Generalized Weinberg's Sum Rules in Deconstructed QCD

(and its Application to Higgsless Electroweak Symmetry Breaking Models)

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based on

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by R.S. Chivukula, M. K., M. Tanabashi

and

hep-ph/0406077

by R.S. Chivukula, H.-J. He, M. K., E.H. Simmons, M. Tanabashi

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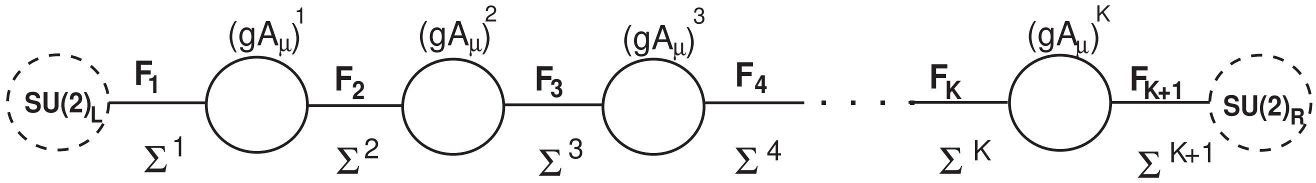
The open moose model

Low energy effective field theory of QCD proposed by
D.T. Son and M.A. Stephanov, hep-ph/0304182



- $SU(2)_L \otimes SU(2)_R$ **global symmetry**
- $[SU(2)]^K$ **local symmetry :** $(gA_\mu)^k \equiv g_k A_\mu^k\ ,\ \ (k = 1\ \sim\ K)$
- $K{+}1$ **nonlinear** σ **models :** $\Sigma^k = \exp\left(\dfrac{2i\pi^k}{F_k}\right)\ ,\ \ (k = 1\ \sim\ K+1)$

$$\begin{aligned}
\Sigma^1 &\rightarrow L\ \Sigma^1\ U_1^\dagger(x) \\
\Sigma^k &\rightarrow U_{k-1}(x)\ \Sigma^k\ U_k^\dagger(x) \qquad (k = 2 \sim K) \\
\Sigma^{K+1} &\rightarrow U_K(x)\ \Sigma^{K+1}\ R
\end{aligned}$$

- $O(p^2)$ **Lagrangian :**

$$\mathcal{L} = \frac{1}{4}\sum_{k=1}^{K+1} F_k^2\ \mathbf{Tr}\left|D_\mu \Sigma^k\right|^2 \ - \ \frac{1}{2}\sum_{k=1}^{K}\mathbf{Tr}\left(F_{\mu\nu}^k\right)^2$$

$$\begin{aligned}
D_\mu \Sigma^1 &= \partial_\mu \Sigma^1 && + \ i\Sigma^1 (gA_\mu)^1 \\
D_\mu \Sigma^k &= \partial_\mu \Sigma^k \ - i(gA_\mu)^{k-1}\Sigma^k && + \ i\Sigma^k (gA_\mu)^k \\
D_\mu \Sigma^{K+1} &= \partial_\mu \Sigma^{K+1} - i(gA_\mu)^K \Sigma^{K+1}
\end{aligned}$$

- **The linear combination of σ model fields**

$$\Sigma = \Sigma^1 \Sigma^2 \cdots \Sigma^{K+1} = \exp\left(2i \sum_{k=1}^{K+1} \frac{\pi^k}{F_k} \right)$$

transforms only under the global symmetry groups

$\Sigma \;\rightarrow\; L\, \Sigma R^\dagger \quad \leftarrow$ **identified with pion** (π)

⬇

$$\sum_{k=1}^{K+1} \frac{\pi^k}{F_k} = \frac{\pi}{f_\pi} \qquad \left(\frac{1}{f_\pi^2} = \sum_{k=1}^{K+1} \frac{1}{F_k^2} \right)$$

. . . .

- K **linear conbinations of the** π^k **are "eaten" by the gauge fields**

⬇

All the gauge fields A_μ^k **become massive**

⬇

Identified with vector mesons of QCD

$(\rho,\ a_1,\ \rho',\ a_1',\ \cdots)$

- **The vector meson mass-squared matrix**

$$M^2 = \frac{1}{4}\left(\begin{array}{c|c|c|c|c|c}
g_1^2(F_1^2+F_2^2) & -g_1g_2F_2^2 & & & & \\ \hline
-g_1g_2F_2^2 & g_2^2(F_2^2+F_3^2) & -g_2g_3F_3^2 & & & \\ \hline
 & -g_2g_3F_3^2 & g_3^2(F_3^2+F_4^2) & -g_3g_4F_4^2 & & \\ \hline
 & & \ddots & \ddots & \ddots & \\ \hline
 & & & -g_{K-2}g_{K-1}F_{K-1}^2 & g_{K-1}^2(F_{K-1}^2+F_K^2) & -g_{K-1}g_KF_K^2 \\ \hline
 & & & & -g_{K-1}g_KF_K^2 & g_K^2(F_K^2+F_{K+1}^2)
\end{array}\right)$$

$$M^2|\ \hat{\ell}\ \rangle = m_{\hat{\ell}}^2|\ \hat{\ell}\ \rangle$$

$|\ \hat{\ell}\ \rangle$ **: eigen vectors ,** $m_{\hat{\ell}}^2$ **: eigen values**

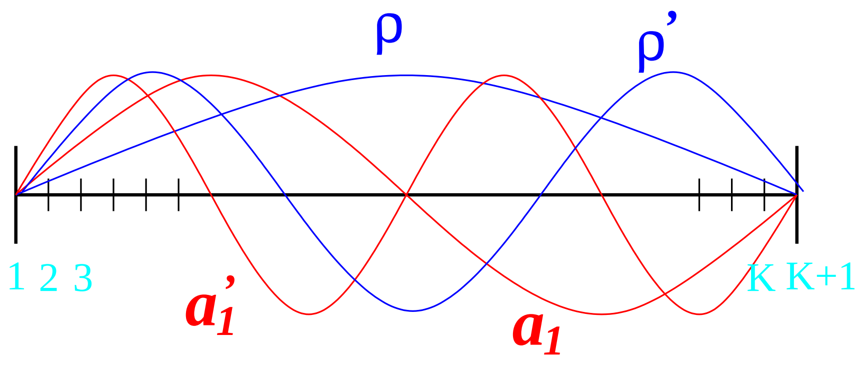
- **The mass eigen state fields :** $A_\mu^{\hat{\ell}}$ $(\hat{\ell} = 1 \sim K)$

$$A_\mu^{\hat{\ell}} = \sum_k A_\mu^k \ \langle k|\hat{\ell}\rangle$$

Example $f_k = f$ **(const.),** $g_k = g$ **(const.)**

$$M^2 = \frac{f^2g^2}{4}\left(\begin{array}{c|c|c|c|c|c}
2 & -1 & & & & \\ \hline
-1 & 2 & -1 & & & \\ \hline
 & -1 & 2 & -1 & & \\ \hline
 & & \ddots & \ddots & \ddots & \\ \hline
 & & & -1 & 2 & -1 \\ \hline
 & & & & -1 & 2
\end{array}\right)$$

- **The wave function of the mass eigenstates**



Parity $\rho,\ \rho',\ \cdots\ :\ +$ $\quad a_1,\ a_1',\ \cdots\ :\ -$

Weinberg's 1st and 2nd Sum rules

- **Vector meson decay constant**

$$\langle 0|J^a_{V\mu}(0)|A^{b\hat{l}}\rangle = g_{\hat{l}V}\delta^{ab}\varepsilon_\mu \quad \& \quad \langle 0|J^a_{A\mu}(0)|A^{b\hat{l}}\rangle = g_{\hat{l}A}\delta^{ab}\varepsilon_\mu$$

$$J^a_{V\mu}(x) = \bar{\psi}(x)\frac{\lambda^a}{2}\gamma_\mu\psi(x), \quad J^a_{A\mu}(x) = \bar{\psi}(x)\frac{\lambda^a}{2}\gamma_\mu\gamma_5\psi(x)$$

$$\Downarrow$$

$$g_{\hat{l}V,A} = \frac{F_1^2 g_1}{4}[\langle 1|\hat{l}\rangle \pm \langle K|\hat{l}\rangle]$$

Here, the parity symmetry is assumed.

$$F_k = F_{K+2-k}, \quad g_k = g_{K+1-k}$$

$$\Downarrow$$

$$g_{\hat{\ell}V} = 0, \quad \textbf{for even } \hat{\ell}$$

$$g_{\hat{\ell}A} = 0, \quad \textbf{for odd } \hat{\ell}$$

- **Current correlator**

$$\langle J^a_{V,A\,\mu}(q)\, J^b_{V,A\,\nu}(-q)\rangle = \delta^{ab}\,(-q^2\eta_{\mu\nu} + q_\mu q_\nu)\;\Pi_{V,A}(-q^2)$$

$$\Downarrow$$

$$\Pi_V(Q^2) = \sum_{\hat{l}} \frac{g^2_{\hat{l}V}}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)}$$

$$\Pi_A(Q^2) = \frac{f_\pi^2}{Q^2} + \sum_{\hat{l}} \frac{g^2_{\hat{l}A}}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)}$$

$$\Downarrow$$

$$\begin{aligned}
\Pi_{V-A}(Q^2) &\equiv \Pi_V(Q^2) - \Pi_A(Q^2) \\
&= -\frac{f_\pi^2}{Q^2} + \sum_{\hat{l}} \frac{g^2_{\hat{l}V} - g^2_{\hat{l}A}}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)} \\
&= -\frac{f_\pi^2}{Q^2} + \frac{F_1^4 g_1^2}{4} \sum_{\hat{l}} \langle 1|\hat{\ell}\rangle \frac{1}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)} \langle\hat{\ell}|K\rangle \\
&= -\frac{f_\pi^2}{Q^2} + \frac{F_1^4 g_1^2}{4} \left\langle 1 \left| \frac{1}{M^2(Q^2\mathcal{I} + M^2)} \right| K \right\rangle
\end{aligned}$$

- **Expanding $\Pi_{V-A}(Q^2)$ in powers of $1/Q^2$**

$$\Pi_{V-A}(Q^2) = -\frac{f_\pi^2}{Q^2} + \frac{F_1^4 g_1^2}{4Q^2}\left\langle 1 \left| \frac{1}{M^2}\left\{\mathcal{I} - \frac{M^2}{Q^2} + \left(\frac{M^2}{Q^2}\right)^2 - \cdots\cdots \right\}\right| K \right\rangle$$

$$= \frac{1}{Q^2}\left(-f_\pi^2 + \frac{F_1^4 g_1^2}{4}\left\langle 1 \left| \frac{\mathcal{I}}{M^2} \right| K \right\rangle\right) \qquad \Longrightarrow 0$$

$$- \frac{1}{Q^4}\frac{F_1^4 g_1^2}{4}\langle 1 | K \rangle \qquad \Longrightarrow 0$$

$$+ \frac{1}{Q^6}\frac{F_1^4 g_1^2}{4}\left\langle 1 \left| M^2 \right| K \right\rangle$$

$$- \frac{1}{Q^8}\frac{F_1^4 g_1^2}{4}\left\langle 1 \left| (M^2)^2 \right| K \right\rangle$$

$$+ \bullet \bullet \bullet \bullet \bullet \bullet$$

- **Comparing with**

$$\Pi_{V-A}(Q^2) = -\frac{f_\pi^2}{Q^2} + \sum_{\hat{l}} \frac{g_{\hat{l}V}^2 - g_{\hat{l}A}^2}{m_{\hat{l}}^2(m_{\hat{l}}^2 + Q^2)}$$

$$\begin{aligned}
&= \frac{1}{Q^2}\left\{ -f_\pi^2 + \sum_{\hat{l}} \left(\frac{g_{\hat{l}V}^2}{m_{\hat{l}}^2} - \frac{g_{\hat{l}A}^2}{m_{\hat{l}}^2} \right) \right\} \\
&- \frac{1}{Q^4}\left\{ \sum_{\hat{l}} \left(g_{\hat{l}V}^2 - g_{\hat{l}A}^2 \right) \right\} \\
&+ \frac{1}{Q^6}\left\{ \sum_{\hat{l}} \left(g_{\hat{l}V}^2 - g_{\hat{l}A}^2 \right) m_{\hat{l}}^2 \right\} \\
&- \frac{1}{Q^8}\left\{ \sum_{\hat{l}} \left(g_{\hat{l}V}^2 - g_{\hat{l}A}^2 \right) (m_{\hat{l}}^2)^2 \right\} \\
&+ \bullet \bullet \bullet \bullet \bullet \bullet
\end{aligned}$$

Weinberg's 1st and 2nd sum rules are always satisfied in this model!!

$$\sum_{\hat{l}} \left(\frac{g_{\hat{l}V}^2}{m_{\hat{l}}^2} - \frac{g_{\hat{l}A}^2}{m_{\hat{l}}^2} \right) = f_\pi^2, \qquad \sum_{\hat{l}} \left(g_{\hat{l}V}^2 - g_{\hat{l}A}^2 \right) = 0$$

$\boxed{\textbf{Generalized Weinberg's Sum rules}}$

- **From the tridiagonal form of M^2,**

$$\left\langle 1 \left| (M^2)^j \right| K \right\rangle = 0, \qquad \textbf{for} \quad j = 0 \sim K-2$$

$$\Downarrow$$

$$\boxed{\begin{array}{c}\textbf{There is a family of} \\ \textbf{generalized sum rules !!}\end{array}}$$

$$\sum_{\hat{l}} \left(g_{\hat{l}V}^2 - g_{\hat{l}A}^2 \right) (m_{\hat{l}}^2)^j = 0$$

$$(j = 0 \sim K-2)$$

$$\Downarrow$$

$$\Pi_{V-A}(Q^2) \sim \frac{1}{(Q^2)^{K+1}}$$

$$\left(\textbf{cf} \ : \Pi_{V-A}(Q^2) \sim \frac{\langle \bar{\psi}\psi \rangle}{Q^6} \quad \textbf{in QCD} \right)$$

- **Unfortunately, the open moose model does not behave like QCD.**

$\Downarrow$

- **However, the model has an interesting aspect : The generalized sum rules allow for a convenient expression of $\Pi_{V-A}(Q^2)$ in this model.**

$$\Pi_{V-A}(Q^2) = \frac{P(Q^2)}{Q^2 \cdot (Q^2 + m_{\hat{1}}^2) \ldots (Q^2 + m_{\hat{K}}^2)} ,$$

$P(Q^2)$ is a polynomial in Q^2 at most of order K. However, generalized sum rules implies that $\Pi_{V-A}(Q^2)$ must fall like $1/(Q^2)^{K+1}$.

$\Downarrow$

$$P(Q^2) \;=\; \textbf{const}.$$

Requiring that the pion pole has the correct residue,

$$\Pi_{V-A}(Q^2) = -\frac{f_\pi^2}{Q^2} \prod_{\hat{l}} \frac{m_{\hat{l}}^2}{(Q^2 + m_{\hat{l}}^2)}$$

$\boxed{\textbf{QCD } S \textbf{ parameter}}$

$$
\begin{aligned}
S &= 4\pi \frac{d}{dQ^2}\left[Q^2\,\Pi_{V-A}(Q^2)\right]_{Q^2=0} \\
&= 4\pi f_\pi^2 \left(\sum_{\hat{l}} \frac{1}{m_{\hat{l}}^2}\right) \quad > \; 0
\end{aligned}
$$

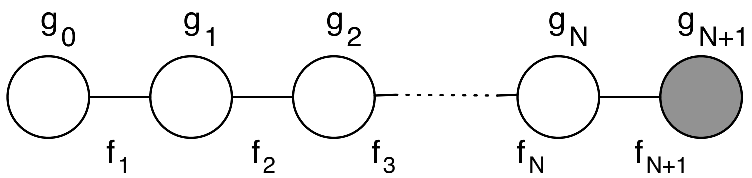
$\boxed{\pi^+ - \pi^0 \textbf{ mass difference}}$

$$
\begin{aligned}
\Delta m_\pi^2 &= \frac{3\alpha_{em}}{4\pi f_\pi^2}\int_0^\infty dQ^2\,(-Q^2)\,\Pi_{V-A}(Q^2) \qquad (\; > \; 0 \;) \\
&= -\frac{3\alpha_{em}}{4\pi f_\pi^2}\sum_{\hat{l}}(g_{\hat{l}V}^2 - g_{\hat{l}A}^2)\log m_{\hat{l}}^2 \\
&= -\frac{3\alpha_{em}F_1^4 g_1^2}{16\pi f_\pi^2}\left\langle 1 \left| \log M^2 \right| K \right\rangle
\end{aligned}
$$

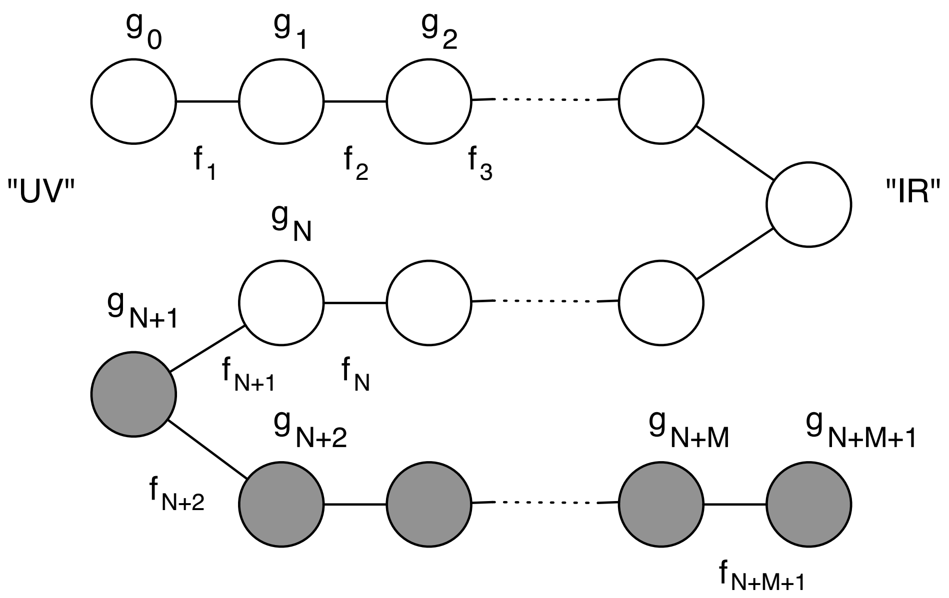
Collective symmetry breaking !!

(cf : Mass of the little Higgs)

The model is also applicable to the deconstructed version of the Higgsless electroweak symmetry breaking models



R. Foadi et al. JHEP 03 (2004) 042 ; J. Hirn et al. hep-ph/0401032



C. Csaki et al. hep-ph/0305237; PRL 92 (2004) 101802 ; K. Agashe et al. JHEP 08 (2003) 050 ; Y. Nomura JHEP 11 (2003) 050 ; R. Barbieri et al. hep-ph/0310285 ; H. Davoudiasl et al. hep-ph/0312193; JHEP 05 (2004) 015 ; G. Burdman et al. hep-ph/0312247 ; G. Cacciapaglia et al. hep-ph/0401160

We can calculate the corrections to electroweak parameters in large class of Higgsless models !!

hep-ph/0406077