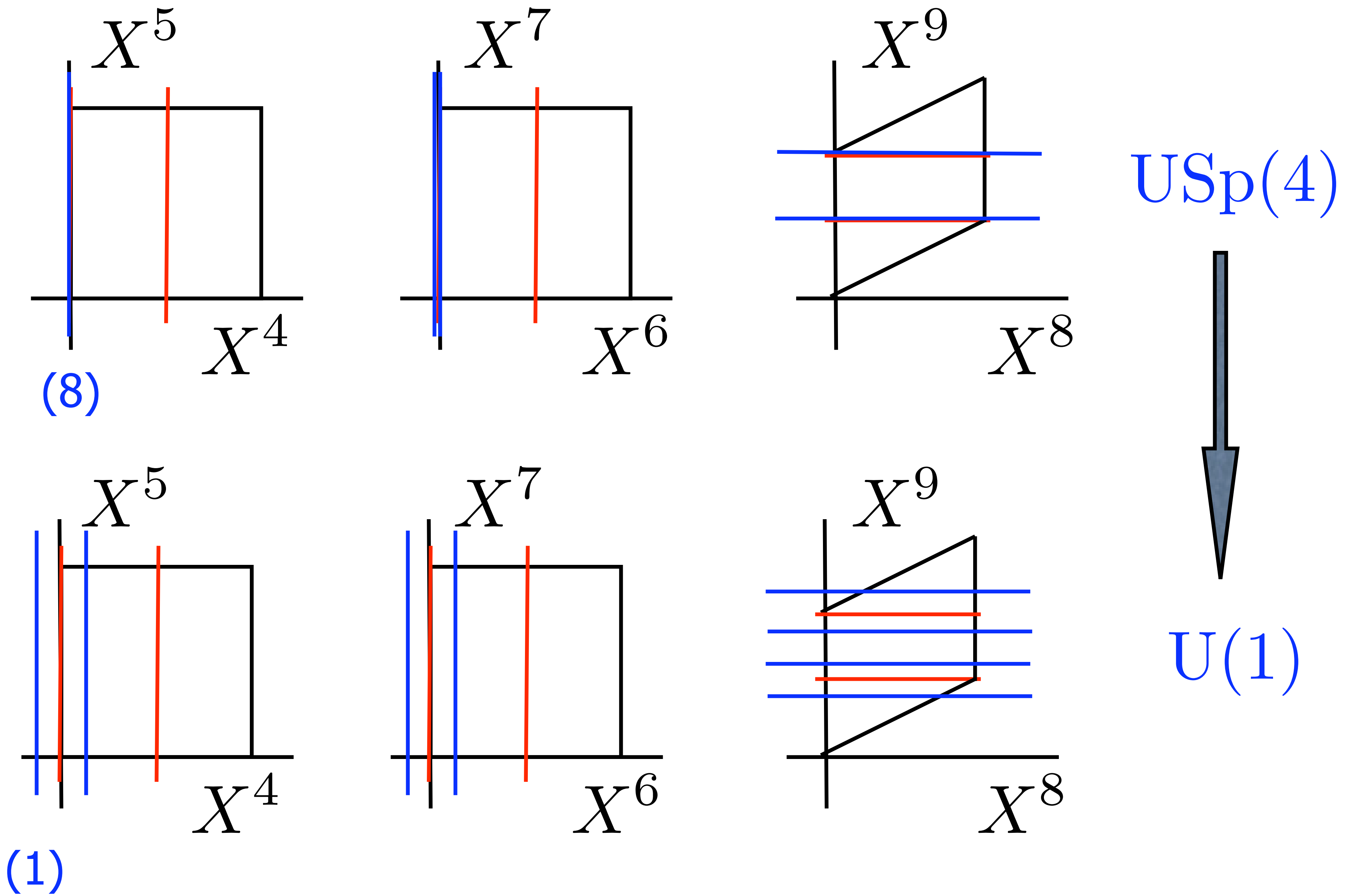
# a schematic picture of the configuration

(the relative place of each D-brane has no meaning)



# USp(8) breaking





$X^5$
$X^4$
(8)
$X^7$
$X^6$
$X^9$
$X^8$
USp(4)
U(1)
$X^5$
$X^4$
(1)
$X^7$
$X^6$
$X^9$
$X^8$

# definition of hypercharge

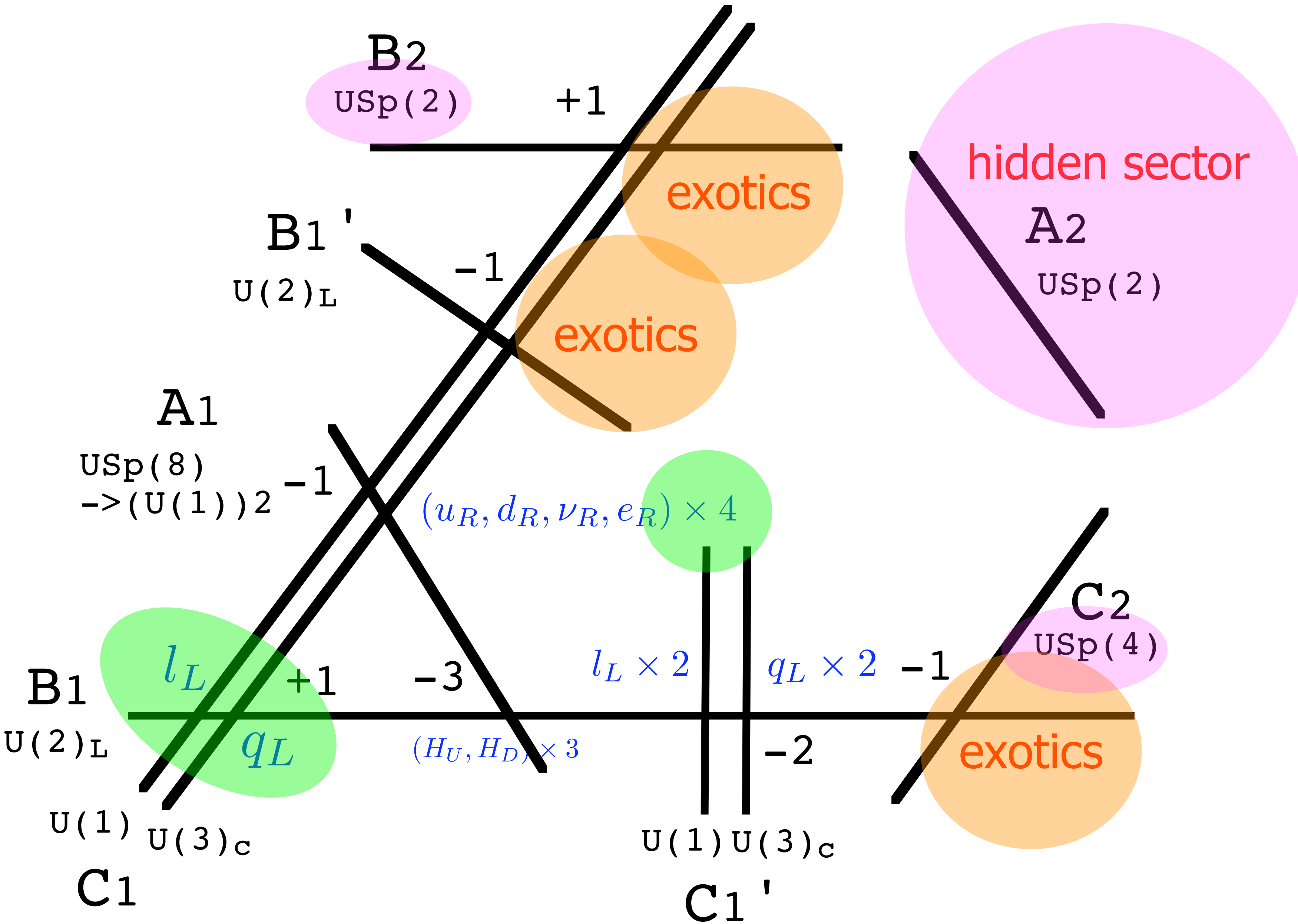
$$Q_Y \equiv \frac{1}{6}Q_3 - \frac{1}{2}Q_1 + \frac{1}{2}\left(Q_8 + Q_8'\right)$$

$\mathrm{U}(3)_c \times \mathrm{U}(1)$ of $C_1$ brane

Phenomenology of this model is discussed in
Cvetic-Langacker-Shiu, PR D66 (2002) 066004.

The Yukawa couplings of this model is discussed in
Cvetic-Langacker-Shiu, NP B642 (2002) 139.

# difficulties of this model



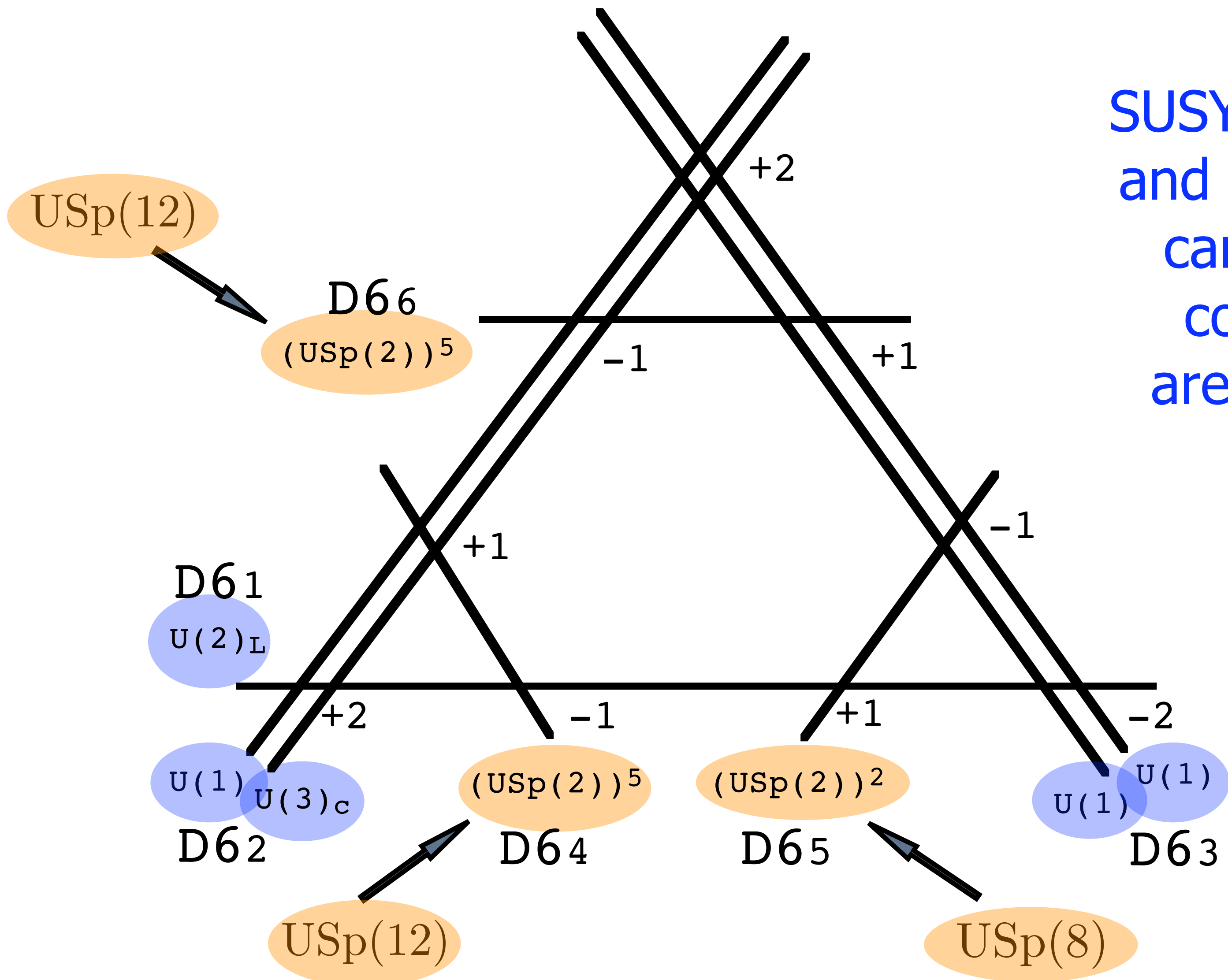
# A model of dynamical Yukawa coupling generation

## D6-branes and their winding numbers

| D6-brane | winding number | multiplicity |
|---|---|---|
| $D6_1$ | $[(1,-1),(1,1),(1,0)]$ | $4$ |
| $D6_2$ | $[(1,1),(1,0),(1,-1)]$ | $6+2$ |
| $D6_3$ | $[(1,0),(1,-1),(1,1)]$ | $2+2$ |
| $D6_4$ | $[(1,0),(0,1),(0,-1)]$ | $12$ |
| $D6_5$ | $[(0,1),(1,0),(0,-1)]$ | $8$ |
| $D6_6$ | $[(0,1),(0,-1),(1,0)]$ | $12$ |

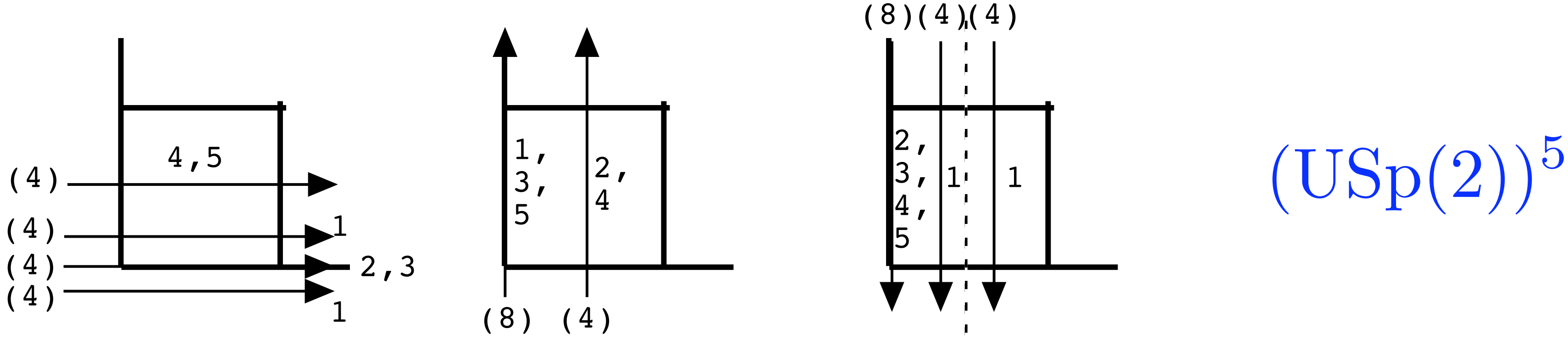
# a schematic picture of the configuration

(the relative place of each D-brane has no meaning)

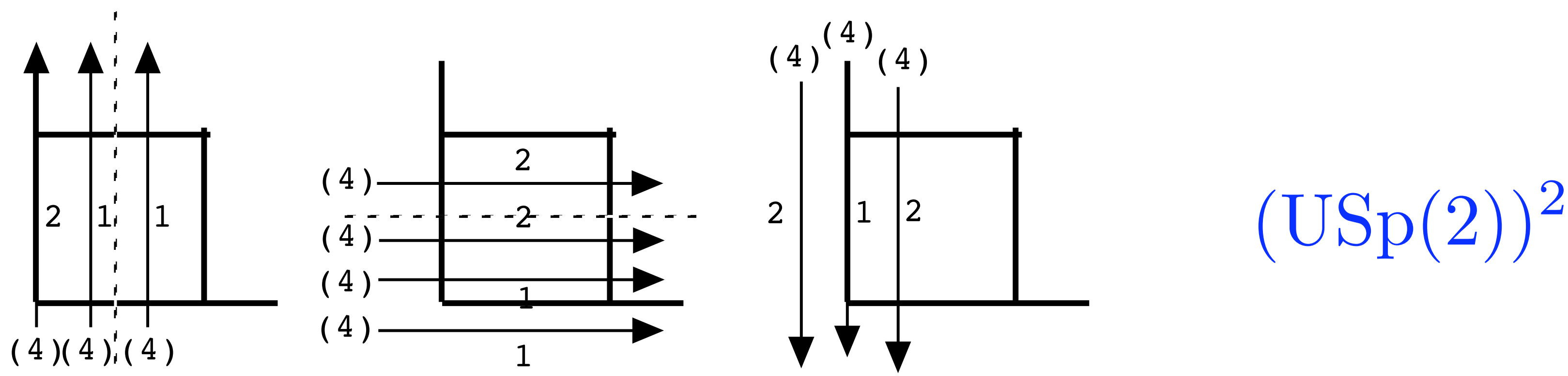



SUSY conditions
and RR tadpole
cancellation
conditions
are satisfied.

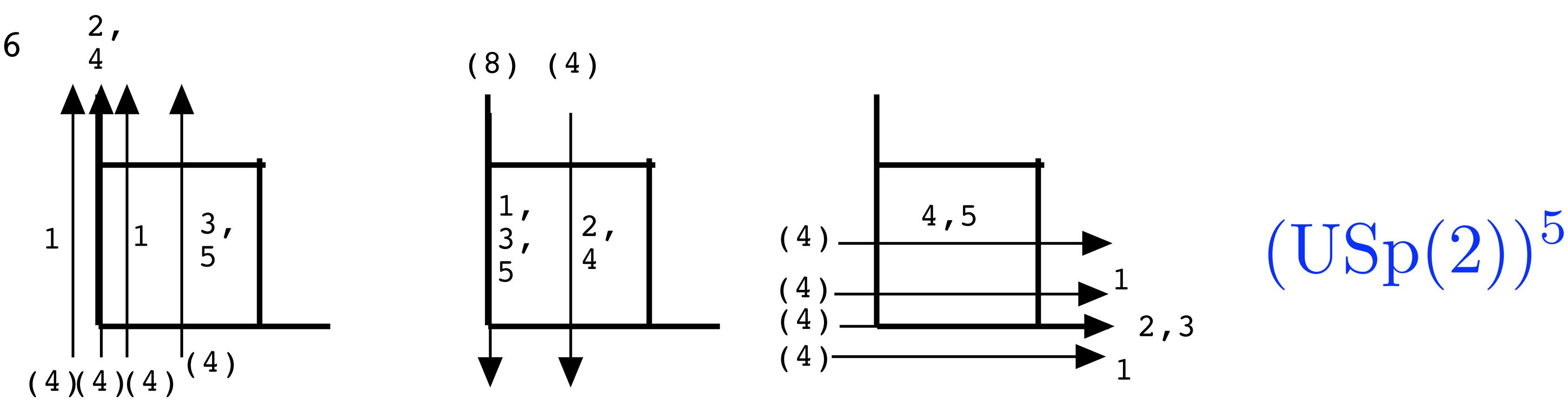
D6$_4$




D6$_5$




D6$_6$



# The values of gauge coupling constants at the string scale

$$\alpha_{\rm U} = \frac{\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{\sqrt{V_6}}{V_3} = \frac{2\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{\chi^{3/2}}{1+\chi^2}$$

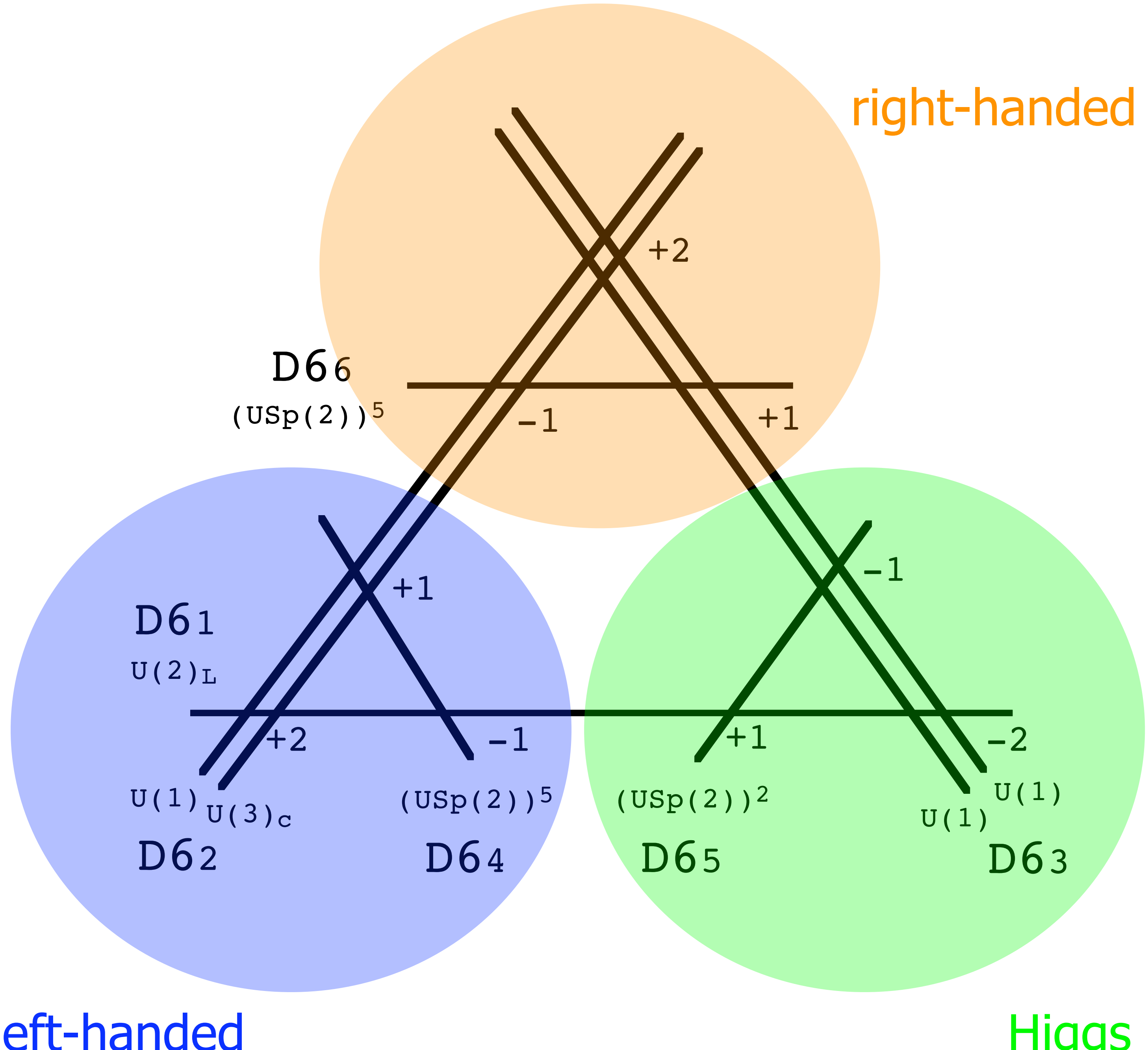
$$\alpha_{\rm USp} = \frac{2\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{\sqrt{V_6}}{V_3} = \frac{4\kappa_4 M_s}{\sqrt{4\pi}} \cdot \frac{1}{\sqrt{\chi}}$$

in case of $\chi \equiv \frac{R_2^{(1)}}{R_1^{(1)}} \sim 0.1$ and $\kappa_4 M_s \sim 1$ , for example

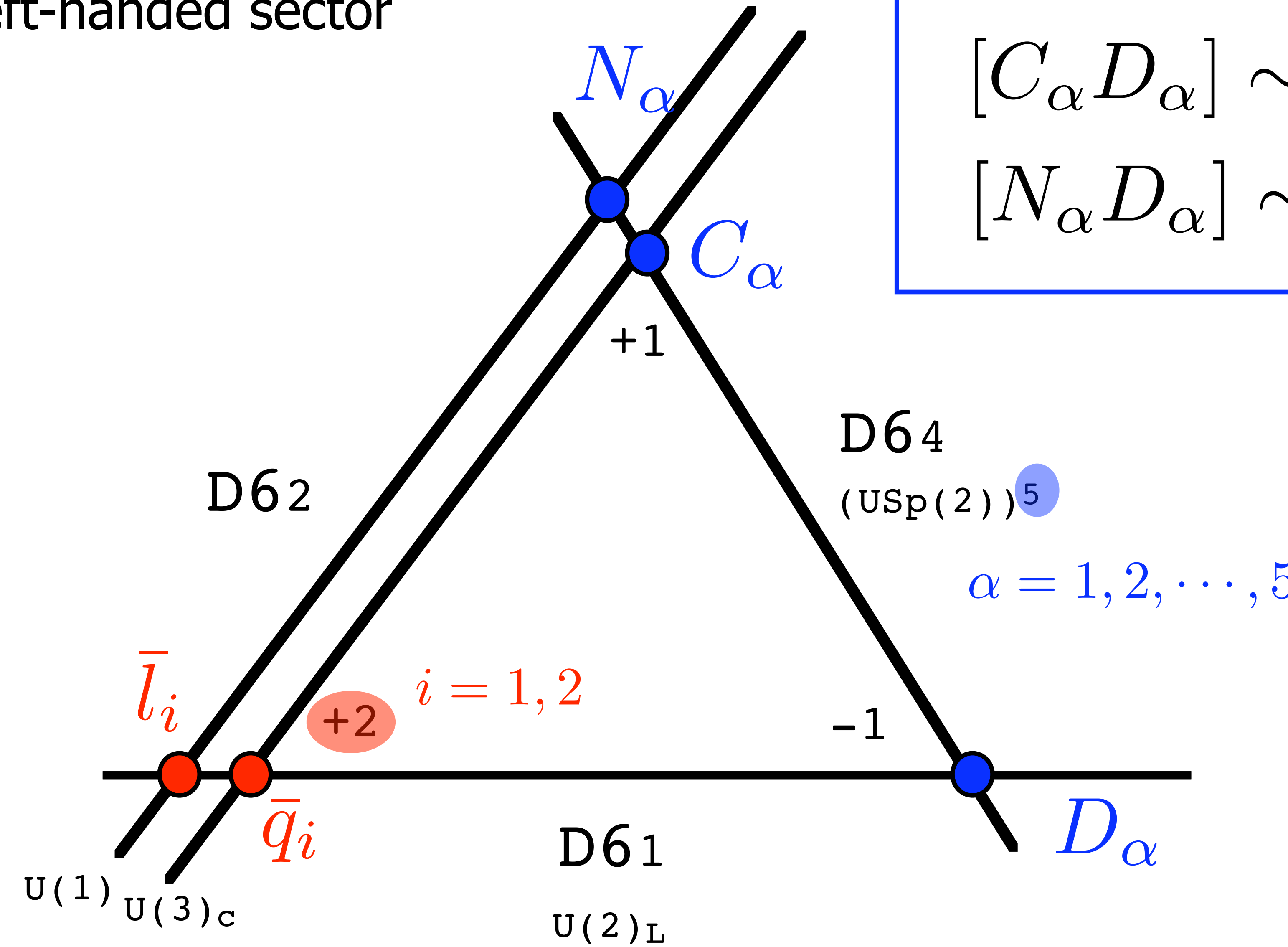
$\alpha_{\rm U} \sim 0.01$ reasonable

$\alpha_{\rm USp} \sim 1$ strong coupling $\Lambda \sim M_s$

# Three Sectors



left-handed sector




$$[C_\alpha D_\alpha] \sim q_\alpha$$
$$[N_\alpha D_\alpha] \sim l_\alpha$$

The origin of the generation is not the intersection number, but the number of D6-branes of USp(2)

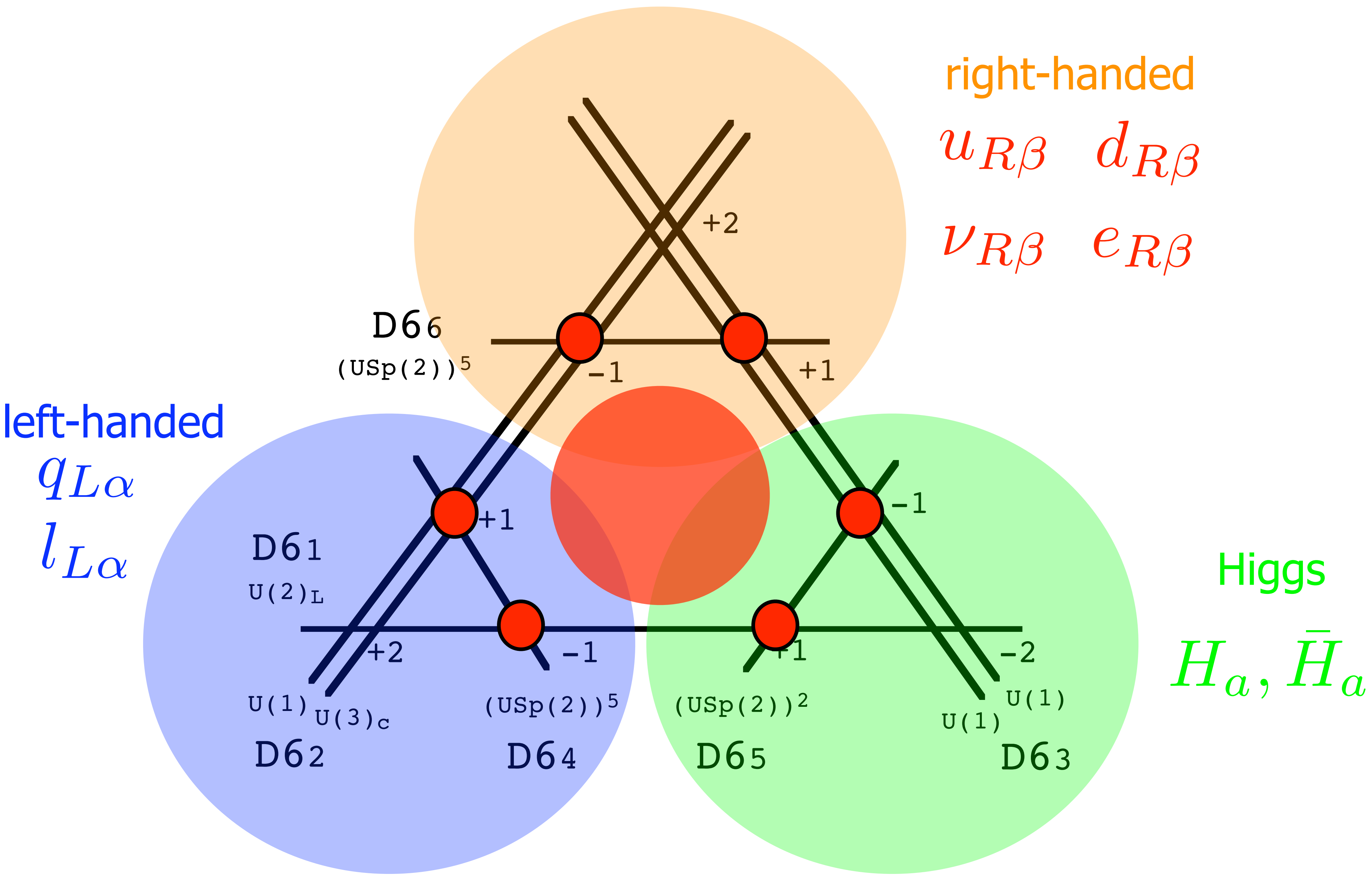
# tree-level Yukawa couplings

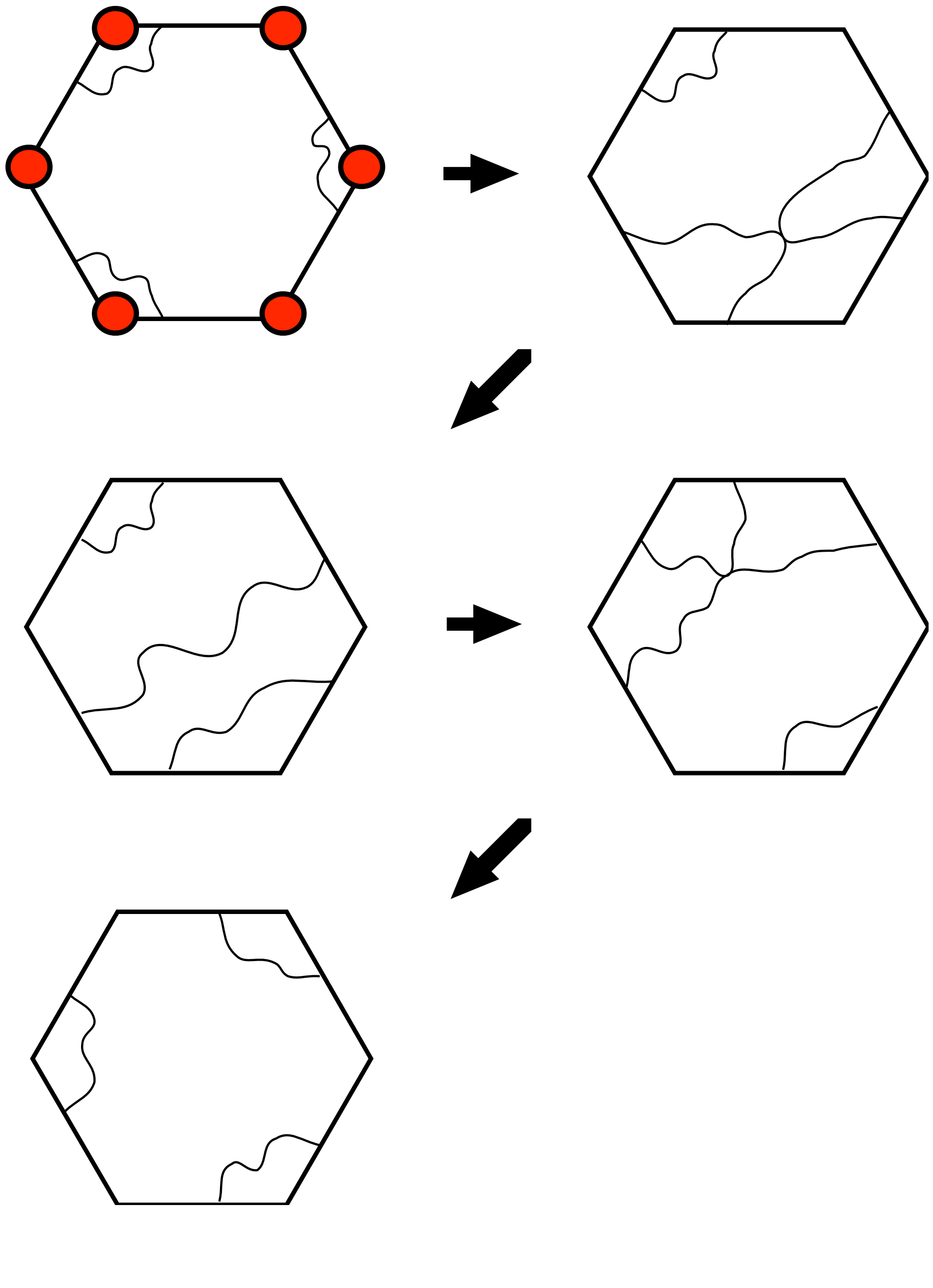
$$W_{\mathrm{tree}} = \sum_{i=1,2} \sum_{\alpha=1}^{5} g^{q}_{i\alpha} \bar{q}_i [C_\alpha D_\alpha] + \sum_{i=1,2} \sum_{\alpha=1}^{5} g^{l}_{i\alpha} \bar{l}_i [N_\alpha D_\alpha]$$

$$[C_\alpha D_\alpha] \rightarrow \Lambda_L q_\alpha \qquad [N_\alpha D_\alpha] \rightarrow \Lambda_L l_\alpha$$

Three generations of $q_L$ and $l_L$ remain.

The similar happens in right-handed and Higgs sectors.





# dynamical generation of Yukawa couplings

$$\sum_{\alpha,\beta=1}^{5}\sum_{a=1,2}\frac{g^u_{\alpha\beta a}}{M_s^3}[C_\alpha D_\alpha][\bar{C}_\beta \bar{D}_\beta^{(-)}][T_a T_a^{(+)}] \sim \sum_{\alpha,\beta=1}^{5}\sum_{a=1,2} g^u_{\alpha\beta a}\frac{\Lambda_L\Lambda_R\Lambda_H}{M_s^3} q_\alpha u_\beta H_a^{(2)},$$

$$\sum_{\alpha,\beta=1}^{5}\sum_{a=1,2}\frac{g^d_{\alpha\beta a}}{M_s^3}[C_\alpha D_\alpha][\bar{C}_\beta \bar{D}_\beta^{(+)}][T_a T_a^{(-)}] \sim \sum_{\alpha,\beta=1}^{5}\sum_{a=1,2} g^d_{\alpha\beta a}\frac{\Lambda_L\Lambda_R\Lambda_H}{M_s^3} q_\alpha d_\beta \bar{H}_a^{(1)},$$

$$\sum_{\alpha,\beta=1}^{5}\sum_{a=1,2}\frac{g^\nu_{\alpha\beta a}}{M_s^3}[N_\alpha D_\alpha][\bar{N}_\beta \bar{D}_\beta^{(-)}][T_a T_a^{(+)}] \sim \sum_{\alpha,\beta=1}^{5}\sum_{a=1,2} g^\nu_{\alpha\beta a}\frac{\Lambda_L\Lambda_R\Lambda_H}{M_s^3} l_\alpha \nu_\beta H_a^{(2)},$$

$$\sum_{\alpha,\beta=1}^{5}\sum_{a=1,2}\frac{g^e_{\alpha\beta a}}{M_s^3}[N_\alpha D_\alpha][\bar{N}_\beta \bar{D}_\beta^{(+)}][T_a T_a^{(-)}] \sim \sum_{\alpha,\beta=1}^{5}\sum_{a=1,2} g^e_{\alpha\beta a}\frac{\Lambda_L\Lambda_R\Lambda_H}{M_s^3} l_\alpha e_\beta \bar{H}_a^{(1)}.$$

The Yukawa coupling matrices depend on the geometrical configurations of D6-branes, especially for the place of USp(2) D6-branes.

↑ expected to be O(1)

The scales of dynamics of USp(2) can naturally be very large.

# 4. Summary

- A brief and intuitive introduction to the system of intersecting D-branes.
- Very brief introductions to one typical model and a composite model.

It would be very interesting to explorer more realistic models in this framework.

I am looking forward to your creative researches about string models.