Phenomenological analysis of lepton and quark Yukawa couplings in SO(10) two Higgs model

1. *Introduction*
2. *Relationship between quark and lepton*
3. *Numerical analysis*
4. *Summary*
5. *Discussion (about Neutrino Osc.)*

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Introduction

SO(10) GUT model seems to us the most attractive model when we take the unification of the quarks and leptons into consideration.

- However, in order to reproduce the observed quark and lepton masses and mixings, usually, a lot of Higgs scalars
- So it is the very crucial problem to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra and mixings.
- If the quark and lepton mass matrices are composed by only one matrix.

$$M_u^0 = c_u M_0^0, \qquad M_d^0 = c_d M_0^0, \qquad M_e^0 = c_e M_0^0,$$

the CKM Matrix must be diagonalized. $\rightarrow \times$

Question

How many parameters included in the mass matrix can be reduced?

- It is difficult that the mass matrices are unified into the one at $m = \Lambda_X$. but can the mass matrices be unified into the two matrices M_1^0 ,M_2^0?

$$M_d^0 = c_{d0} M_0^0 + c_{d1} M_1^0, \quad M_u^0 = c_{u0} M_0^0 + c_{u1} M_1^0, \quad M_e^0 = c_{e0} M_0^0 + c_{e1} M_1^0,$$

$$(M_D^0 = c_{D0} M_0^0 + c_{D1} M_1^0, \quad M_\nu^0 = c_R^{-1} M_D^0 {M_R^0}^{-1} M_D^{0T}.)$$

- In the case where two Higgs scalars, ϕ_{10} and ϕ_{126}, are incorporated in the SO(10) model, the symmetric mass matrices of quarks and charged leptons have the following relations,

$$c_{d0} : c_{d1} : c_{e0} : c_{e1} = c_{u0} : c_{u1} : c_{D2} : c_{D1} = 1 : 1 : 1 : -3$$

- I want to know the probability that such models will be realized without fine tuning.
- Moreover, the effect of RGE is exactly taken into consideration. In this talk, we distinguish between the values at $\mu = \Lambda_X$ and $\mu = m_Z$ by using the superscript "0" or not.

Input Parameters

- The 10,000 sets of the random numbers which become the normal distribution are substituted for each masses and CKM mixing parameters.
  (Particle Data Group, D.E. Groom et al., Eur. Phys. J. **C 15**, 1 (2002);
  M. Jamin et.al, Eur.Phys.J. **C 24**, 273 (2002))

$$|m_u\,(2\mathrm{GeV})| = 2.9 \pm 0.6\mathrm{MeV}, \qquad |m_d\,(2\mathrm{GeV})| = 5.2 \pm 0.9\mathrm{MeV},$$

$$|m_s\,(2\mathrm{GeV})| = 80 - 155\mathrm{MeV}, \qquad |m_c\,(m_c)| = 1.0 - 1.4\mathrm{GeV},$$

$$m_b\,(m_b) = 4.0 - 4.5\mathrm{GeV}, \qquad m_t^{\mathrm{direct}} = 174.3 \pm 5.1\mathrm{GeV},$$

$$\left|m_e^{\mathrm{pole}}\right| = 0.510998902 \pm 0.000000021\mathrm{MeV},$$

$$\left|m_\mu^{\mathrm{pole}}\right| = 105.658357 \pm 0.00005, \qquad m_\tau^{\mathrm{pole}} = 1776.99 \pm 0.29\mathrm{MeV}$$

$$\sin\theta_{12} = 0.2229 \pm 0.0022, \qquad \sin\theta_{23} = 0.0412 \pm 0.0020,$$

$$\sin\theta_{13} = 0.0036 \pm 0.0007, \qquad \delta = (59 \pm 13)^\circ$$

- We estimate the evolution effect about these values from $\mu = m_Z$ to $\mu = \Lambda_X$ by using of RGE.
  (In this work, we suppose MSSM @$tan\beta = 10$)
  (H.Fusaoka and Y.Koide, PRD57 (1998) 3986)

Review

We begin with the short review of our previous work.

- In the case where two Higgs scalars, ϕ_{10} and ϕ_{126}, are incorporated in the SO(10) model, the mass matrices of quarks and charged leptons have the following relations,

$$M_u^0 = c_{u0} M_0^0 + c_{u1} M_1^0, \qquad M_d^0 = M_0^0 + M_1^0, \qquad M_e^0 = M_0^0 - 3M_1^0,$$

$$\longmapsto M_e^0 = c_u M_u^0 + c_d M_d^0 = c_u (M_u^0 + \kappa M_d^0),$$

$$M_0^0 = \frac{3M_d^0 + M_e^0}{4}, \qquad M_1^0 = \frac{M_d^0 - M_e^0}{4},$$

$$c_{u0} = -\frac{c_d - 1}{c_u}, \qquad c_{u1} = -\frac{c_d + 3}{c_u}.$$

- Because M_0^0 and M_1^0 are symmetric at the unification scale $\mu = \Lambda_X$ in the model with one 10 and one 126 Higgs scalars, M_u^0, M_d^0, and M_e^0 are also symmetric.

The diagnosis of our model $M_e^0 = c_u(M_u^0 + \kappa M_d^0)$.

1. We suppose the mass matrices are complex and symmetric.
2. We can take a basis on which the up-quark mass matrix is diagonal without loss of generality in order to compare with the experiment values. Hence, we eliminate parameters without red.

$$M_u^0 \equiv A^T \widetilde{M_u^0} A \equiv A^T \begin{pmatrix} m_u^0 & 0 & 0 \\ 0 & m_c^0 & 0 \\ 0 & 0 & m_t^0 \end{pmatrix} A,$$

$$M_d^0 \equiv A^T \widetilde{M_d^0} A \equiv A^T V^{0\dagger} \begin{pmatrix} m_d^0 & 0 & 0 \\ 0 & m_s^0 & 0 \\ 0 & 0 & m_b^0 \end{pmatrix} V^{0*} A,$$

$$M_e^0 \equiv A^T \widetilde{M_e^0} A \equiv A^T U_e^{0\dagger} \begin{pmatrix} m_e^0 & 0 & 0 \\ 0 & m_\mu^0 & 0 \\ 0 & 0 & m_\tau^0 \end{pmatrix} U_e^{0*} A.$$

Without loss of generality, we can make the masses of third generation positive real number.
Although the remaining masses are complex under the ordinary circumstances,
we assume that all masses are real in order to simplify the problem.

The diagnosis of our model $\widetilde{M_e^0} = c_u(\widetilde{M_u^0} + \kappa\widetilde{M_d^0})$.

1. We suppose the mass matrices are complex and symmetric.
2. We can take a basis on which the up-quark mass matrix is diagonal without loss of generality in order to compare with the experiment values. Hence, we eliminate parameters without red.
3. By using "Tr" and "det", U_{e0} is eliminated.

$$\left(m_e^0\right)^2 + \left(m_\mu^0\right)^2 + \left(m_\tau^0\right)^2 = \mathrm{Tr}\left(\widetilde{L}\right) = c_u^2 \mathrm{Tr}\left\{\widetilde{Q}\left(\kappa\right)\right\},$$

$$\left(m_e^0 m_\mu^0 m_\tau^0\right)^2 = \det\left(\widetilde{L}\right) = c_u^6 \det\left\{\widetilde{Q}\left(\kappa\right)\right\},$$

$$2(\left(m_e^0 m_\mu^0\right)^2 + \left(m_\mu^0 m_\tau^0\right)^2 + \left(m_\tau^0 m_e^0\right)^2) = \left\{\mathrm{Tr}\left(\widetilde{L}\right)\right\}^2 - \mathrm{Tr}\left\{\left(\widetilde{L}\right)^2\right\}$$

$$= c_u^4 \left[\left\{\mathrm{Tr}\left(\widetilde{Q}\left(\kappa\right)\right)\right\}^2 - \mathrm{Tr}\left\{\left(\widetilde{Q}\left(\kappa\right)\right)^2\right\}\right]$$

$$\widetilde{Q}\left(\kappa\right) \equiv \left(\widetilde{M_u^0} + \kappa\widetilde{M_d^0}\right)\left(\widetilde{M_u^0} + \kappa\widetilde{M_d^0}\right)^\dagger, \qquad \widetilde{L} \equiv \widetilde{M_e^0}\widetilde{M_e^0}^\dagger$$

From here, the signs of the charged lepton mass are negligible.

The diagnosis of our model $\widetilde{M_e^0} = c_u(\widetilde{M_u^0} + \kappa\widetilde{M_d^0})$.

1. We suppose the mass matrices are complex and symmetric.
2. We can take a basis on which the up-quark mass matrix is diagonal without loss of generality in order to compare with the experiment values. Hence, we eliminate parameters without red.
3. By using "Tr" and "det", U_{e0} is eliminated.
4. c_u is eliminated

$$A(\kappa) \equiv \frac{\left(\left(m_e^0 m_\mu^0\right)^2 + \left(m_\mu^0 m_\tau^0\right)^2 + \left(m_\tau^0 m_e^0\right)^2\right)}{\left(\left(m_e^0\right)^2 + \left(m_\mu^0\right)^2 + \left(m_\tau^0\right)^2\right)^2} \frac{2\left[\mathrm{Tr}\left\{\widetilde{Q}(\kappa)\right\}\right]^2}{\left\{\mathrm{Tr}\left(\widetilde{Q}(\kappa)\right)\right\}^2 - \mathrm{Tr}\left\{\left(\widetilde{Q}(\kappa)\right)^2\right\}} \to 1$$

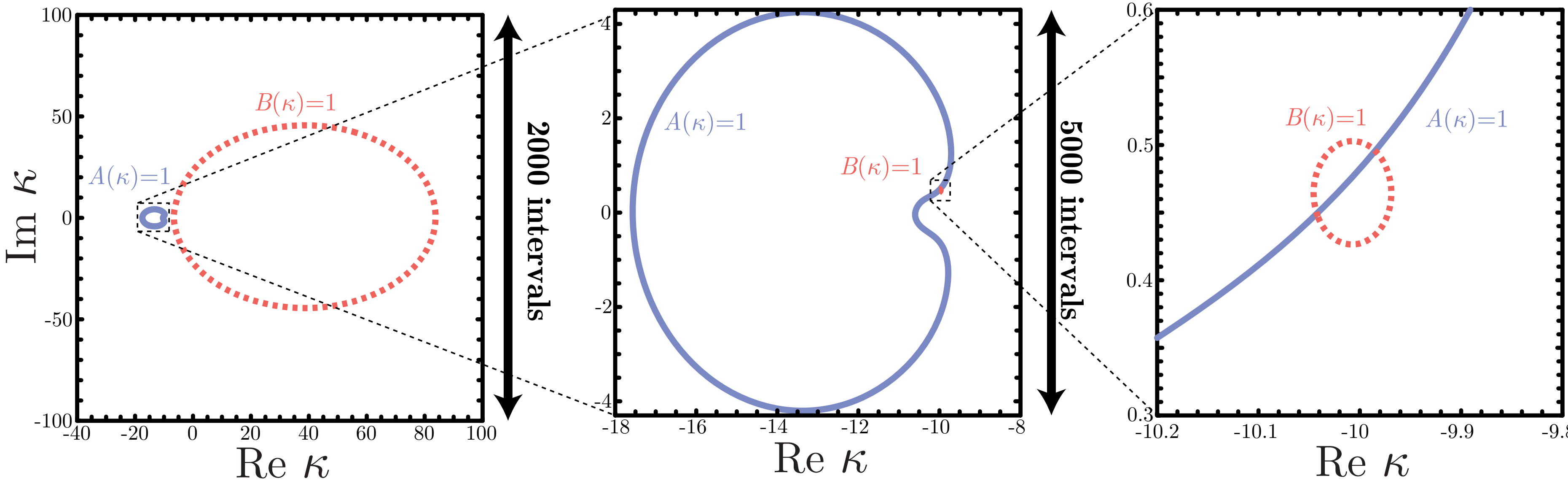
$$B(\kappa) \equiv \frac{\left(m_e^0 m_\mu^0 m_\tau^0\right)^2}{\left(\left(m_e^0\right)^2 + \left(m_\mu^0\right)^2 + \left(m_\tau^0\right)^2\right)^3} \frac{\left[\mathrm{Tr}\left\{\widetilde{Q}(\kappa)\right\}\right]^3}{\det\left\{\widetilde{Q}(\kappa)\right\}} \to 1$$

$$\widetilde{Q}(\kappa) \equiv \left(\widetilde{M_u^0} + \kappa\widetilde{M_d^0}\right)\left(\widetilde{M_u^0} + \kappa\widetilde{M_d^0}\right)^\dagger, \quad \widetilde{L} \equiv \widetilde{M_e^0}\widetilde{M_e^{0\dagger}}$$

We look for κ which sets $A(\kappa)$ and $B(\kappa)$ to 1 simultaneously.

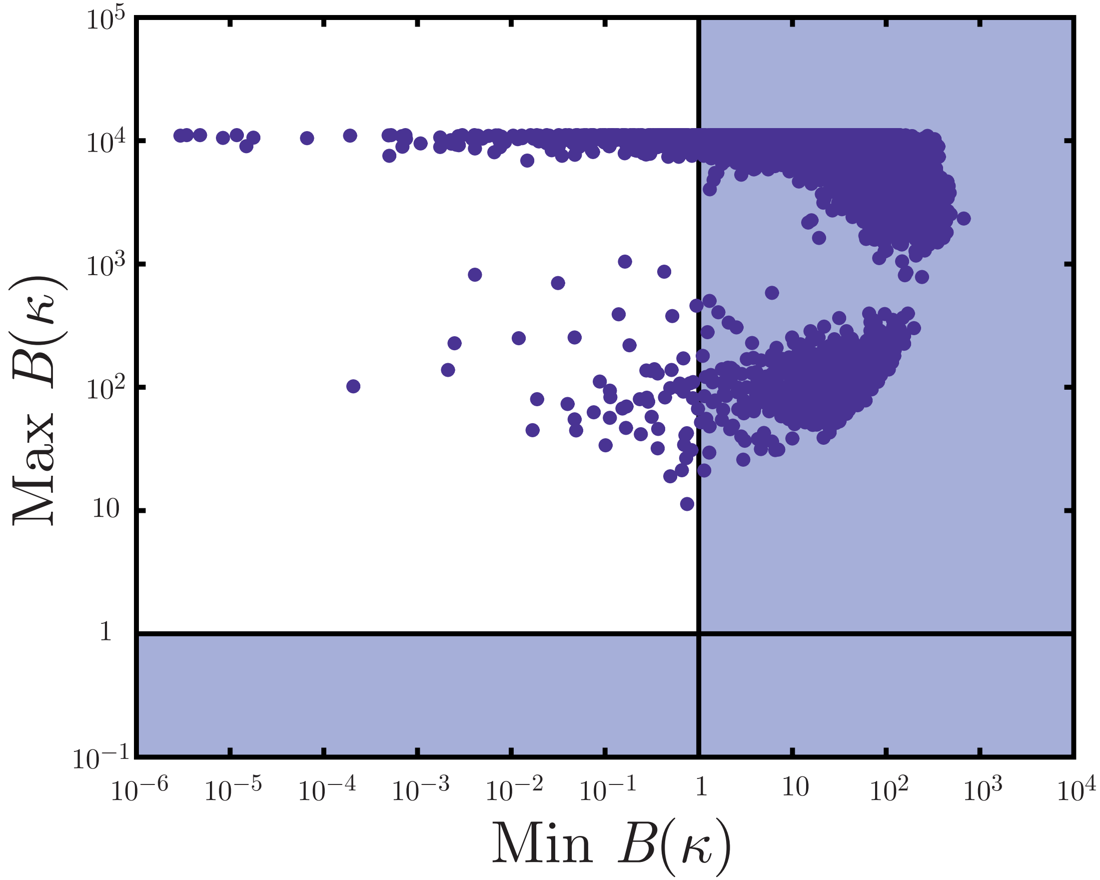
Contour Plot

We scan the range $A(\kappa) = 1$ by changing Im(κ) from -100 to 100 at 2000 equal intervals. Moreover, we get the maximum and minimum of $B(\kappa)$ on the line of $A(\kappa) = 1$ by changing Im(κ) at 5000 equal intervals.



The solid line show $A(\kappa) = 1$ and the dotted line $B(\kappa) = 1$.
This is an example which is given as follows:
D_u^0 = diag$(-4.3036 \times 10^{-6}, -0.0013986, +0.61064)$,
D_d^0 = diag$(-5.4757 \times 10^{-5}, -1.0839 \times 10^{-3}, +0.053905)$,
D_e^0 = diag$(1.8763 \times 10^{-5}, 3.9602 \times 10^{-3}, 0.067624)$,
$\theta_{12}^0 = 0.22329$, $\theta_{23}^0 = 0.035861$, $\theta_{31}^0 = 3.6715 \times 10^{-3}$, and $\delta^0 = 0.85149$.

Variation in $B(\kappa)$



The maximum and minimum of $B(\kappa)$ on the line of $A(\kappa) = 1$ in the case of (15) in Table 1. Because $B(\kappa)$ is continuous, there is the κ which sets $A(\kappa)$ and $B(\kappa)$ to 1 simultaneously when Min$(B(\kappa)) < 1 <$ Max$(B(\kappa))$. There are 10000 dots in all area, and 482 dots in the white area as tabulated in Table 1.

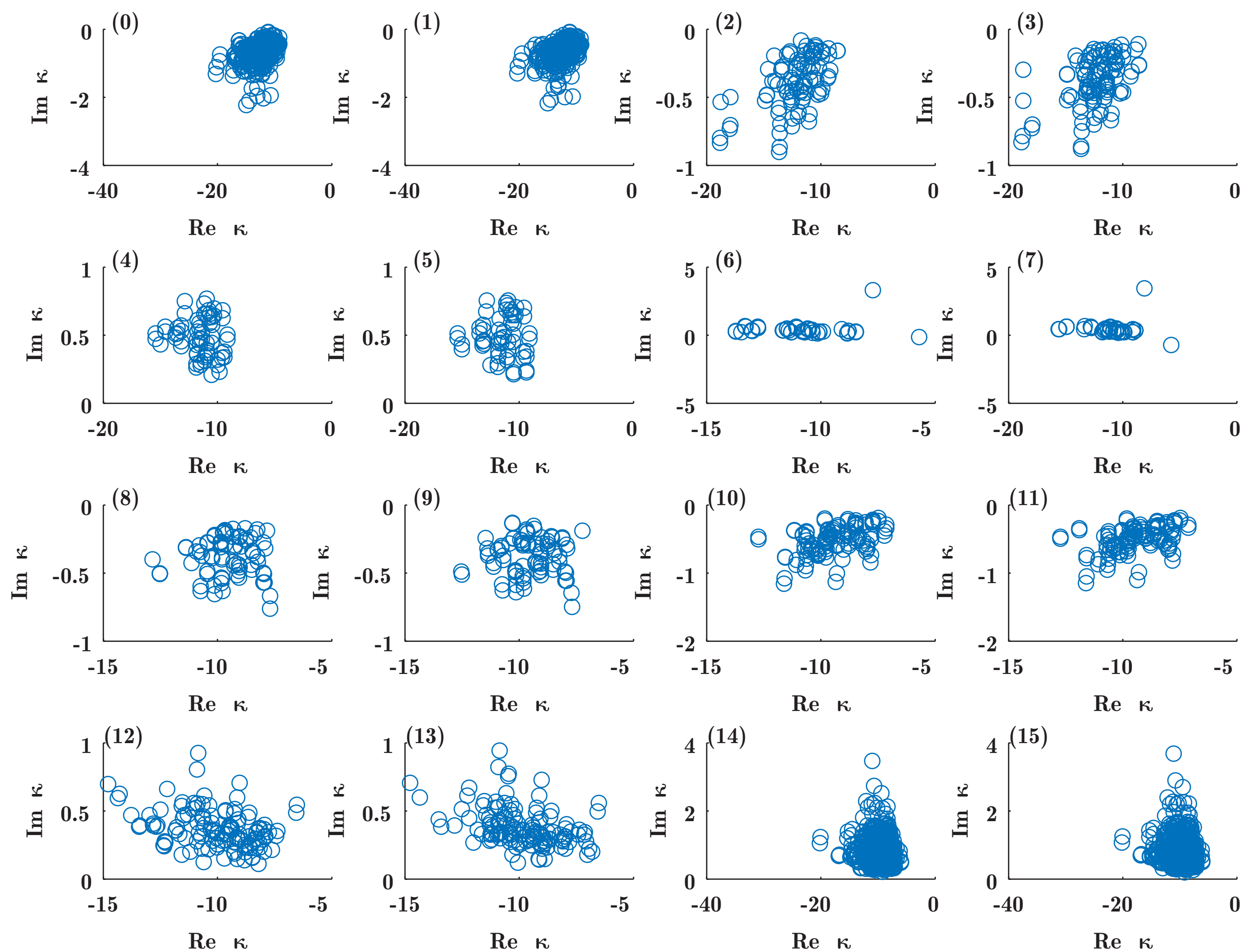
The combinations of the signs

	(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum		(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum
(0)	$(+++)$	$(+++)$	344	(8)	$(+++)$	$(+-+)$	56
(1)	$(-++)$	$(+++)$	328	(9)	$(-++)$	$(+-+)$	60
(2)	$(+-+)$	$(+++)$	225	(10)	$(+-+)$	$(+-+)$	54
(3)	$(--+)$	$(+++)$	209	(11)	$(--+)$	$(+-+)$	56
(4)	$(+++)$	$(-++)$	34	(12)	$(+++)$	$(--+)$	283
(5)	$(-++)$	$(-++)$	30	(13)	$(-++)$	$(--+)$	294
(6)	$(+-+)$	$(-++)$	35	(14)	$(+-+)$	$(--+)$	470
(7)	$(--+)$	$(-++)$	35	(15)	$(--+)$	$(--+)$	482

The combinations of the signs of (m_u, m_c, m_t) and (m_d, m_s, m_b). The signs of the charged lepton are negligible. And the total number of the cases conforming to the requirements $A(\kappa)$=$B(\kappa)$=1 after the 10,000 substitutions.

- This probability will increase if the signs of m_d, and m_s are same.
- The probability that the model will be realized without fine tuning is about 5% if we select the appropriate signs (14) or (15) of the masses.

The distribution of κ



The distribution of κ in the complex plane. Each circle shows the value of κ to meet the requirement $A(\kappa) = B(\kappa) = 1$

The combinations of the signs

	(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum		(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum
(0)	$(+++)$	$(+++)$	344	(8)	$(+++)$	$(+-+)$	56
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- This probability will increase if the signs of m_d, and m_s are same.
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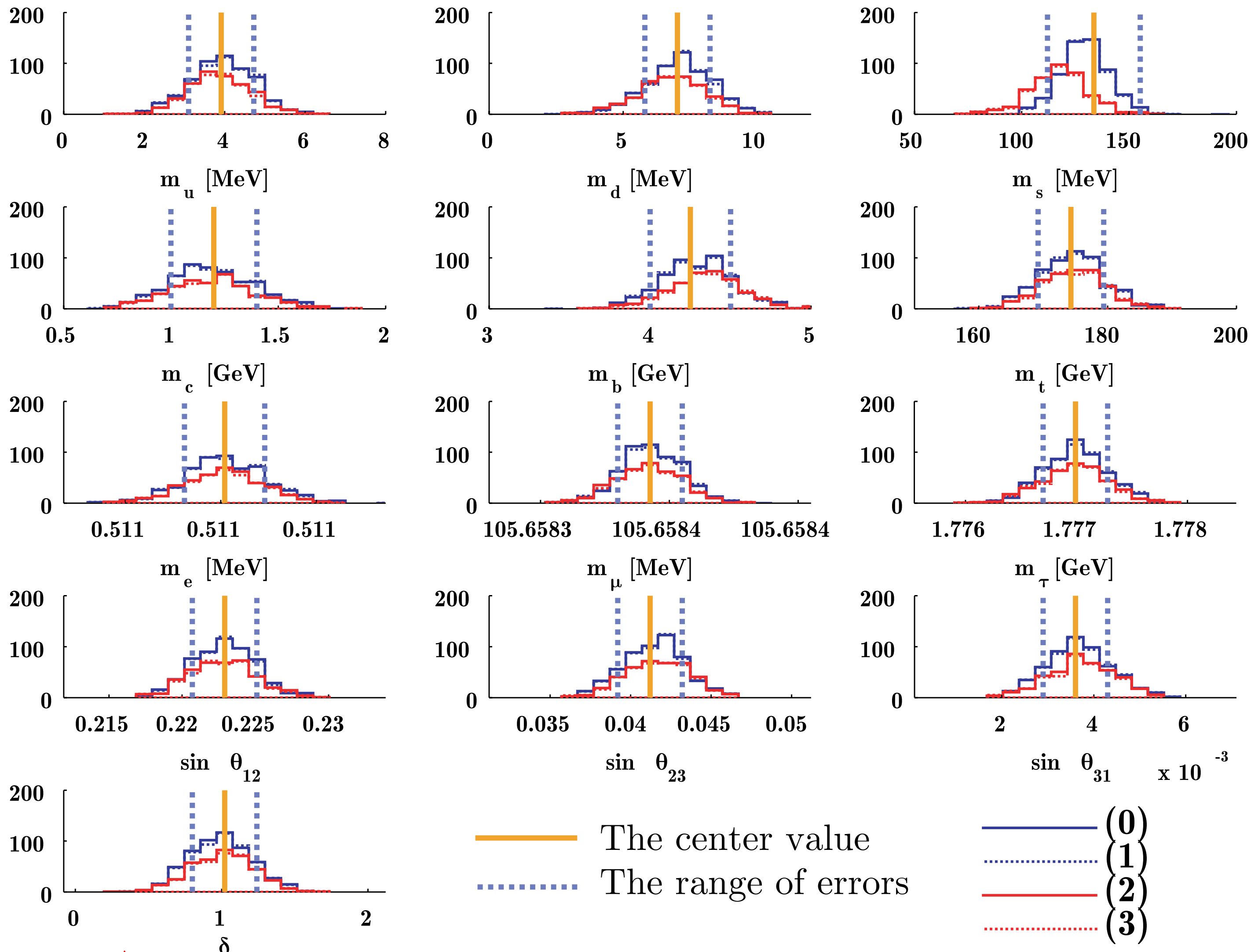
Results : up(±++) down(+++) and up(±−+) down(+++)



The combinations of the signs

	(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum		(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum
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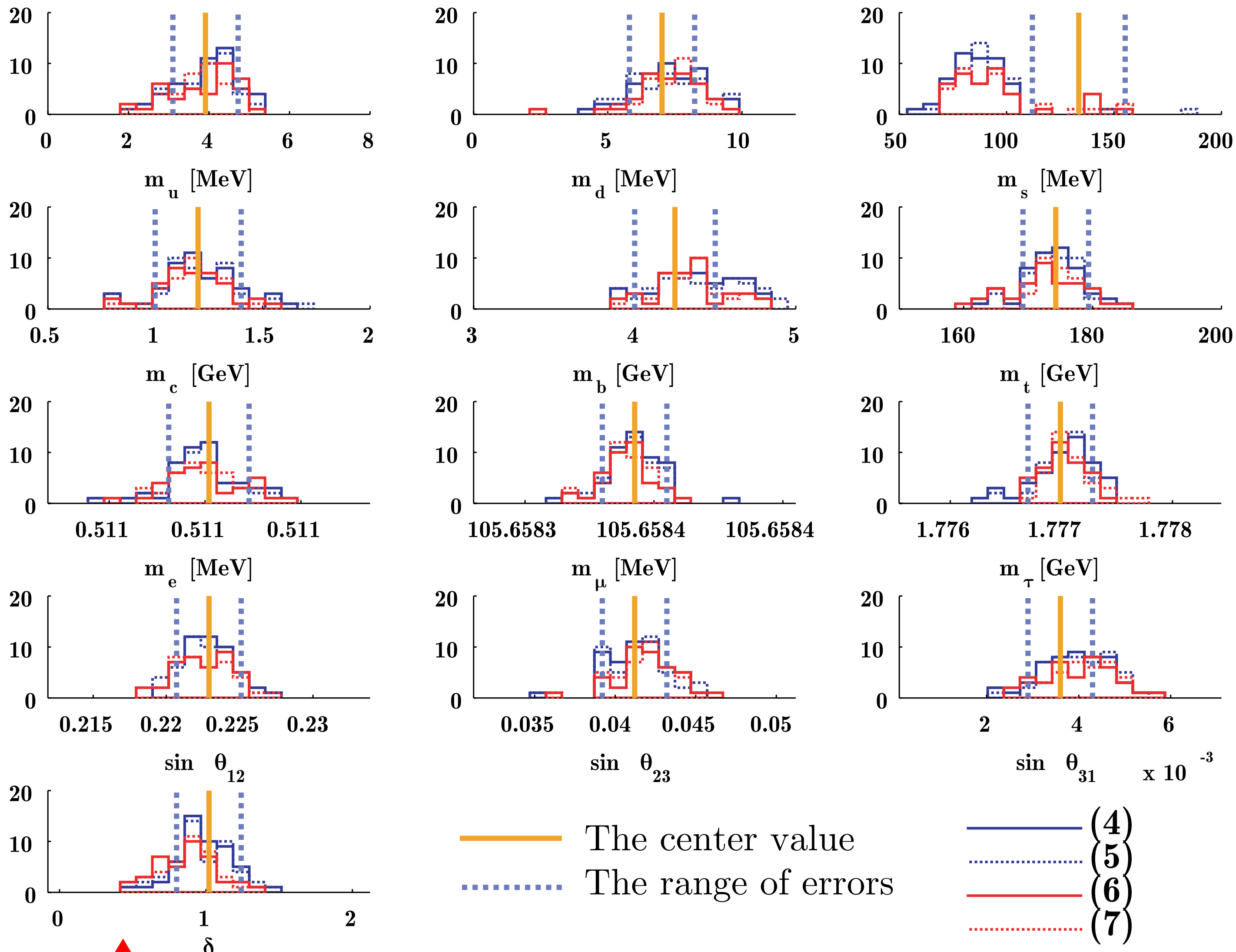
Results : up(±++) down(−++) and up(±−+) down(−++)



The combinations of the signs

	(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum		(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum
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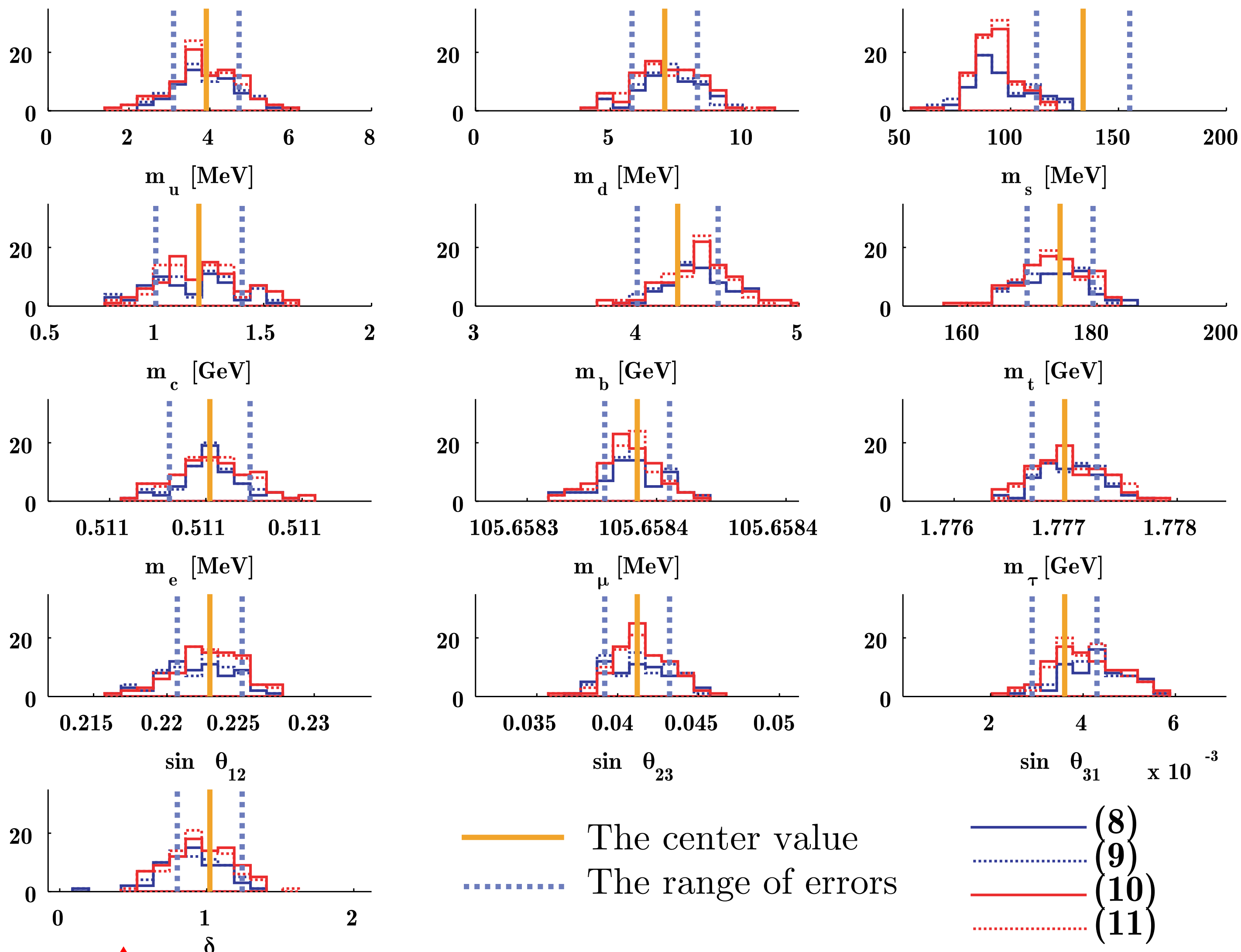
- This probability will increase if the signs of m_d, and m_s are same.
- The probability that the model will be realized without fine tuning is about 5% if we select the appropriate signs (14) or (15) of the masses.

Results : up($\pm$++) down(+−−+) and up($\pm$−+) down(+−−+)



The combinations of the signs

	(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum		(m_u, m_c, m_t)	(m_d, m_s, m_b)	sum
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- This probability will increase if the signs of m_d, and m_s are same.
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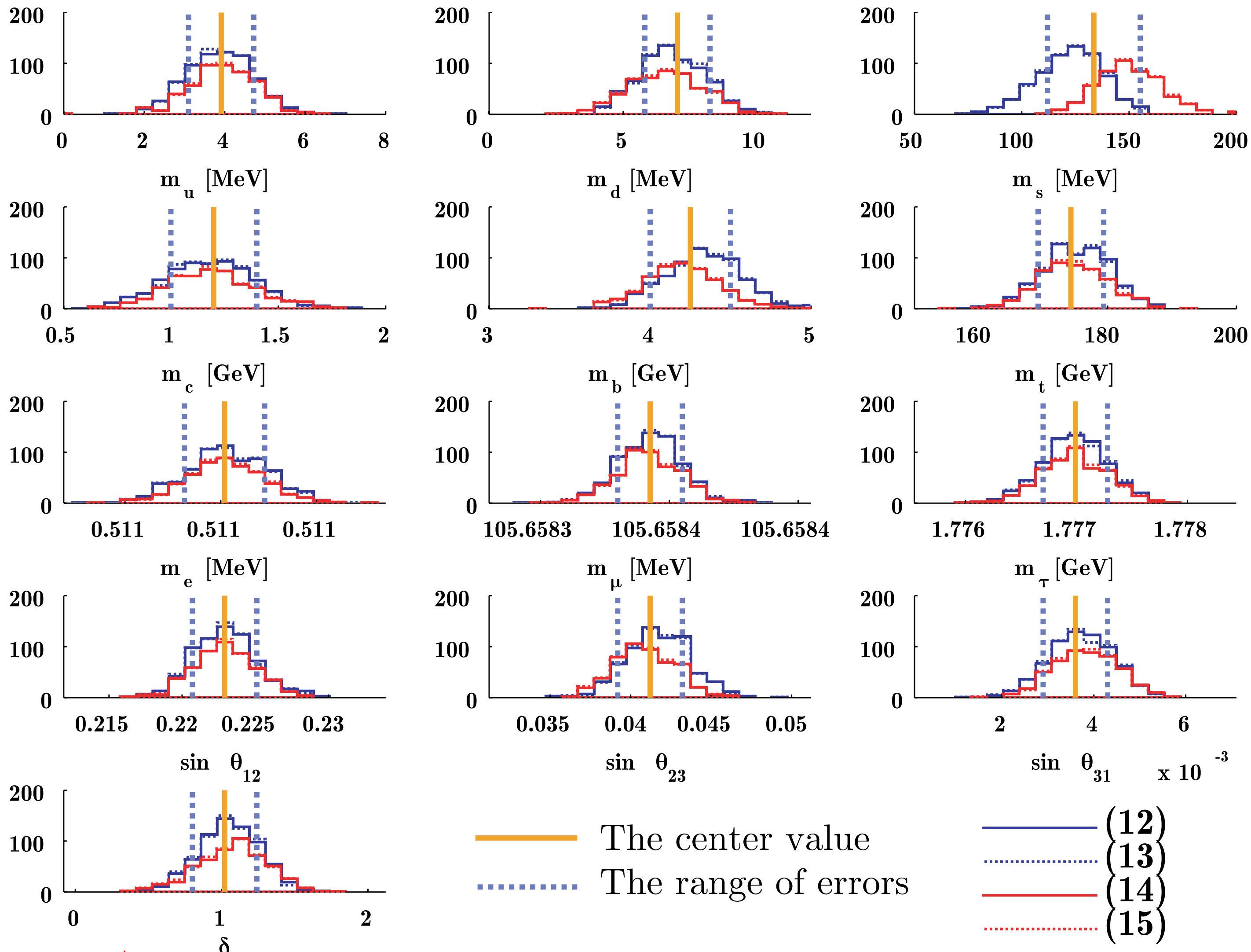
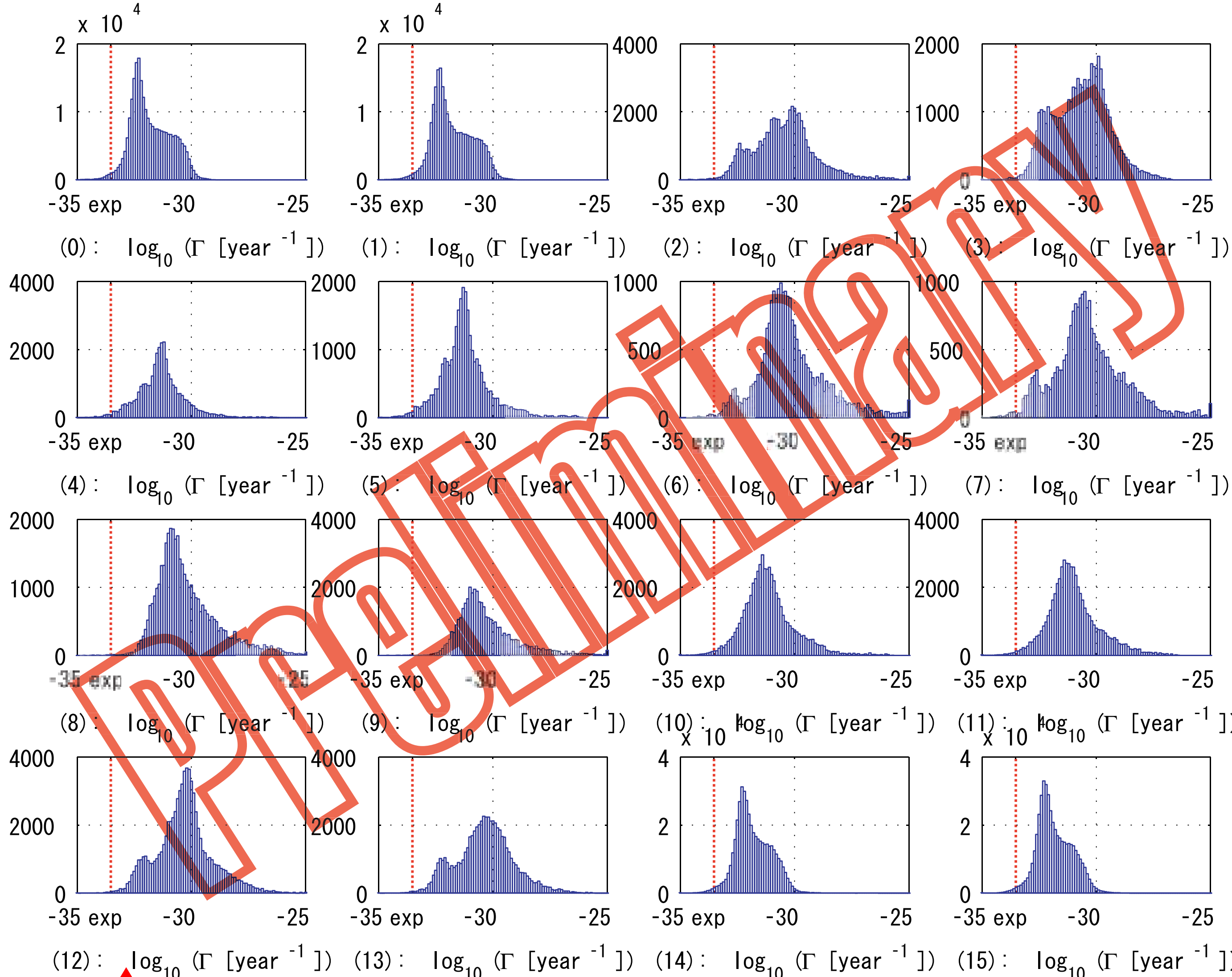
Results : up(±++) down(−−+) and up(±−+) down(−−+)



Proton Decay($p \to K^+\bar{\nu}$)



Conclusion

- We have discussed the probability that the our model will be realized without fine tuning.
- The random numbers which become normal distributions have been substituted for each physical value.
- And we have taken the RGE effect between $\mu \sim m_Z$ and Λ_X into consideration. In this way, the search for κ which sets $A(\kappa)$ and $B(\kappa)$ to 1 simultaneously has been repeated 10,000 times.

In this way, we have arrived at three conclusions:

- From the histograms, this probability will increase if we make m_s somewhat larger or smaller than the present experiment value properly.
- This probability will increase if the signs of m_d, and m_s are same. This gives the suggestion to the texture model. For example, a model with a texture $(M_d)_{11} = 0$ on the nearly diagonal basis of the up-quark Yukawa coupling M_u is denied because these model leads to $m_d/m_s < 0$.
- The probability that the model will be realized without fine tuning is about 5% if we select the appropriate signs (14) or (15) of the masses.

Discussions 1

- In the minimal SO(10) GUT model, the mass matrices of quarks and leptons have the following forms:

$$M_d^0 = M_0^0 + M_1^0, \quad M_u^0 = c_{u0} M_0^0 + c_{u1} M_1^0, \quad M_e^0 = M_0^0 - 3M_1^0,$$
$$M_R^0 = c_R M_1^0, \quad M_D^0 = c_{u0} M_0^0 - 3c_{u1} M_1^0, \quad M_\nu^0 = c_R^{-1} M_D^0 {M_R^0}^{-1} M_D^{0T}.$$

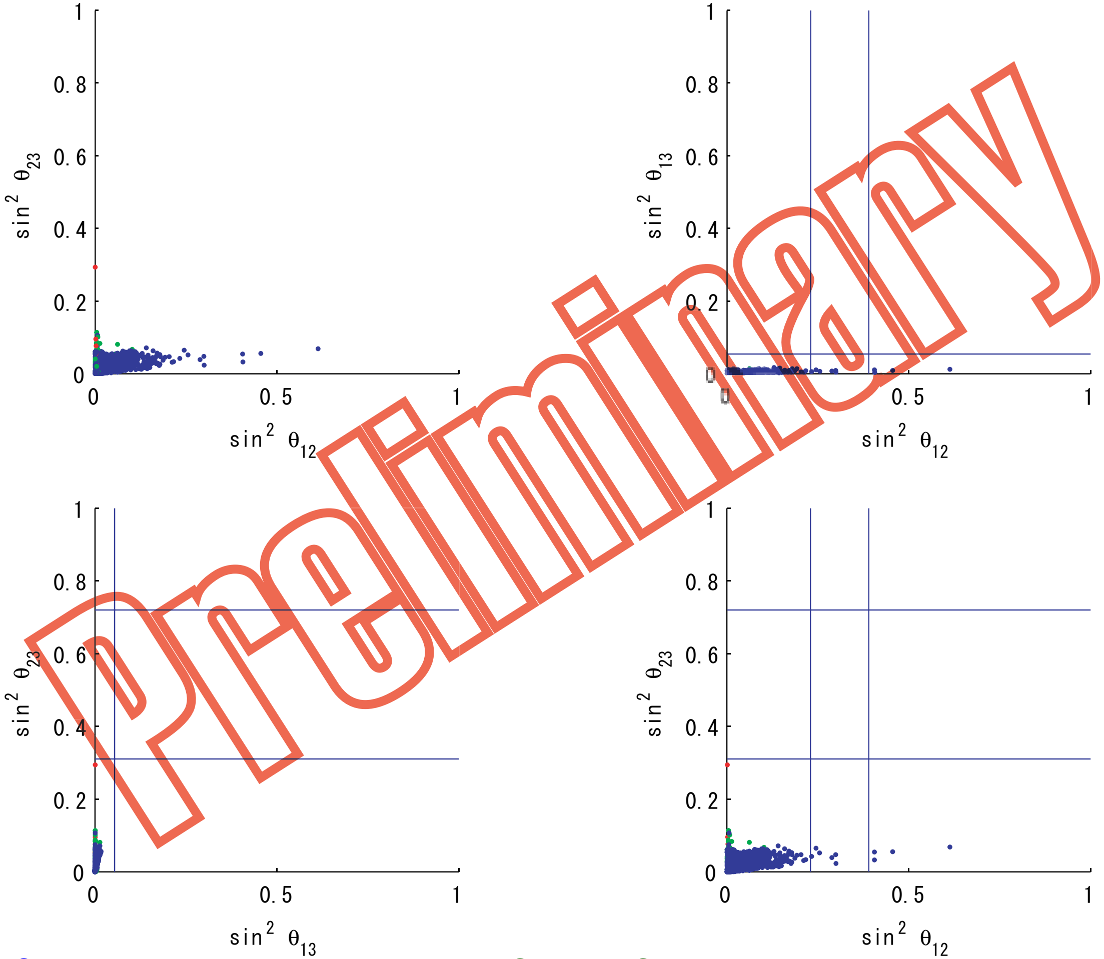
- These relations are rewritten as follows:

$$M_e^0 = c_u (M_u^0 + \kappa M_d^0) = |c_u| e^{i\sigma} (M_u^0 + \kappa M_d^0)$$

Here, κ and $|c_u|$ can be constrained by the charged lepton mass ratios and absolute values, respectively. However, one parameter σ remains unknown. Therefore we move σ between 0 and 2π.

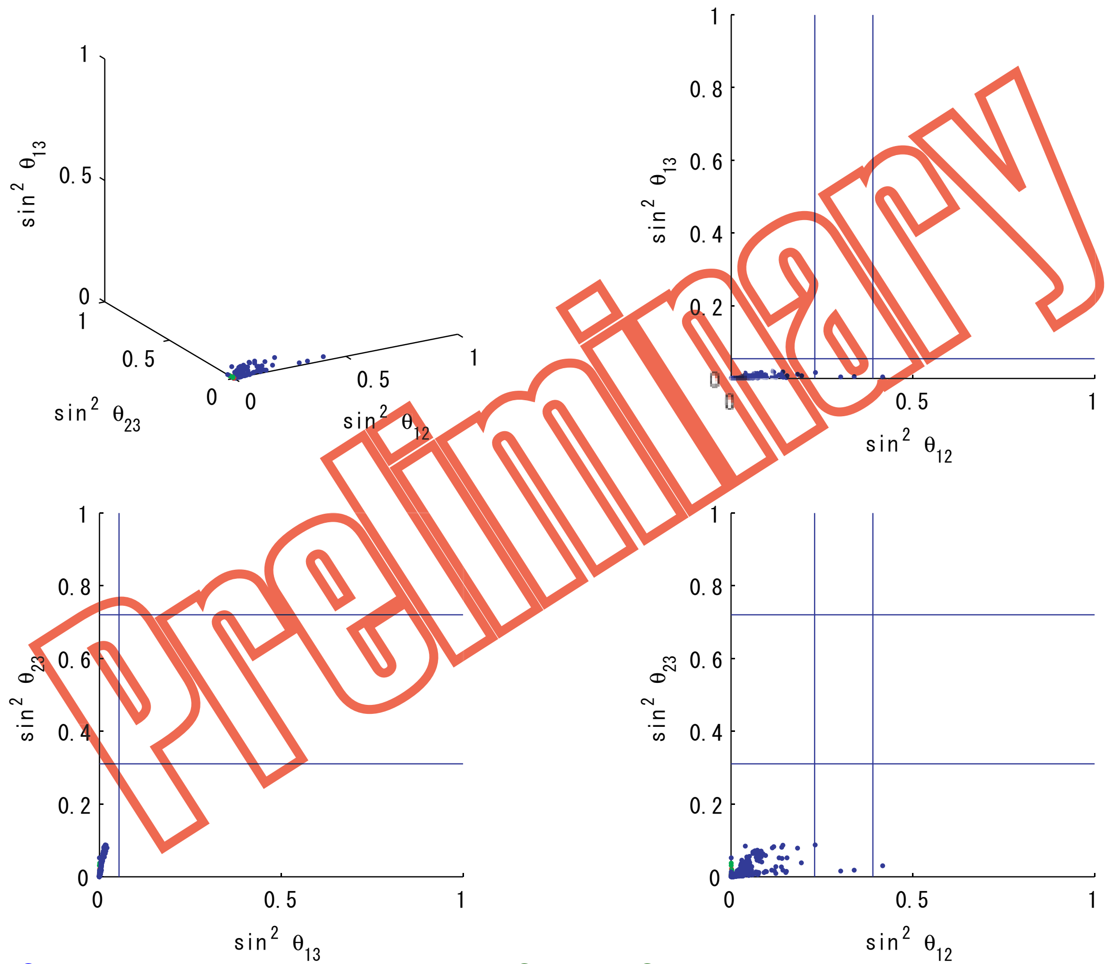
$$M_\nu^0 \propto (c_{u0} M_0^0 - 3c_{u1} M_1^0) {M_1^0}^{-1} (c_{u0} M_0^0 - 3c_{u1} M_1^0)^T.$$
$$c_{u0} = \frac{1}{|c_u| e^{i\sigma}} - \kappa, \qquad c_{u1} = -\frac{3}{|c_u| e^{i\sigma}} - \kappa$$

Neutrino : up$(\pm++)$ *down*$(+++) \rightarrow$ *344*



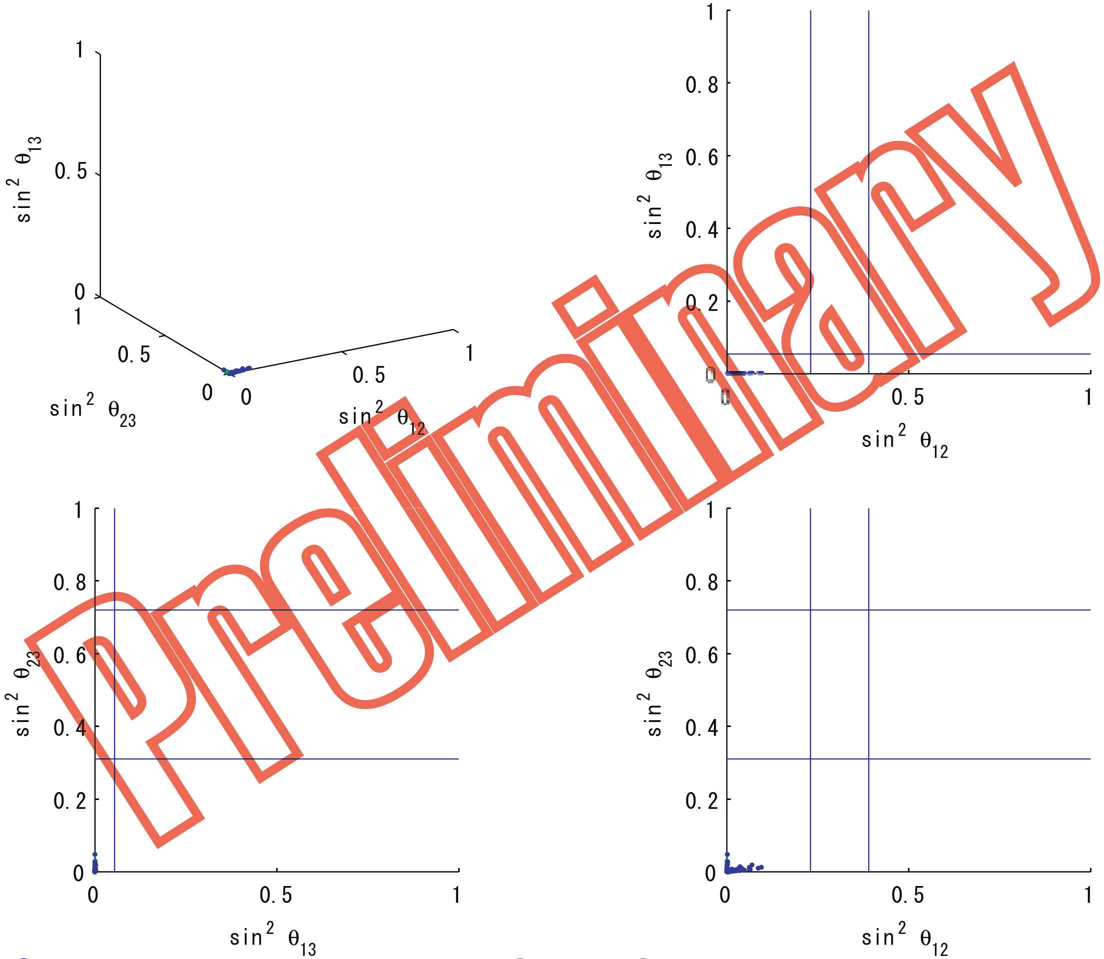
$\delta m_{12}^2/\delta m_{13}^2 < 0.018$, $0.018 < \delta m_{12}^2/\delta m_{13}^2 < 0.053$, $0.053 < \delta m_{12}^2/\delta m_{13}^2$

Neutrino : $up(\pm - +)$ $down(+++)$ $\rightarrow 225$



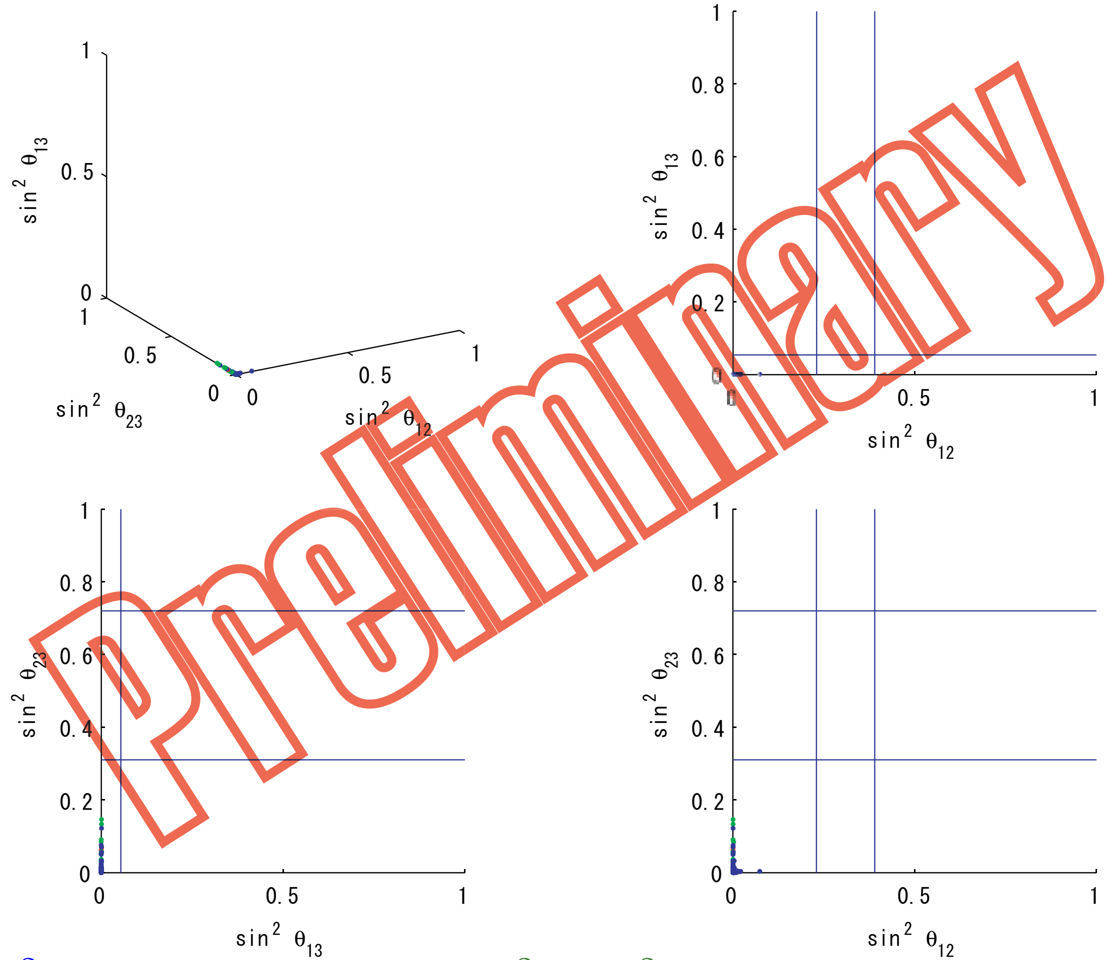
$\delta m^2_{12}/\delta m^2_{13} < 0.018,$ $0.018 < \delta m^2_{12}/\delta m^2_{13} < 0.053,$ $0.053 < \delta m^2_{12}/\delta m^2_{13}$

Neutrino : up$(\pm++)$ *down*$(-++) \rightarrow$ *34*



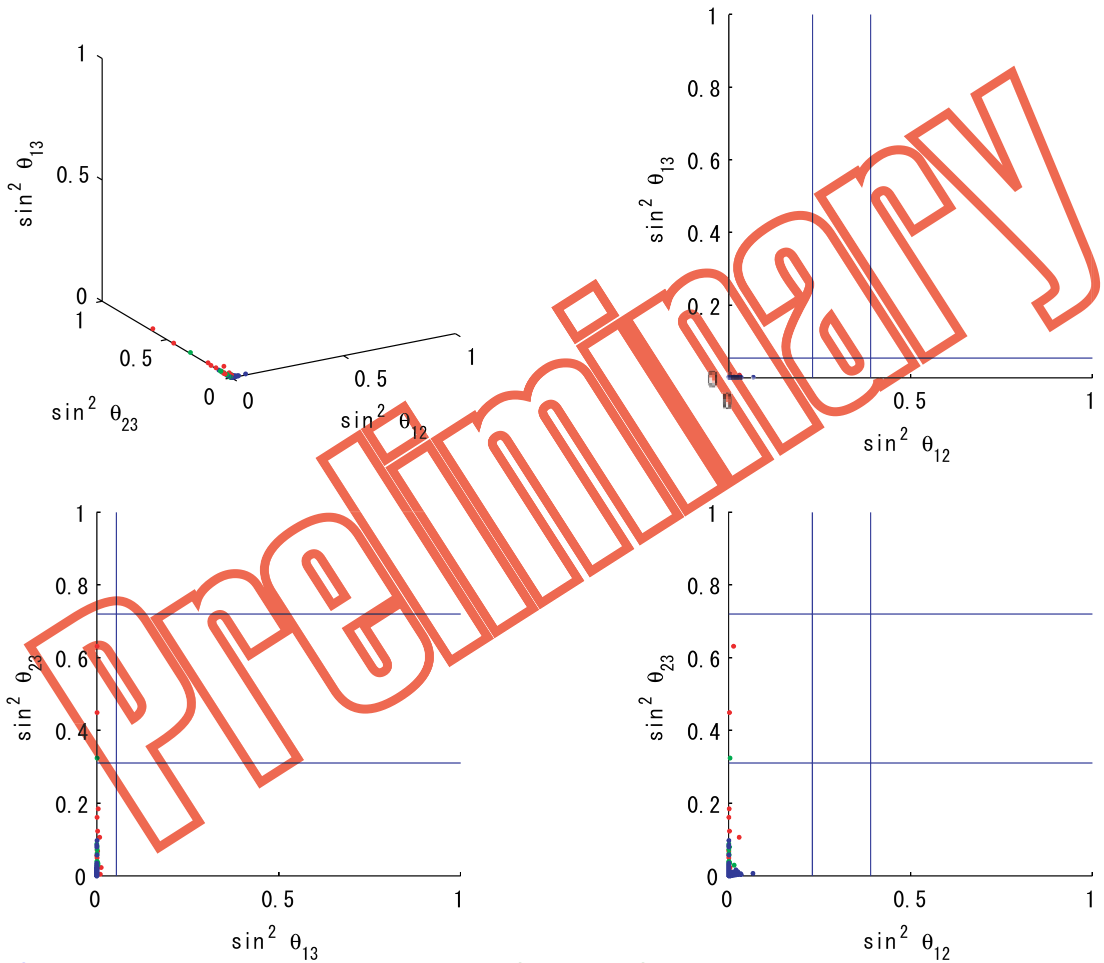
$\delta m^2_{12}/\delta m^2_{13} < 0.018$, $0.018 < \delta m^2_{12}/\delta m^2_{13} < 0.053$, $0.053 < \delta m^2_{12}/\delta m^2_{13}$

Neutrino : up($\pm - +$) down($-++$) $\rightarrow$ 35



$\delta m^2_{12}/\delta m^2_{13} < 0.018$, $0.018 < \delta m^2_{12}/\delta m^2_{13} < 0.053$, $0.053 < \delta m^2_{12}/\delta m^2_{13}$

Neutrino : up$(\pm + +)$ down$(+ - +) \rightarrow 56$



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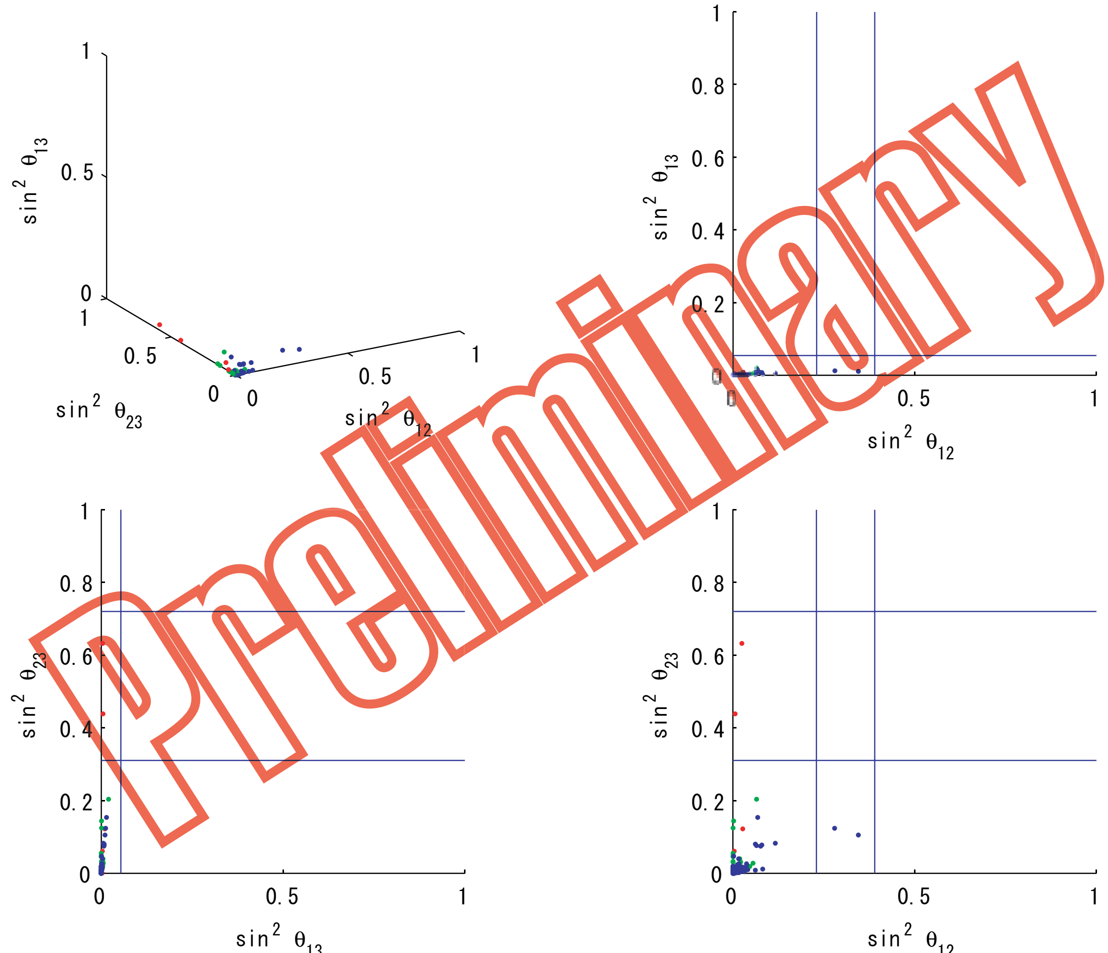
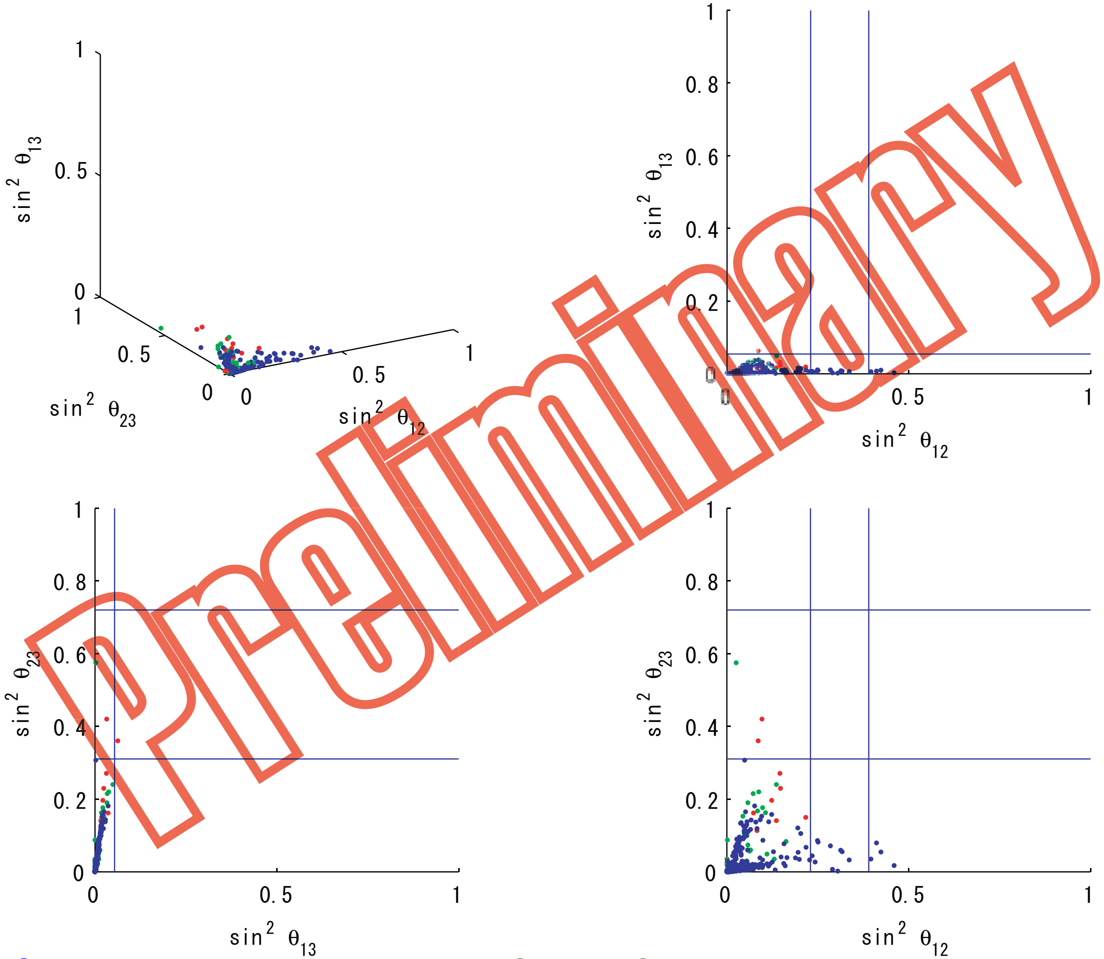
Neutrino : up(± − +) down(+−+) → 54



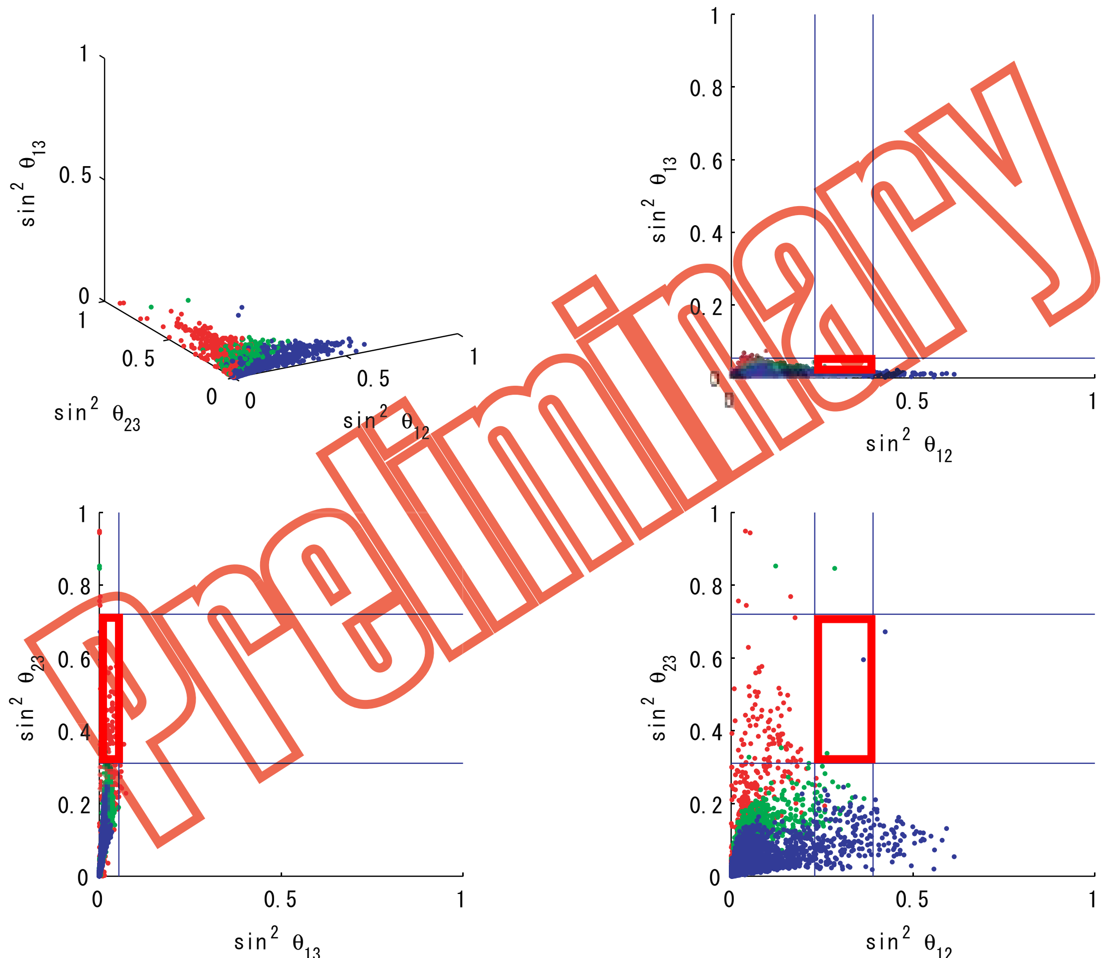
$\delta m^2_{12}/\delta m^2_{13} < 0.018,\quad 0.018 < \delta m^2_{12}/\delta m^2_{13} < 0.053,\quad 0.053 < \delta m^2_{12}/\delta m^2_{13}$

Neutrino : up$(\pm + +)$ *down*$(- - +) \rightarrow$ *283*



$\delta m^2_{12}/\delta m^2_{13} < 0.018$, $0.018 < \delta m^2_{12}/\delta m^2_{13} < 0.053$, $0.053 < \delta m^2_{12}/\delta m^2_{13}$

Neutrino : *up*$(\pm - +)$ *down*$(- - +) \rightarrow$ *470*



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Discussions 2

- In the minimal SO(10) GUT model, the mass matrices of quarks and leptons have the following forms:

$$M_d^0 = M_0^0 + M_1^0, \quad M_u^0 = c_{u0} M_0^0 + c_{u1} M_1^0, \quad M_e^0 = M_0^0 - 3 M_1^0,$$
$$M_R^0 = c_R M_1^0, \quad M_D^0 = c_{u0} M_0^0 - 3 c_{u1} M_1^0, \quad M_\nu^0 = c_R^{-1} M_D^0 {M_R^0}^{-1} M_D^{0T}.$$

- These relations are rewritten as follows:

$$M_e^0 = c_u (M_u^0 + \kappa M_d^0) = |c_u| e^{i\sigma} (M_u^0 + \kappa M_d^0)$$
$$M_\nu^0 \propto (c_{u0} M_0^0 - 3 c_{u1} M_1^0) {M_1^0}^{-1} (c_{u0} M_0^0 - 3 c_{u1} M_1^0)^T.$$

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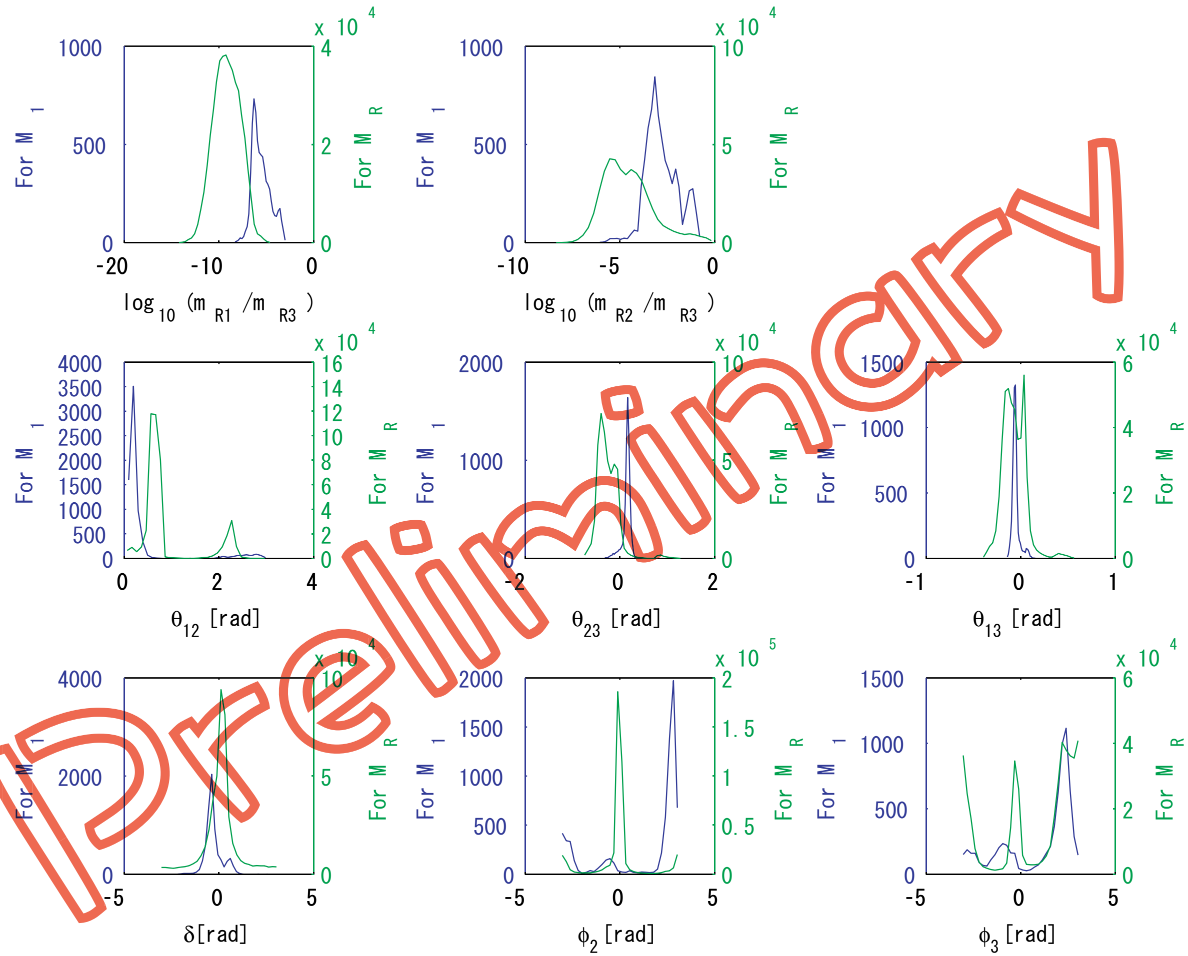
- Conversely, We can predict the pattern of M_R by using the experimental values of ν oscillations; (M.C Gonzalez-Garcia et al., hep-ph/0406056)

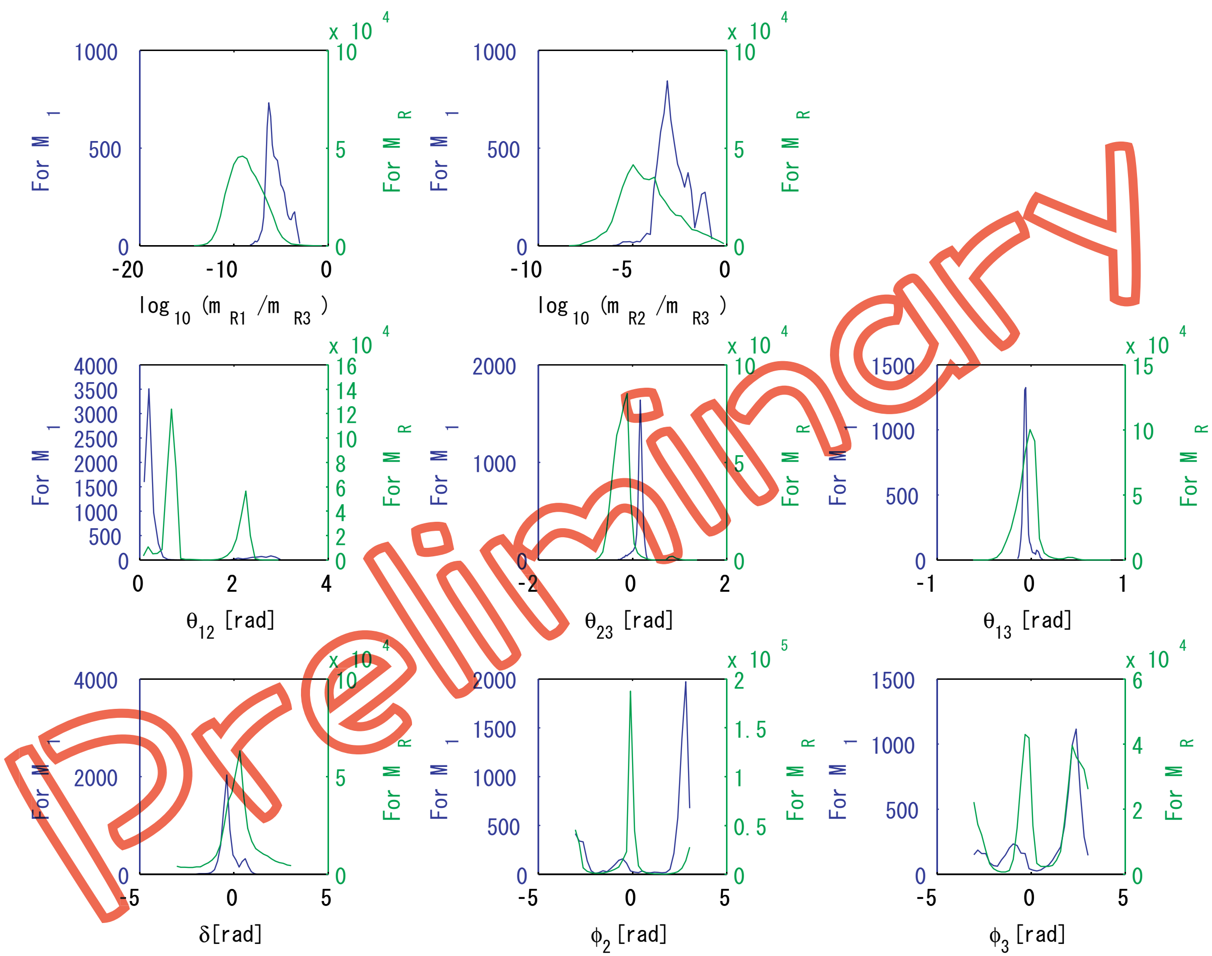
$$\Delta m_{12}^2 = 7.1 \times 10^{-5} \text{eV}^2, \qquad \Delta m_{23}^2 = 2.2 \times 10^{-3} \text{eV}^2,$$
$$\tan^2 \theta_{12} = 0.41, \qquad \tan^2 \theta_{12} = 0.41, \qquad \tan^2 \theta_{13} = 0,$$
$$\phi_2, \phi_3 = 0 - 2\pi \quad \leftarrow \text{Majorana phases.}$$

up$(\pm - +)$ *down*$(- - +)$ *and* $m_1 = 10^{-3}$eV



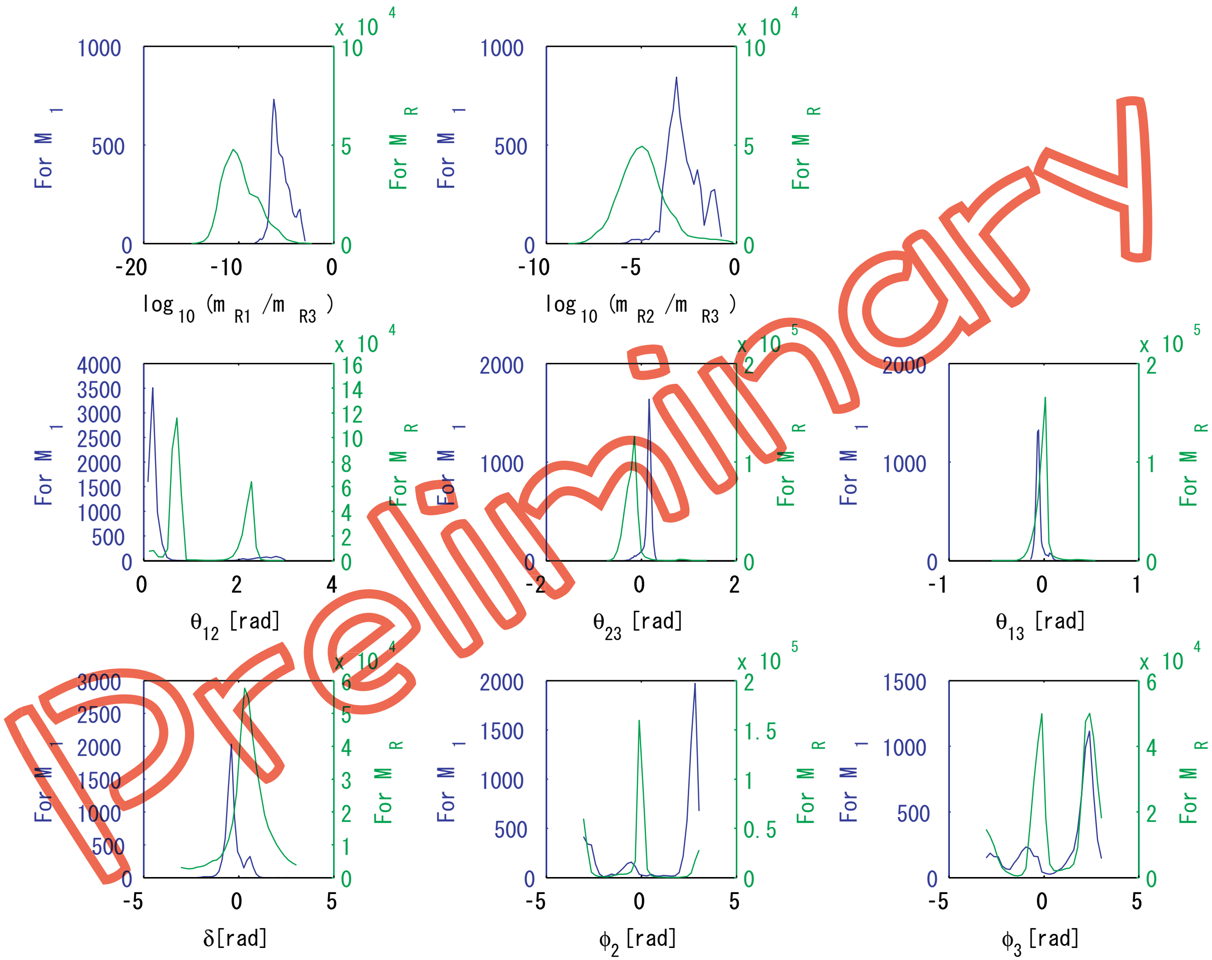
In the basis on which the charged lepton mass matrix is diagonal.

$up(\pm - +)$ $down(- - +)$ and $m_1 = 10^{-2}$eV



In the basis on which the charged lepton mass matrix is diagonal.

$up(\pm - +)$ $down(- - +)$ and $m_1 = 10^{-1}$eV



In the basis on which the charged lepton mass matrix is diagonal.

The number of parameters in our model

$$M_u^0 = M_0^0 + M_1^0, \quad M_d^0 = M_0^0 + M_1^0, \quad M_e^0 = M_0^0 - 3M_1^0, \quad M_e^0 = M_0^0 - 3M_1^0,$$

$$\longmapsto M_e^0 = c_u M_u^0 + c_d M_d^0 = c_u (M_u^0 + \kappa M_d^0), \qquad (1)$$

$$M_\nu^0 = c_R^{-1} M_D^0 \, M_1^{0\,-1} M_D^{0\,T} \qquad (2)$$

Free pmt. of Model

	D_u, D_d, D_e, D_ν	3×4 =12
	$c_u, \lvert c_R \rvert, \kappa$	$2+1+2$ = 5
+)	V_{CKM} , A_e, A_ν	$4+9+9$ =22
	# of pmt.	39
−)	# of eqs.	24
	# of free pmt.	15

Input

The 2(=15-13) parameter still remain as free pmt.

$$\sigma = \arg c_u, \ \lvert c_R \rvert$$

The ν masses and MNS parameters are derived from Eq.(2) when σ and $\lvert c_R \rvert$ are substituted.

Observable pmt.

m_u, m_c, m_t	3
m_d, m_s, m_b	3
CKM:$\theta_{12}, \theta_{23}, \theta_{13}, \delta$	4
m_e, m_μ, m_τ	3
# of input pmt.	13

$m_{\nu e}, m_{\nu \mu}, m_{\nu \tau}$	3
MNS:$\theta_{12}, \theta_{23}, \theta_{13}, \delta, \beta, \rho$	6
# of output pmt.	9

see PRD **65** 033008.

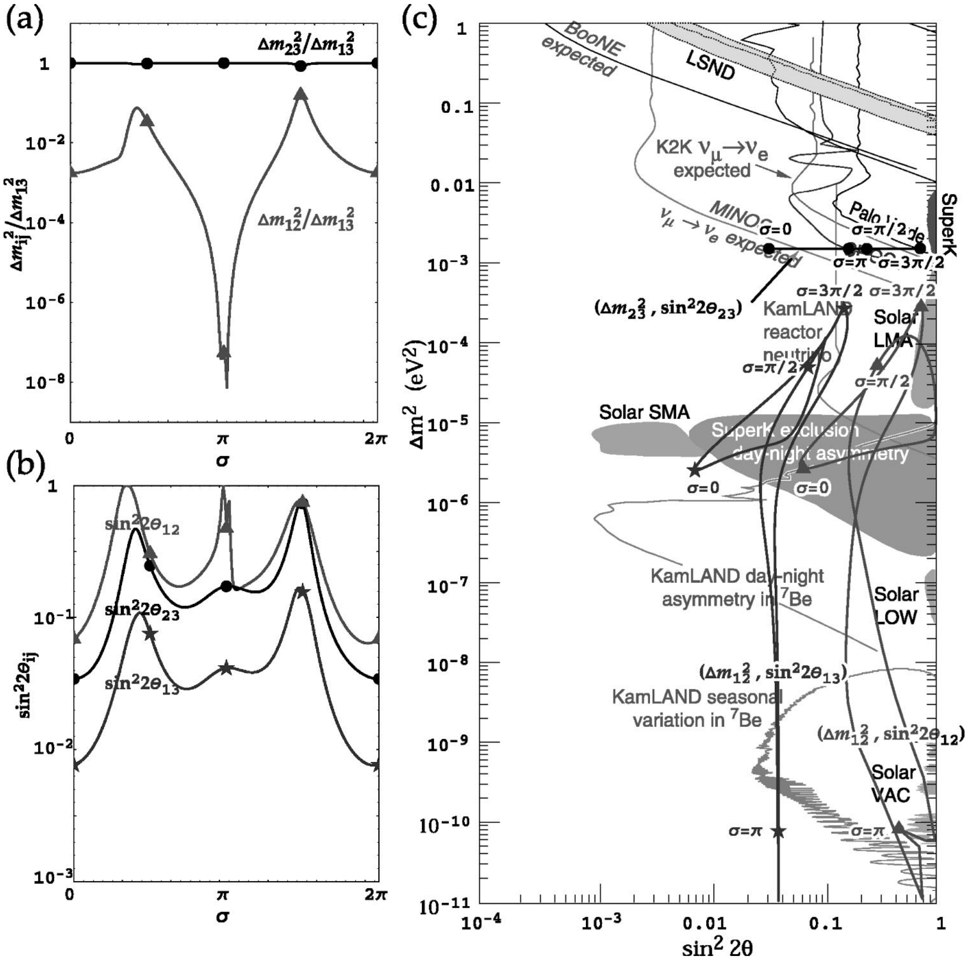


FIG. 1. The relation between our results and the two-flavor oscillation analysis [14] when σ is moved. (a) The circles and triangles indicate the values of $\Delta m^2_{23}/\Delta m^2_{13}$ and $\Delta m^2_{12}/\Delta m^2_{13}$ at every $\pi/2$ of σ. (b) The circles, triangles, and stars indicate the values of $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\sin^2 2\theta_{13}$ at every $\pi/2$ of σ. (c) The circles, triangles, and stars indicate the values of $(\Delta m^2_{23}, \sin^2 2\theta_{23})$, $(\Delta m^2_{12}, \sin^2 2\theta_{12})$, and $(\Delta m^2_{12}, \sin^2 2\theta_{13})$ at every $\pi/2$ of σ. Here we have set $\Delta m^2_{23}=1.5\times 10^{-3}$ eV^2 in every case.

The purpose of the present paper is to study the general tendency of the fitting and not to pursue a precise data fitting, for the data themselves are not definitive, and there are theoretical ambiguities not incorporated in the present data fitting, such as the renormalization group effect.

Using the values of Eq. (3.1), we have

$$|c_d| = 3.16, \tag{3.2}$$

$$c_0 = \frac{1-c_d}{c_u} = 54.84 e^{-20.24^\circ i}, \tag{3.3}$$

$$c_1 = -\frac{3+c_d}{c_u} = 70.54 e^{+41.90^\circ i}. \tag{3.4}$$

In this case, Eqs. (2.11)–(2.13) are rewritten on the basis of $M_u = D_u$ [see Eq. (1.8)] as

$$M_0 = \frac{3V_q D_d V_q^T + c_d(\kappa D_u + V_q D_d V_q^T)}{4}$$
$$= 2.1646\times 10^3 e^{+10.48^\circ i}\begin{pmatrix} -0.00405 e^{-57.29^\circ i} & -0.00753 e^{-56.24^\circ i} & -0.00533 e^{+65.46^\circ i} \\ -0.00753 e^{-56.24^\circ i} & -0.02986 e^{-51.59^\circ i} & +0.06358 e^{-57.64^\circ i} \\ -0.00533 e^{+65.46^\circ i} & +0.06358 e^{-57.64^\circ i} & +1.00000 \end{pmatrix} \text{ MeV}, \tag{3.5}$$

$$M_1 = \frac{V_q D_d V_q^T - c_d(\kappa D_u + V_q D_d V_q^T)}{4}$$
$$= 9.5127\times 10^2 e^{-24.44^\circ i}\begin{pmatrix} -0.00715 e^{+95.23^\circ i} & -0.01333 e^{+96.54^\circ i} & +0.00944 e^{+38.23^\circ i} \\ -0.01333 e^{+96.54^\circ i} & -0.04878 e^{+90.73^\circ i} & +0.11247 e^{+95.13^\circ i} \\ +0.00944 e^{+38.23^\circ i} & +0.11247 e^{+95.13^\circ i} & +1.00000 \end{pmatrix} \text{ MeV}, \tag{3.6}$$