Supersymmetric Radius Stabilization in Warped Extra Dimensions

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Introduction

Recent attention to physics of Extra Dimensions is motivated by an alternative solution to the gauge hierarchy problem without SUSY

● **Large Extra Dimensions**: Arkani-Hamed-Dimopoulos-Dvali (98)

$$S_{4+n} = (M_{4+n})^{2+n} \int d^4x d^n y \sqrt{-g_{4+n}} R_{4+n} \supset M_4^2 \int d^4x \sqrt{-g_4} R_4$$

$$M_4^2 = (2\pi r)^n (M_{4+n})^{2+n}$$

(compactified on n-dim torus with radius "r")

For $M_{4+n} \sim 1TeV$ ➡

~~$r \sim 10^{13} cm (n=1)$~~, $r \sim 10^{-1} mm (n=2)$

$r \sim 10^{-6} mm (n=3) \cdots$

Gauge hierarchy problem ⇔ Large compactification radius

●Warped Extra Dimensions: Randall-Sundrum (99)

Metric: $ds^2 = e^{-2kry}\eta_{\mu\nu}dx^\mu dx^\nu + r^2 dy^2 \left(0 \le y \le \pi\right)$

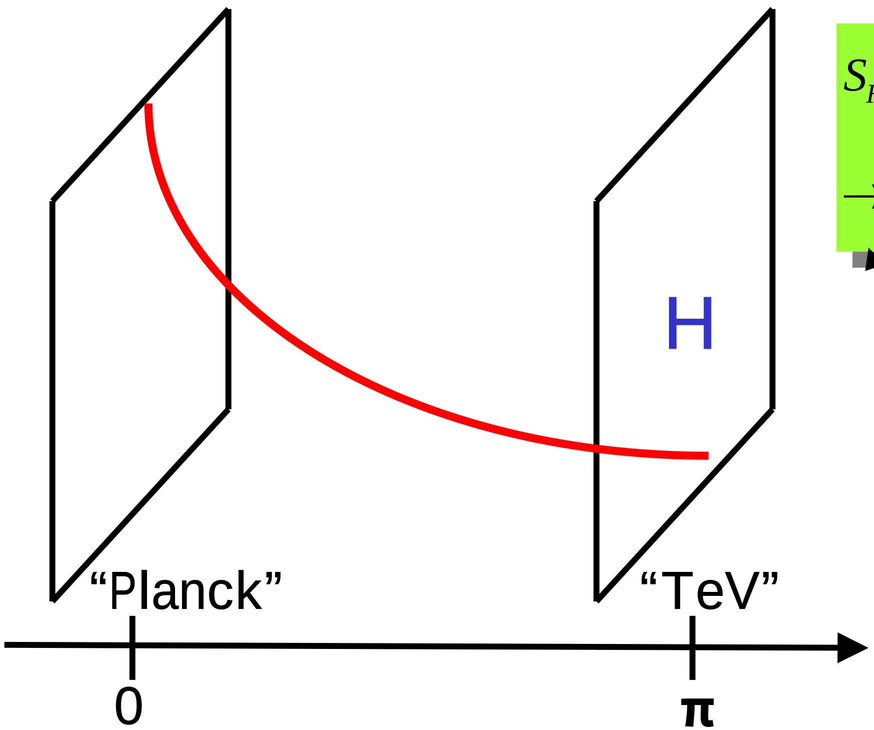
$$S = \int d^5x \left[\sqrt{-G}\left(2M_5^3 R - \Lambda\right) + \delta(y)\sqrt{-g_{y=0}}\left(L_{y=0} - \Lambda_{y=0}\right) + \delta(y-\pi)\sqrt{-g_{y=\pi}}\left(L_{y=\pi} - \Lambda_{y=\pi}\right)\right]$$

$$\Lambda = -\Lambda_{y=0} = \Lambda_{y=\pi} = -24M_5^3 k^2$$

➡ Vanishing 4D cosmological constant

$$M_4^2 = \frac{M_5^3}{k}\left(1 - e^{-2rk\pi}\right) \Rightarrow M_4 \sim M_5 \sim k$$

4D Planck ~ 5D Planck



$$S_{Higgs} = \int d^4x \sqrt{-g_{y=\pi}}\left[g^{\mu\nu}_{y=\pi}\left(D_\mu H\right)^\dagger \left(D_\nu H\right) - \lambda\left(H^\dagger H - v_0^2\right)^2\right]$$

$$\to \int d^4x \left[\eta^{\mu\nu}\left(D_\mu H\right)^\dagger \left(D_\nu H\right) - \lambda\left(H^\dagger H - \underbrace{v_0^2 e^{-2kr\pi}}_{v_{y=\pi}^2}\right)^2\right]$$

$$\sqrt{-g_{y=\pi}}\, g^{\mu\nu}_{y=\pi} = e^{-2kr\pi}\eta^{\mu\nu} \Rightarrow H \to e^{rk\pi} H$$

$$v_0 \sim M_4, kr \sim 12 \Rightarrow v_{y=\pi} \sim 100GeV$$

“Alternative Motivation” to consider Extra Dimensions

4D SUSY Phenomenology:
SUSY breaking spectrum severely constrained
to be almost flavor blind & CP invariant

In 4D SUGRA, once SUSY is broken,
SUSY breaking is mediated to the visible sector
by Planck suppressed contact terms

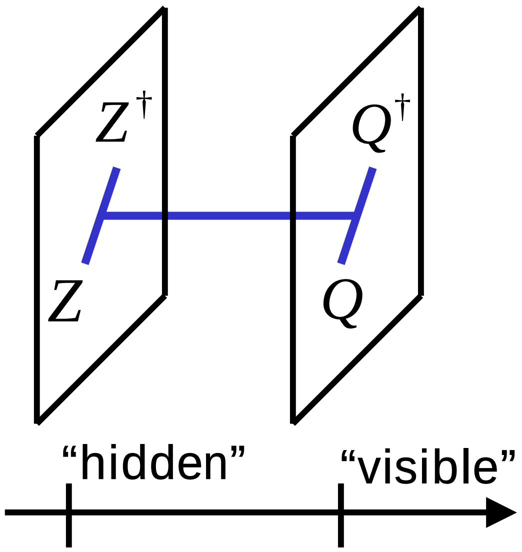
$$\int d^4\theta c_{ij} \frac{Z^\dagger Z Q_i^\dagger Q_j}{M_4^2} \Rightarrow c_{ij} m_{3/2}^2 \tilde{Q}_i^\dagger \tilde{Q}_j$$

i,j: flavor index
Q: MSSM superfield
Z: hidden sector superfield

$c_{ij} \neq \delta_{ij}$ in general,
No symmetry reason to be flavor diagonal
$\Rightarrow$ SUSY flavor problem

If visible sector & hidden sector are separated in extra dimensional spaces, the contact terms are strongly suppressed by the locality in higher dimensional theory

Randall-Sundrum (98); Luty-Sundrum (99)



SUSY breaking spectrum is induced by superconformal anomaly (Anomaly Mediation)

Randall-Sundrum (98); Giudice et al. (98)

$$M_{\lambda_i} = \frac{\alpha_i}{4\pi} b_i m_{3/2}, \tilde{m}^2 = 2C_i b_i \frac{\alpha_i^2}{4\pi} m_{3/2}^2$$

Flavor blind!!

(There are many proposals for tachyonic slepton problem)

Not the end of the story ⇒ Radius Stabilization

Phenomenological viable Brane World Scenario
= Compactification Radius should be stabilized

No radion potential since the radion is a moduli
⇒ Size of radius is undetermined
⇒ Once SUSY is broken,
nontrivial radion potential is generated
⇒ but such a potential usually destabilize the radius
⇒ Some bulk fields are introduced to stabilize the radius
⇒ New flavor-violating soft SUSY breaking
>> Anomaly Mediation

Luty-Okada (02)

Seems to be generic to consider SUSY breaking, its mediation & radius stabilization all together in brane world model building

⇒ makes realistic model construction very hard

We propose a very simple model of radius stabilization independent of SUSY breaking & its mediation

⇒ greatly simplify brane world model building

Simple Model of Radius Stabilization

Hypermultiplet on $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + r^2 dy^2, \left(\sigma(y) = k|y| \, 0 \le y \le \pi; \mu,\nu = 0,1,2,3\right)$

$$\int d^4\theta \left[\frac{T+T^\dagger}{2} e^{-(T+T^\dagger)\sigma}\left(-6M_5^3 + |H|^2 + |H^c|^2\right)|\phi|^2\right] + \left[\int d^2\theta \phi^3 e^{-3T\sigma}\left\{H\left(-\partial_y + \left(\frac{3}{2}+c\right)T\sigma'\right)H^c + W_b(y)\right\} + h.c.\right]$$

$$\downarrow \quad \left(H(+), H^c(-)\right) \to \omega^{-1}\left(H, H^c\right), \omega \equiv \phi e^{-T\sigma} \qquad W_b \equiv J_0 H\delta(y) - J_\pi H\delta(y-\pi)$$

$$\int d^4\theta \left[-3M_5^3\left(T+T^\dagger\right)|\omega|^2 + \frac{T+T^\dagger}{2}\left(|H|^2 + |H^c|^2\right)\right] + \left[\int d^2\theta \underbrace{\omega H\left(-\partial_y + \left(\frac{1}{2}+c\right)T\sigma'\right)H^c + \omega^2 W_b(y)}_{W} + h.c.\right]$$

r: radius of 5th dimension (real part of scalar component of T); $\phi = 1 + \theta^2 F_\phi$
k: AdS curvature scale; T: radion chiral multiplet; **Φ**: compensating multiplet

SUSY config. ← F-flat conditions

$$0 = \partial W/\partial H = -\partial_y \tilde{H}^c + \left(c + \frac{1}{2}\right)T\sigma' \tilde{H}^c$$

$$0 = \partial W/\partial H^c = \partial_y H + \left(c - \frac{1}{2}\right)T\sigma' H$$

$$\tilde{H}^c(0) = \frac{J_0}{2}, \tilde{H}^c(\pi) = \frac{J_\pi}{2}e^{-Tk\pi} \left(H^c(y) = \varepsilon(y)\tilde{H}^c(y)\right)$$

$$0 = \partial W/\partial T \to H(y) = 0$$

→

$$H(y) = C_1 e^{(1/2-c)T\sigma} = 0$$

$$H^c(y) = C_2 \varepsilon(y) e^{(c+1/2)T\sigma}$$

$$J_0 = J_\pi e^{-(c+3/2)Tk\pi}$$

4D Effective Action
&
Radion Mass

It is useful to describe our model via 4D effective theory with only the light hypermultiplet

Putting $H(x,y)=h(x)e^{(1/2-c)T\sigma}, H^c(x,y)=h^c(x)e^{(c+1/2)T\sigma}$ & y-integration leads to

$$L_{4Deff}=\int d^4\theta\left[f\left(T,T^\dagger\right)|\phi|^2+K\left(T,T^\dagger\right)|h|^2+K^c\left(T,T^\dagger\right)\left|h^c\right|^2\right]+\left[\int d^2\theta\phi^2 W(h,T)+h.c.\right]$$

$$f\left(T,T^\dagger\right)=-\frac{3M_5^3}{k}\left[1-e^{-\left(T+T^\dagger\right)k\pi}\right], W=h\left[J_0-J_\pi e^{-(c+3/2)Tk\pi}\right]$$

$$K\left(T,T^\dagger\right)=\frac{e^{(1/2-c)\left(T+T^\dagger\right)k\pi}-1}{(1-2c)k}, K^c\left(T,T^\dagger\right)=\frac{e^{(1/2+c)\left(T+T^\dagger\right)k\pi}-1}{(1+2c)k}$$

Radion potential:

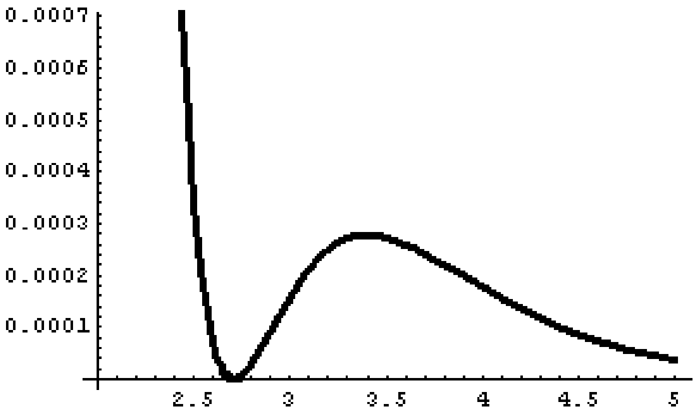
$$V_{radion}=K\left(T,T^\dagger\right)^{-1}\left|\frac{\partial W(h,T)}{\partial h}\right|^2=\frac{(1-2c)k}{e^{(1/2-c)\left(T+T^\dagger\right)k\pi}-1}\left|J_0-J_\pi e^{-(c+3/2)Tk\pi}\right|^2\geq 0$$

Minimum:

$$\partial W/\partial h=0\Rightarrow J_0-J_\pi e^{-(c+3/2)Tk\pi}=0$$

$$\partial W/\partial T=0\Rightarrow h=0, h^c$$ Flat direction

Flat limit (k→0) ⇒ $V\sim 1/\left(T+T^\dagger\right)$
"Runaway potential"



Radion Mass:

$$m_{radion}^2 \sim \left(\frac{\partial^2 f\left(T,T^\dagger\right)}{\partial T^\dagger \partial T} \right)^{-1} \frac{\partial^2 V_{radion}}{\partial T^\dagger \partial T}\Bigg|_{T=T_0} = \frac{1-2c}{e^{(1/2-c)\left(T+T^\dagger\right)k\pi}-1} \left(\frac{\left(3/2+c\right)^2 \left|J_\pi\right|^2}{3M_5^3} \right) k^2 e^{-(1/2+c)\left(T+T^\dagger\right)k\pi}\Bigg|_{T=T_0} > 0$$

Ex. $c = 1/2,\, e^{-T_0 k\pi} \sim 10^{-2},\, J_\pi \sim \left(0.1\times M_5\right)^{3/2},\, k \sim 0.1\times M_5$

$$m_{radion}^2 \sim \left(10^{-5}\times M_4\right)^2 \gg m_{3/2}^2\left(\sim 10TeV\right), F_{hidden}\left(\sim m_{3/2} M_4\right)$$

Expectation:

SUSY breaking effects does not affect the radion potential
The radion is not destabilized even with SUSY breaking

⬇

Let us check!!

Radius Stability
under SUSY Breaking Effects

- In the previous sections,
  the radius is stabilized in a supersymmetric way
- We have to check whether or not
  the radius is destabilized after SUSY breaking
- But, it is hard to solve EOMs with $F_\phi \neq 0$
- So, the stability will be approximately shown

4D effective Lagrangian:

$$L_{4Deff}=\int d^4\theta\left[f\left(T,T^\dagger\right)|\phi|^2+K\left(T,T^\dagger\right)|h|^2+K^c\left(T,T^\dagger\right)\left|h^c\right|^2\right]+\left[\int d^2\theta\phi^2W\left(h,T\right)+h.c.\right]$$

Lagrangian for auxiliary fields:

$$\begin{aligned}L_{aux}=&\\ &F_T^\dagger\left[\left(f_{TT^\dagger}+K^c_{TT^\dagger}\left|h^c\right|^2+K_{TT^\dagger}|h|^2\right)F_T+\left(K^c_Th^c\right)^\dagger F^c+\left(K_Th\right)^\dagger F+W_T^\dagger+f_{T^\dagger}F_\phi\right]\\ &+F^{c\dagger}\left[\left(K^c_Th^c\right)F_T+K^cF^c\right]+F^\dagger\left[\left(K_Th\right)F_T+KF+W_h^\dagger\right]\\ &+FW_h+F_TW_T+2\left(F_\phi W+h.c.\right)+F_\phi^\dagger f_TF_T+\left|F_\phi\right|^2f\end{aligned}$$

Eqs. of motion for auxiliary fields expanded around SUSY vacuum

(up to 1st order of SUSY breaking) $h=0, h^c = const, T=T_0$

$$0=\frac{\partial L_{aux}}{\partial F_T^\dagger}=\left(f_{TT^\dagger}+K^c_{TT^\dagger}\left|h^c\right|^2+K_{TT^\dagger}\left|h\right|^2\right)F_T+\left(K_T^c h^c\right)^\dagger F^c+\left(K_T h\right)^\dagger F+W_T^\dagger+f_{T^\dagger}F_\phi$$

$$\sim\left(f_{TT^\dagger}+K^c_{TT^\dagger}\left|h^c\right|^2\right)F_T+\left(K_T^c h^c\right)^\dagger F^c+W^\dagger_{h^\dagger T^\dagger}\delta h^\dagger+f_{T^\dagger}F_\phi$$

$$0=\frac{\partial L_{aux}}{\partial F^{c\dagger}}=\left(K_T^c h^c\right)F_T+K^cF^c$$

$$0=\frac{\partial L_{aux}}{\partial F^\dagger}=\left(K_T h\right)F_T+KF+W_h^\dagger\sim KF+W^\dagger_{h^\dagger T^\dagger}\delta T^\dagger\ \left(\delta T=T-T_0\right)$$

Solution:

$$F_T\sim-\left.\frac{W^\dagger_{h^\dagger T^\dagger}\delta h^\dagger+f_{T^\dagger}F_\phi}{f_{TT^\dagger}+\left(K^c_{TT^\dagger}-\left|K^c_T\right|^2/K^c\right)\left|h^c\right|^2}\right|_{T=T_0,h=0}$$

$$F\sim-\frac{1}{K}W^\dagger_{h^\dagger T^\dagger}\delta T^\dagger\Big|_{T=T_0,h=0},\quad F^c\sim-\frac{1}{K^c}\left(K_T^c h^c\right)F_T$$

Scalar potential up to 2nd order of SUSY breaking:

$$\Delta V = -FW_h - F_T W_T - 2\left(F_\phi W + h.c.\right) - F_\phi^\dagger f_T F_T - \left|F_\phi\right|^2 f$$

$$\sim \frac{1}{K}\left|W_{hT}\delta T\right|^2 + \frac{\left|W_T^\dagger \delta h^\dagger - f_{T^\dagger} F_\phi\right|^2}{f_{TT^\dagger} + \left(K^c_{TT^\dagger} - \left|K^c_T\right|^2 / K^c\right)\left|h^c\right|^2} - \left|F_\phi\right|^2 f$$

$$0 = \frac{\partial \Delta V}{\partial \delta T} \Rightarrow \delta T \sim 0$$

$$0 = \frac{\partial \Delta V}{\partial \delta h} \Rightarrow \delta h \sim \frac{f_T\left(T_0\right)}{W_T\left(T_0\right)} F_\phi^\dagger \sim \frac{M_5^3 F_\phi^\dagger}{J_0 k} e^{-\left(T_0 + T_0^\dagger\right)k\pi}$$

Thus, the radius is stable even with SUSY breaking!!

Scalar Masses Induced by Bulk Fields

Scalar Masses by 1-loop Bulk Hypermultiplet

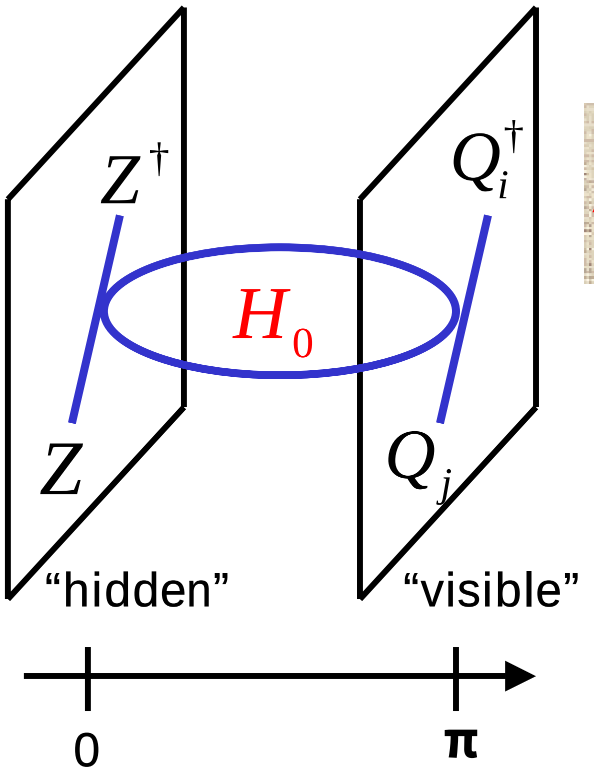
0-mode of H can couple directly to both the visible & hidden sectors
⇒ flavor-violating scalar masses

$$K_{hidden} \sim \frac{1}{|N_0|^2}\int d^4\theta \frac{h_0^\dagger h_0 Z^\dagger Z}{M_5^3}, K_{visible} \sim c_{ij}\frac{e^{(3/2-c)\left(T_0+T_0^\dagger\right)k\pi}}{|N_0|^2}\int d^4\theta \frac{h_0^\dagger h_0 Q_i^\dagger Q_j}{M_5^3}$$

⬇

$$\left(H_0(x,y) = \frac{1}{N_0} e^{(3/2-c)T_0 k|y|} h_0(x), |N_0|^2 = \frac{e^{(1/2-c)\left(T_0+T_0^\dagger\right)k\pi} - 1}{(1-2c)k} \right)$$

$$\Delta\tilde{m}_{ij}^2 \sim \frac{1}{16\pi^2} c_{ij} m_{3/2}^2 \left(\frac{k}{M_4}\right)^2 \left(\frac{1-2c}{e^{(1/2-c)\left(T_0+T_0^\dagger\right)k\pi} - 1}\right)^2 e^{(3/2-c)\left(T_0+T_0^\dagger\right)k\pi}$$



To be suppressed...

c > 3/2: H_0 localized@hidden brane

c < -1/2: H_0 localized@visible brane

c∼3/2,-1/2: $k \ll M_5$

Ex. $k \sim 0.1 M_5, M_4^2 \simeq 10^3 k^2 \Rightarrow \frac{\Delta\tilde{m}_{ij}^2}{\tilde{m}_{AMSB}^2} \sim 10^{-3} \ll 1$

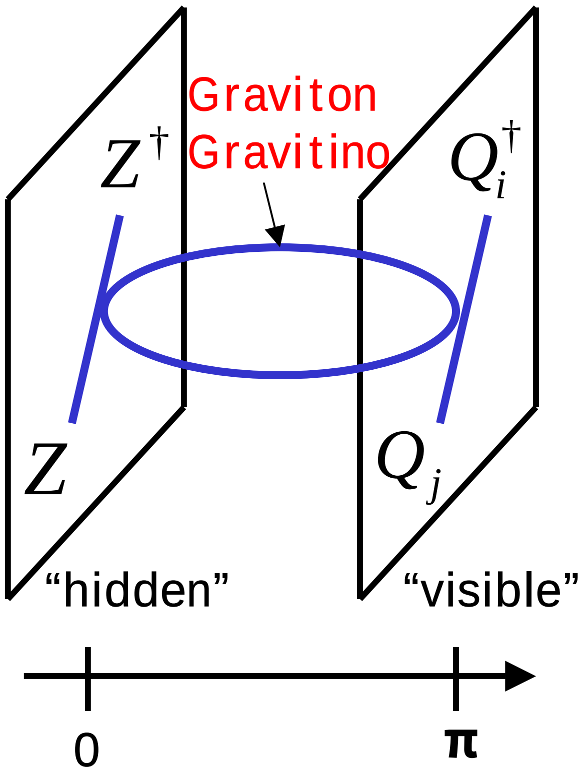
Scalar Masses by 1-loop Gravity Multiplet

Flavor blind but tachyonic ⇒ should be suppressed

Although not yet been explicitly calculated,

naïve guess can be made from the result in the flat case

Gherghetta–Riotto (01)
Rattazzi-Scrucca-Strumia (03)
Buchbinder et al. (03)



$$\Delta \tilde{m}^2_{5Dflat} \sim -\frac{1}{16\pi^2} m^2_{3/2} \frac{1}{\left(M_4 r_0\right)^2}$$

$$M_4^2 = M_5^3 r_0 \left(flat \right)$$

$$M_4^2 = \frac{M_5^3}{k}\left(1 - e^{-2kr_0\pi}\right) \sim \frac{M_5^3}{k}\left(warped \right)$$

$$\Delta \tilde{m}^2_{5Dwarped} \sim -\frac{1}{16\pi^2} m^2_{3/2} \left(\frac{k}{M_4}\right)^2$$

Ex.

$$\Delta \tilde{m}^2_{5Dwarped} / \tilde{m}^2_{AMSB} \sim 10^{-3} \ll 1$$

$$\left(k \sim 0.1 M_5 \Rightarrow M_4^2 \simeq 10^3 k^2\right)$$

Summary & Discussion

Summary

✧We have proposed a simple SUSY model of
radius stabilization in SUSY Randall-Sundrum model
with a bulk hypermultiplet

✧Radius is stabilized by SUSY vacuum conditions
with appropriate sources & bulk mass
⇒ remarkable advantage for model building !!

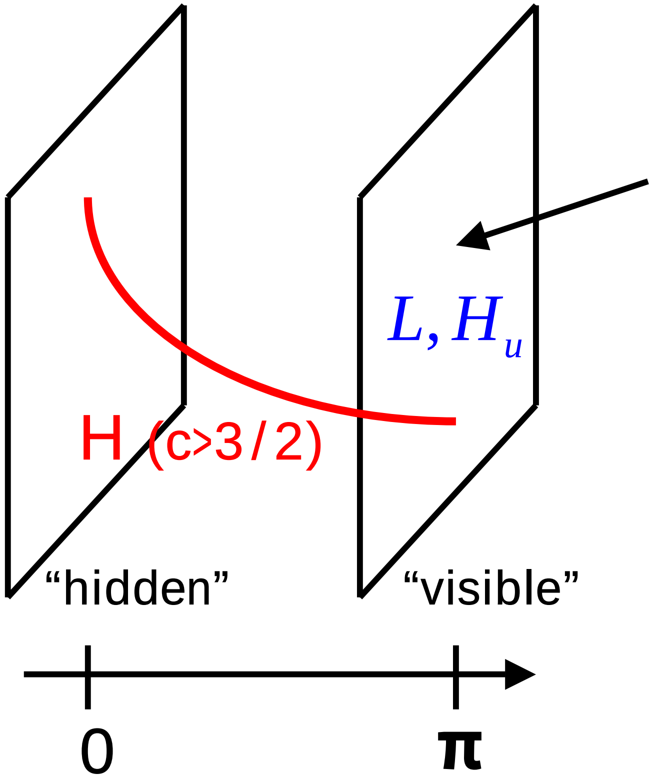
✧Radion Mass >> SUSY breaking scale
Radion potential is not modified so significantly
by SUSY breaking

✧New unwanted scalar masses can be negligible
⇒ Anomaly Mediation dominated (No SUSY flavor problem)

Bonus of Our Model:

If the bulk hypermultiplet "H" is identified with the right-handed **ν** & has Yukawa couplings on the visible brane @y=**π**, a tiny **ν**-mass can be naturally obtained by the mechanism of Grossman & Neubert

Grossman & Neubert (99)



$$W = \delta(y-\pi)\int d^2\theta \sqrt{-g_{vis}}\, Y L H_u H$$

Exponentially suppressed!!

Dynamical Origin of Constant Source $J_{0,\pi}$

Consider SUSY SU(2) gauge theory
with 4 doublets ($V_i\,(i=1\sim 4)$) on each branes
& superpotential $$W=\frac{1}{\sqrt{M_5}}V_iV_jH \ @\ y=0,\pi$$

It is known that in this theory, below **Λ**
meson composite superfields $\left(V_iV_j\right)$ have VEV
through the constraint $Pf\left(V_iV_j\right)=\Lambda^4$

Seiberg (94)

i.e. $$W=\frac{1}{\sqrt{M_5}}V_iV_jH\rightarrow W=\underbrace{\frac{\Lambda^2}{\sqrt{M_5}}}_{J_{0,\pi}}H$$

Goldberger-Wise mechanism

Goldberger-Wise (99)

$$S=\int d^5x\left[\frac{1}{2}\sqrt{-G}\left(G^{AB}\partial_A\Phi\partial_B\Phi-m^2\Phi^2\right)-\sum_{y_i=0,\pi}\delta\left(y-y_i\right)\sqrt{-g_{y=y_i}}\lambda_i\left(\Phi^2-v_{y=y_i}^2\right)^2\right]$$

5D scalar

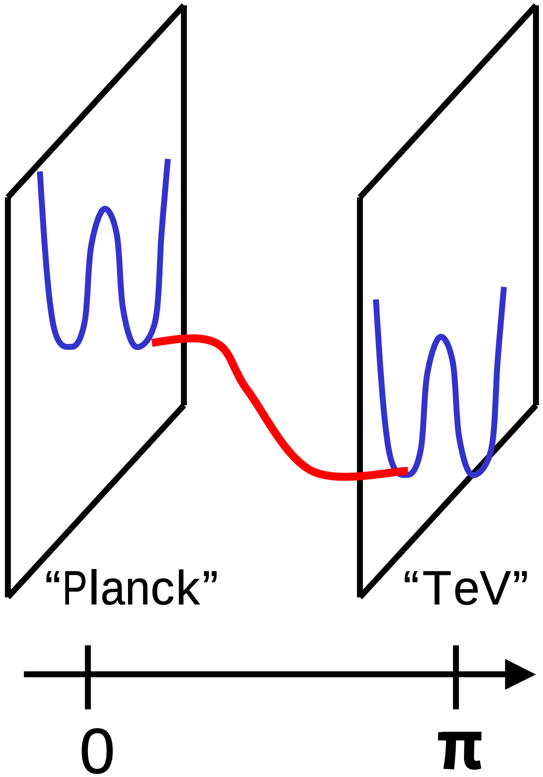
Classical solution:

$$\Phi(y)=e^{2rky}\left[Ae^{\nu rky}+Be^{-\nu rky}\right];\Phi(0)=v_{y=0},\Phi(\pi)=v_{y=\pi},\nu\equiv\sqrt{4+m^2/k^2}$$

$$A=v_{y=0}e^{-(2+\nu)rk\pi}-v_{y=\pi}e^{-2\nu rk\pi},B=-v_{y=0}e^{-(2+\nu)rk\pi}+v_{y=\pi}\left(1+e^{-2\nu rk\pi}\right)\Leftarrow rk\gg 1$$

Potential:

$$V(r)=k\varepsilon v_{y=0}^2+4ke^{-4rk\pi}\left(v_{y=\pi}-v_{y=0}e^{-\varepsilon rk\pi}\right)^2\left(1+\frac{\varepsilon}{4}\right)$$
$$-k\varepsilon v_{y=0}e^{-(4+\varepsilon)rk\pi}\left(2v_{y=\pi}-v_{y=0}e^{-\varepsilon rk\pi}\right)+O\left(\varepsilon^2\right),\varepsilon\sim m^2/4k^2$$



$$rk=\frac{4k^2}{\pi m^2}\ln\left(\frac{v_{y=0}}{v_{y=\pi}}\right);\frac{v_{y=0}}{v_{y=\pi}}=1.5,\frac{m}{k}=0.2\Rightarrow rk\sim 12$$

Kinetic energy ⇒ large “r”

Potential energy ⇒ small “r”

Extension to 3-flavors case

To avoid parity anomaly,
minimal matter content is two bulk right-handed fermions

Mass matrix:

$$M = v\varepsilon^{\nu_2-1/2}\begin{pmatrix} \varepsilon^{\nu_1-\nu_2} & 1 \\ \varepsilon^{\nu_1-\nu_2} & 1 \\ \varepsilon^{\nu_1-\nu_2} & 1 \end{pmatrix} \Rightarrow \begin{cases} m_{\nu_1}^2 \sim 0 \\ m_{\nu_2}^2 \sim M^2\left(v/M\right)^{2\nu_1+1} \\ m_{\nu_3}^2 \sim M^2\left(v/M\right)^{2\nu_2+1} \end{cases}$$

in the basis $\psi_L^\nu = \left(\nu_e^L, \nu_\mu^L, \nu_\tau^L\right), \psi_R^\nu = \left(\psi_0^{R,1}, \psi_0^{R,2}\right); \nu_i = m_i/k\left(i=1,2\right)$

(for simplicity, KK modes are neglected)

$$\nu_1 \approx 1.34-1.37, \nu_2 \approx 1.27-29 \Rightarrow \begin{cases} \Delta m_{21}^2 \approx 10^{-6}-10^{-5} eV^2\left(MSW\right) \\ \Delta m_{32}^2 \approx 5\times 10^{-4} - 6\times 10^{-3} eV^2 \end{cases}$$

Observed hierarchy of **ν** -masses

⇔ small difference of the bulk fermion masses

Superconformal gravity effective Lagrangian
@2-derivative order

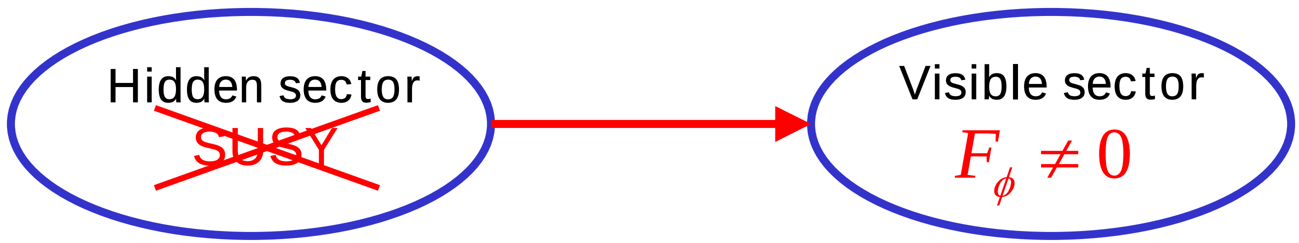
$$L_{4Deff} = \int d^4\theta \phi^\dagger \phi f\left(T^\dagger, T\right)$$
$$= \sqrt{-g}\left[-\frac{1}{6} fR(g) - \frac{1}{4f}\left(f_T \partial^\mu T - h.c.\right)\left(f_T \partial_\mu T - h.c.\right) - f_{T^\dagger T}\partial^\mu T^\dagger \partial_\mu T + fermions\right]$$

(gravity supermultiplet + radion chiral supermultiplet)

Suppose that the hidden (visible) sector resides on the brane @y=0(π)

Bulk-Brane coupling:

$$L_{brane} = \underbrace{\delta(y)\int d^2\theta\phi^3 W_{hidden}}_{\text{No T dependence!!}} + \delta(y-\pi)\int d^2\theta\phi^3 e^{-3Tk\pi} W_{visible}$$



All SUSY breaking effects appear in the visible sector only through $F_\phi \neq 0$

Since it is hard to solve EOMs with $F_\phi \neq 0$,
We will show the radius stability approximately