Construction Of Composite Models Based On The Complementarity Principle

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Motivation

****************** $\boxed{\text{Standard Model}}$ *******************

The model

- gauge group : $\mathbf{SU(2)_L \times U(1)_Y} (\times \mathbf{SU(3)_c})$

Higgs VEV $\Rightarrow$ $\Downarrow$

S.S.B.: $\boxed{\mathbf{SU(2)_L \times U(1)_Y \to U(1)_{e.m.}}}$

- fermion: $q\,,l$
- gauge boson: $W^{\pm}\,,Z$

Massive physical spectra

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$\boxed{\textbf{'07: LHC} \to \textbf{Higgs may be discovered?}}$

$\Downarrow$

$\boxed{\textbf{Now we should give predictions to "Higgs physics" !}}$

Consider $\Downarrow$

$\boxed{\textbf{Composite Higgs Model}}$

Contents

1. Introduction

–Gauge-Higgs Sector In The S.M.

1.1 Kinematics

gauge-higgs sector in the S.M.

$$\mathcal{L} = |D_\mu \phi|^2 - m^2|\phi|^2 - \lambda \left(|\phi|^2\right)^2 - \frac{1}{2}\mathrm{tr}[A_{\mu\nu}]^2$$

- $D_\mu = \partial_\mu - igA_\mu \cdots SU(2)_L$ covariant derivative
- $A_\mu = A_\mu^a \frac{\sigma^a}{2} \cdots SU(2)_L$ gauge bosons
- $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \cdots$ Field strength

Using

$$\begin{aligned} \tilde{\phi} &= i\sigma_2\phi^* = \begin{pmatrix} \phi_2^* \\ -\phi_1^* \end{pmatrix} \quad \text{and} \\ M &= \left(\tilde{\phi}\,,\,\phi\right) = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}, \end{aligned}$$

rewrite $\mathcal{L}$ as follows

$$\mathcal{L}^{higgs} = \frac{1}{2}\mathrm{tr}[D_\mu M^\dagger D^\mu M] - \frac{1}{2}m^2\mathrm{tr}[M^\dagger M] - \frac{1}{4}\lambda\left(\mathrm{tr}[M^\dagger M]\right)^2.$$

This Lagrangian is invariant under the transformations:

$$\begin{aligned} M &\rightarrow g_L(x) M g_c^\dagger \quad ; \quad g_L(x) \subset SU(2)_L\,,\, g_c \subset SU(2)_c \\ A_\mu &\rightarrow g_L(x) A_\mu g_L^\dagger(x) - i\partial_\mu g_L(x) g_L^\dagger(x). \end{aligned}$$

$$\Downarrow$$

$$\mathbf{SU(2)_L \times SU(2)_c}$$

symmetry

1.2 Higgs Mechanism And Physical Spectra

The physical spectra are obtained by "**Higgs mechanism**":

$$\phi = \begin{pmatrix} 0 \\ h+v \end{pmatrix}$$

$$\text{non-zero VEV;} \quad <\phi> = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \leftrightarrow \quad <M> = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}$$

S.S.B

$$\boxed{\mathbf{SU(2)_L \times SU(2)_c \rightarrow SU(2)_V}}$$

In particular, gauge bosons become massive;

$$\begin{aligned} \mathcal{L}_{kin}^{higgs} &\sim \frac{g^2}{2}\mathrm{tr}[M^\dagger A_\mu A^\mu M] \\ &\xrightarrow{\text{S.S.B}} \frac{1}{2}g^2 v^2 \mathrm{tr}[A_\mu A^\mu], \end{aligned}$$

which transform as

$$A_\mu \rightarrow g_V \cdot A_\mu \cdot g_V^\dagger \quad ; \quad g_V \subset SU(2)_V.$$

Totally, the spectral properties

Spectra	masses	$SU(2)_V$
3 gauge bosons A_μ	massive	adjoint repr.
1 scalar boson h	massive	singlet

$$\Downarrow$$

The original $\mathbf{SU(2)_L}$ gauge symmetry was broken!

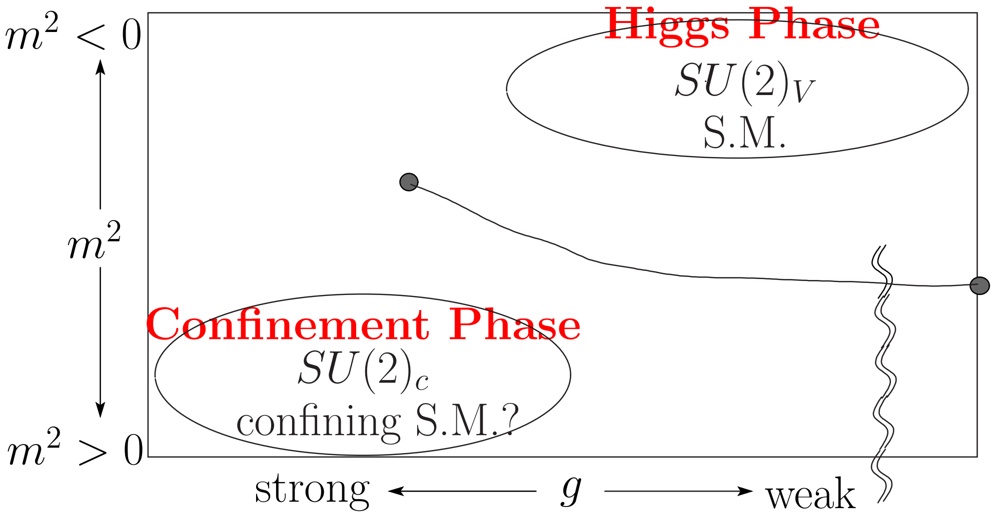
$$\Downarrow$$

$$\boxed{\textbf{Are there other possibilities?}}$$

−The Complementarity Principle

$SU(2)$ gauge-higgs system on lattice.

- K.Osterwalder and E.Seiler, Ann.Phys.**110**(1978).
- E.Fradkin and S.H.Shenker, P.R.D19(1979).



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In the strong gauge coupling region,

there is no phase transition
between higgs and confinement phases.

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" The same spectral properties "

****************** $\boxed{\textbf{Complementarity Principle}}$ ******************

Interpretation. (S.Dimopoulos, S.Raby and L.Susskind, N.P.B173(1980).)

Phases	Global sym.	Spectra	Repr.
Higgs Phase	$SU(2)_V$	3-massive A_μ	adjoint
		1-massive h	singlet
Confinement Phase	$SU(2)_c$	3-massive $\mathbf{W}_\mu$	adjoint
		1-massive $\mathbf{H}$	singlet

"$\mathbf{SU(2)_L}$ invariant W , H in confinement phase "

$\Downarrow$

$\boxed{\textbf{Is this possible in continuum theory?}}$

2. Composite Models

2.1 A *Naive* Formulation Of Confining Model

- L.F.Abbott and E.Farhi, P.L.B101 (1981), N.P.B189 (1981)
- M.Claudson, E.Farhi and R.L.Jaffe, P.R.D34 (1986).
- X.Calmet and H.Fritzsch, P.L.B496 (2000), hep-ph/0206251 , et.al

$SU(2)$ gauge-higgs Lagrangian

$$\mathcal{L} = |D_\mu \phi|^2 - m^2|\phi|^2 - \lambda\left(|\phi|^2\right)^2 - \frac{1}{2g^2}\mathrm{tr}[A_{\mu\nu}]^2$$

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define "**gauge invariant**" composite fields:

$$\Phi \;=\; \Omega^\dagger \cdot \phi \quad , \quad \Omega \;=\; \frac{1}{\sqrt{\phi^\dagger \phi}} M,$$

$$\boxed{\mathbf{\Omega} \quad \rightarrow \quad \mathbf{g_L(x)} \cdot \mathbf{\Omega} \cdot \mathbf{g_c^\dagger} \quad , \quad \mathbf{\Phi} \quad \rightarrow \quad \mathbf{g_c} \cdot \mathbf{\Phi}}$$

$$\mathbf{W}_\mu \;=\; i\Omega^\dagger \cdot D_\mu \Omega \;=\; \Omega^\dagger \cdot A_\mu \cdot \Omega - i\partial_\mu \Omega^\dagger \cdot \Omega$$

$$\boxed{\mathbf{W}_\mu \quad \rightarrow \quad \mathbf{g_c} \cdot \mathbf{W}_\mu \cdot \mathbf{g_c^\dagger}}$$

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$$\Downarrow$$

Rewrite $\mathcal{L}$ as "$SU(2)_L$ invariant" form:

$$\mathcal{L} = |\mathbf{D}_\mu \Phi|^2 - m^2|\Phi|^2 - \lambda\left(|\Phi|^2\right)^2 - \frac{1}{2g^2}\mathrm{tr}[\mathbf{W}_{\mu\nu}]^2.$$

- $\mathbf{D}_\mu \;=\; \partial_\mu - i\mathbf{W}_\mu \cdots SU(2)_c$ covariant derivative
- $\mathbf{W}_{\mu\nu} \;=\; \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - i[\mathbf{W}_\mu, \mathbf{W}_\nu] \cdots$ Field strength

$\boxed{\textbf{Is this a confining model ?}}$

$$\Downarrow$$

$\boxed{\textbf{NO !!}}$

*********************$\boxed{\textbf{The Reasons}}$*************************

- $\Omega = \frac{1}{\sqrt{\phi^\dagger \phi}} \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}$; $\Omega^\dagger \Omega = 1 \quad \Rightarrow \quad$ N.G.B of ϕ
- $\mathbf{W}_\mu = \Omega^\dagger A_\mu \Omega - i\partial_\mu \Omega^\dagger \Omega \quad ; \quad \boxed{\Omega = e^{2i\frac{\pi}{v}}}$

$\Rightarrow$ Non-linear transformation

$\Rightarrow$ $\mathbf{W}$, $A \cdots$ **"unitary gauge equivalent"**

e.g.; mass terms : $\quad \mathrm{tr}[\mathbf{W}_\mu \mathbf{W}^\mu] \quad \overset{\text{unitary gauge}}{\longleftrightarrow} \quad \mathrm{tr}[A_\mu A^\mu]$

S.S.B in **higgs phase** ; $\boxed{\mathbf{SU(2)_L \times SU(2)_c \Rightarrow SU(2)_V}}$

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$$\Downarrow$$

$\boxed{\textbf{What is an appropriate formulation like?}}$

2.2 A Concrete Example For An Appropriate Confining Model

********************** $\boxed{\textbf{Assumptions}}$ ***********************

- $SU(2)_L$ is confined at a low energy scale Λ

  $\Longrightarrow$ well-described by an **effective theory**.

- Analogy to " **N.J.L. model** " in QCD.

*********************** $\boxed{\textbf{A Model}}$ *************************

- **" one gauge boson exchange with MAC "**

  Effective Lagrangian at Λ scale in confinement phase

$$
\begin{aligned}
\mathcal{L}^{eff}_{conf.}(\Lambda) &= \frac{1}{2}\mathrm{tr}[\partial_\mu M^\dagger \partial^\mu M] - \frac{1}{2}m^2 \mathrm{tr}[M^\dagger M] - \frac{\lambda}{4}\left(\mathrm{tr}[M^\dagger M]\right)^2 \\
&\quad -\frac{G_V}{4}\mathrm{tr}[M^\dagger i\overleftrightarrow{\partial}_\mu M]^2 - \frac{G_V}{4}\mathrm{tr}[M^\dagger M]\partial^2 \mathrm{tr}[M^\dagger M] \\
&\quad +\frac{G_V}{4}\Lambda^2\left(\mathrm{tr}[M^\dagger M]\right)^2 + \cdots
\end{aligned}
$$

- **Auxiliary field method:**

$$
\begin{aligned}
(\mathbf{W}_\mu)_{ij} &= G_V\left(M^\dagger i\overleftrightarrow{\partial}_\mu M\right)_{ij} \quad \Rightarrow \quad g_c \mathbf{W}_\mu g_c^\dagger \quad \cdots \quad \text{adjoint repr.} \\
\mathbf{H} &= \Lambda G_V \mathrm{tr}[M^\dagger M] \quad \Rightarrow \quad \chi \quad \cdots \quad \text{unchanged, singlet} \\
G_V &= \frac{3}{4}\cdot\frac{g^2}{8\Lambda^2}
\end{aligned}
$$

- **Rewrite $\mathcal{L}^{eff}_{conf.}$ as $\mathbf{SU(2)_H}$ "hidden local" invariant form via fictitious N.G.B. "$\xi(\mathbf{x})$".**

$$
\begin{aligned}
\xi(x) &= e^{i\sigma(x)} \quad \Rightarrow \quad g_c\xi(x)g_H^\dagger(x) \quad : g_H(x) \subset \mathbf{SU(2)_H} \\
\widetilde{\mathbf{M}} &= M\xi^\dagger \quad \Rightarrow \quad g_L\widetilde{\mathbf{M}}g_H^\dagger(x) \\
\widetilde{\mathbf{W}}_\mu &= \xi^\dagger \mathbf{W}_\mu \xi - i\partial_\mu \xi^\dagger \xi \quad \Rightarrow \quad g_H(x)\widetilde{\mathbf{W}}_\mu g_H^\dagger(x) - i\partial_\mu g_H(x) g_H^\dagger(x)
\end{aligned}
$$

Auxiliary fields $\widetilde{W}$, $\mathbf{H}$ become dynamical ones through the **quantum corrections**.

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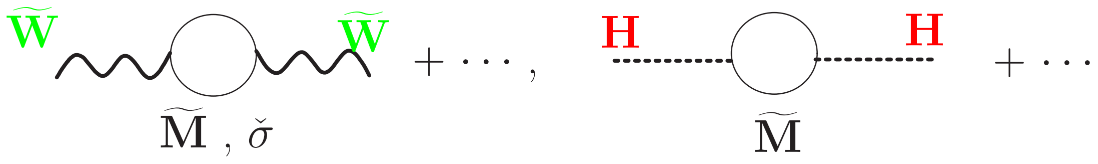
***************** **" Loop expansion"** ********************



- **background field method ;**

$$\xi = \underbrace{\bar{\xi}}_{\text{quantum}} \cdot \underbrace{\check{\xi}}_{\text{classical}}$$

- **proper-time regularization**
- **heat kernel expansion.**
- **We evaluated loop effects up to quadratic order of quantum fields;** $\widetilde{\mathbf{M}}$ **,** $\check{\sigma}$

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As a result, after $\bar{\xi} \longrightarrow 1$,

$$\begin{aligned}
\mathcal{L}^{eff}_{conf.} \;=\; & \frac{1}{2}\mathbf{Z}_{\mathbf{H}}(\partial_\mu \mathbf{H})^2 - \frac{1}{2}\widetilde{\mathbf{m}}^2_{\mathbf{H}}(\mathbf{H})^2 \\
& -\frac{1}{2\mathbf{g}^2_{\mathbf{W}}}\mathrm{tr}[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}] + \widetilde{\mathbf{m}}^2_{\mathbf{W}}\mathrm{tr}[\mathbf{W}_\mu \mathbf{W}^\mu] \\
& + \left(\partial \mathbf{W}^3 \,,\, \mathbf{W}^4\right) \\
& + \left(\mathbf{W}^2\partial^2\mathbf{H} \,,\, \partial^2\mathbf{H}\partial^2\mathbf{H}\right) + \text{finite terms.}
\end{aligned}$$

Note that $\mathbf{W}$, $\mathbf{H}$ posses the spectral properties expected to be obtained in confinement phase !!

$\mathbf{W}$ (or $\widetilde{\mathbf{W}}$) – mass is responsible for

S.S.B in confinement phase!!

$$\boxed{\mathbf{SU(2)_{c'}} \times \mathbf{SU(2)_H} \Rightarrow \mathbf{SU(2)_c}}$$

And we can explicitly check ; (* up to order p^4)

$$\boxed{\mathcal{L}^{\mathbf{eff}}_{\mathbf{conf.}} \overset{\text{no difference}}{\Longleftrightarrow} \mathcal{L}_{\mathbf{higgs}}}$$

" Complementarity " in a qualitative sense

$$\Downarrow$$

" How about a quantitative interpretation? "

$$\Downarrow$$

Wave function renormalization constant of $\mathbf{W}$, i.e; $\mathbf{SU(2)_H}$ gauge coupling of $\widetilde{\mathbf{W}}$

$$\frac{1}{\mathbf{g}^2_{\mathbf{W}}} = \frac{1}{4(4\pi)^2} \log \frac{\Lambda^2}{m^2}$$

If we require $\mathbf{g}^2_{\mathbf{W}} \sim O(1)$; $SU(2)$ weak coupling in the S.M. ,

$$\frac{\Lambda}{m} \sim \mathbf{10^{137}} \text{ !!}$$

$$\Downarrow$$

It is difficult to obtain a quantitative picture in this model!

2.4 Towards A Quantitative Interpretation

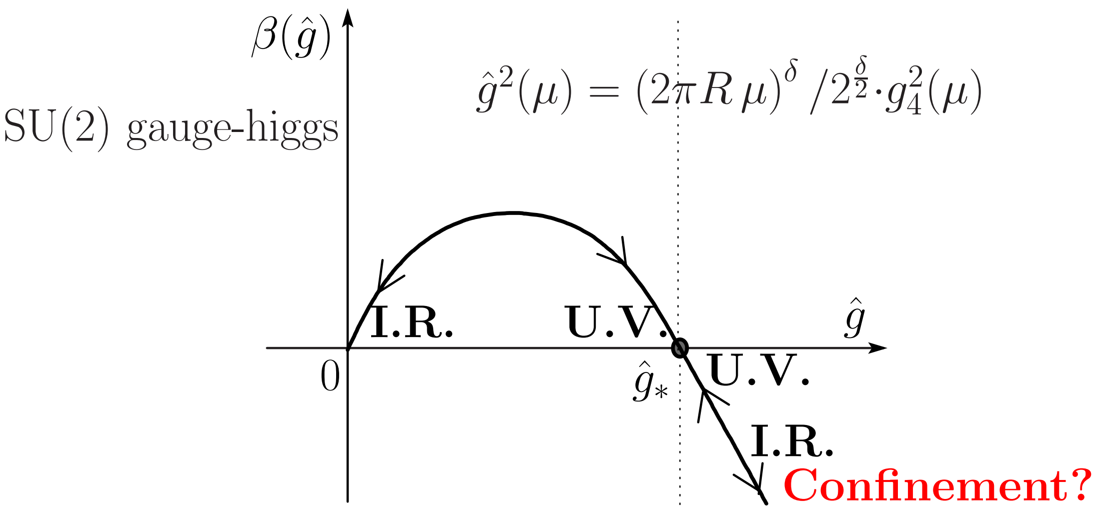
$\boxed{\textbf{Extension to } D = 4 + \delta \textbf{ dimensional model}}$

********************** **Set-up** ************************

- **SU(2) original gauge and scalar fields live in D-dimensional bulk.**
- **δ dimensions are compactified with the same radii R $=$ 1TeV^{-1}**
- **We expect confinement will occur in the strong gauge coupling region.**

For example, from a perturbative RG analysis

M.Hashimoto, M.Tanabashi and K.Yamawaki, P.R.D69 (2004)



- **At a low energy scale (Λ ; $g_D^2(\Lambda) \cdot \Lambda^\delta > \hat{g}_*^2$), effective 4-boson interactions are induced by the strong D-dimensonal gauge dynamics dominated by gauge K-K modes associated with δ extra dimensions.**

As a result, (with identification $\mu_0 = 1/R = m$)

$$\begin{aligned}
\frac{1}{[\mathbf{g}_4^{\mathbf{W}}]^2} &= \frac{1}{4(4\pi)^2}\left[\log\frac{\Lambda^2}{\mu_0^2} + \sum_{k=1}^{\delta} {}_\delta C_k 2^k \sum_{\vec{n}_k=1}^{N_{KK}=[\frac{\Lambda}{\sqrt{\delta}\mu_0}]} \log\frac{\Lambda^2}{\mu_0^2\vec{n}_k^2}\right] \\
\left[\mathbf{F}_\sigma^{1-loop}\right]^2 &= \left[\mathbf{F}_\sigma^{bare}\right]^2 + \frac{3\mu_0^2}{2(4\pi)^2}\left[\log\frac{\Lambda^2}{\mu_0^2} + \sum_{k=1}^{\delta} {}_\delta C_k 2^k \sum_{\vec{n}_k=1}^{N_{KK}} \vec{n}_k^2 \log\frac{\Lambda^2}{\mu_0^2\vec{n}_k^2}\right] \\
&\quad -\frac{3}{2(4\pi)^2}\Lambda^2\left(1 + \sum_{k=1}^{\delta} {}_\delta C_k 2^k \sum_{\vec{n}_k=1}^{N_{KK}}\right) \\
\mathbf{Z}_4^{\mathbf{H}} &= -2\left[\frac{\mathbf{F}_\sigma^{bare}}{\Lambda}\right]^2 + \frac{16}{[\mathbf{g}_4^{\mathbf{W}}]^2}\left(1 - 4\left[\frac{\mathbf{F}_\sigma^{bare}}{\Lambda}\right]^2\lambda_4\right) \\
\frac{\mathbf{m}_{\mathbf{H}}^2}{\Lambda^2} &= \frac{2\left(1 - 4\left[\frac{\mathbf{F}_\sigma^{bare}}{\Lambda}\right]^2\lambda_4\right)}{Z_4^H}\left[\left[\frac{\mathbf{F}_\sigma^{bare}}{\Lambda}\right]^2 - \frac{4}{[\mathbf{g}_4^{\mathbf{W}}]^2}\left(1 - 4\left[\frac{\mathbf{F}_\sigma^{bare}}{\Lambda}\right]^2\lambda_4\right)\right]
\end{aligned}$$

where

$$\begin{aligned}
\left[\mathbf{F}_\sigma^{bare}\right]^2 &= \frac{1}{4G_V(\mu_0/2\pi)^\delta} \quad \cdots \quad \text{decay constant for } \sigma \\
\left[\mathbf{F}_\sigma^{1-loop}\right]^2 &= \frac{\mathbf{m}_{\mathbf{W}}^2}{[\mathbf{g}_4^{\mathbf{W}}]^2} \\
G_V &= \frac{c(\Lambda)}{8\Lambda^2} g_D^2 \cdot \frac{3}{4} \\
c(\Lambda) &\cdots \text{ numerical coefficient associated with } \sharp \text{ of gauge K-K} \\
N_{KK} &= \frac{\Lambda}{\sqrt{\delta}\mu_0} \\
N_{KK}^{\mathbf{tot}} &= 2^\delta \times (N_{KK})^\delta
\end{aligned}$$

********** $\boxed{\textbf{Results of numerical calculations}}$ *********

When we require $[\mathbf{g}_4^{\mathbf{W}}]^2 < O(1)$**,**

- $\delta = 1 \Rightarrow N_{KK} > 1500$, i.e; $\Lambda > 1500$ TeV
  $\Rightarrow \mathbf{m_W} \sim 80$ GeV $\Rightarrow \mathbf{F}_\sigma^{bare}/\Lambda \gg 1$
  $\Rightarrow$ inconsistent.

- $\delta = 2 \Rightarrow N_{KK} > 4$, i.e; $\Lambda > 5$ TeV
  $\Rightarrow \mathbf{m_W} \sim 80$ GeV $\Rightarrow \mathbf{F}_\sigma^{bare}/\Lambda = 0.87184...$
  (* $N_{KK} = 4$)
  $\Rightarrow$ need fine-tuning of 3-4 digits against $\mathbf{m_W}$.

- $\delta \geq 3 \Rightarrow N_{KK} \geq 1$, i.e; $\Lambda \geq 1$ TeV
  $\Rightarrow \mathbf{m_W} \sim 80$ GeV $\Rightarrow \mathbf{F}_\sigma^{bare}/\Lambda \gg 1$
  except for $N_{KK} = 1$
  $\Rightarrow$ inconsistent.

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$\boxed{\textbf{The case of } \delta = 2}$ **; W boson is generated in a quantitative sence.**

$$\Downarrow$$

" How about Higgs? "

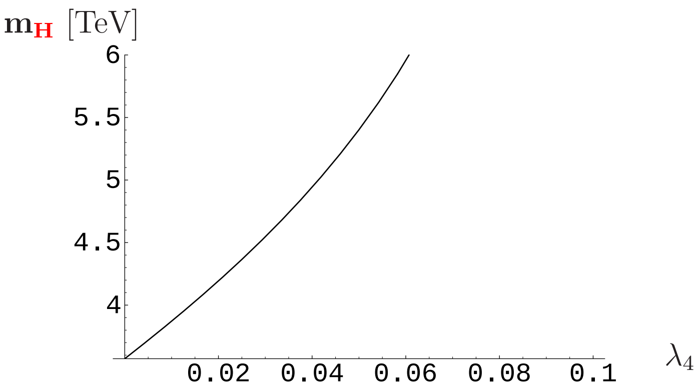

Figure 1: $\mathbf{m_H}$ as a function of λ_4: input $[\mathbf{F}_\sigma^{bare}/\Lambda]^2 = 0.769457$ and $N_{KK}^{\delta=2} = 4$ whose values are respected by $\mathbf{m_W} = 80\,[\mathrm{GeV}]$

" The lower bound " is given by

$$\mathbf{m_H} \geq 3.5\,[\mathrm{TeV}] \quad , \quad 0 \leq \lambda_4 \leq 0.05 - 0.06,$$

This model tells us

If " Complementarity " is confirmed
in a quantitative sence,

$$\Downarrow$$

$$\boxed{\textbf{Higgs may not appear at E.W. scale !?}}$$

3. Summary And Conclusion

- We discussed what an appropriate confining model is like which can represent the Complementarity principle in continuum theory in both a qualitative and quantitative sence.

- Qualitative interpretation ; $\boxed{SU(2)_{c'} \times SU(2)_{H} \Rightarrow SU(2)_{c}}$
- We proposed low energy effective models.

  $*$ D=4 model $\overset{\text{quantitatively}}{\Rightarrow}$ ✗

  $*$ D=4+2 model $\overset{\text{quantitatively}}{\Rightarrow}$ O.K.

  – $m_W = 80$ GeV ,
  – weak gauge coupling: $g_W^2 \sim O(1)$
  – $m_H \sim$ TeV $\Rightarrow$ higgsless at E.W. scale?

- Future works
  – The other S.M. particles (fermions)?
  – Checks universality for gauge coupling g_W^2