クエンチ近似を用いた、ドメインウォールフェルミオンによる中性子電気双極子モーメントの研究

Neutron electric dipole moment with domain-wall fermion in quenched lattice QCD

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(in CP-PACS collaboration)

Introduction

In strong interaction CP symmetry is not good symmetry

- CP violation parameter
  - QCD
    Vacuum angle on nontrivial configuration: θ_{QCD}

$$\mathcal{L}_{QCD} = i\theta_{QCD}\frac{g^2}{32\pi^2}G_{\mu\nu}\widetilde{G}_{\mu\nu}$$

  - Electroweak
    Phase term of CKM matrix M: θ_{CKM}

$$\begin{aligned}\mathcal{L}_{CKM} &= \bar{\psi_R}M\psi_L + \bar{\psi_L}M^*\psi_R \\ \theta_{CKM} &= \arg \det M\end{aligned}$$

CP violation parameter is corresponding to both QCD vacuum effect and CKM matrix phase.

$$\bar{\theta} = \theta_{CKM} + \theta_{QCD}$$

The measurement of CP violation parameter

- Neutron Electric Dipole Moment
  (NEDM: d_n)
  - Electric dipole momnet (EDM) is recieved from CP violation effect
  - Neutron has no charge
  - Recent experimental value

$$|\mathbf{d_n}| < \mathbf{6.3 \times 10^{-26}\, e \cdot cm} \quad (\mathbf{90}\%\,\mathrm{CL})$$

- According to the effective model, $\bar{\theta} < 10^{-9}$
  This value is so small that it is unnatural for theoretical parameter.
  $\Longrightarrow$ strong CP problem

Axion model can solve it, but axion is still not detected.

Motivation

Mearsurment of the effects between NEDM and QCD vacuum by lattice QCD simulation

Lattice QCD

- Based on the first principle
- Non-perturbative method for QCD
  It is possible to simulate in low energy region.

Set up

QCD Lagrangian

$$\mathcal{L}_{QCD} = \mathcal{L}_{\mathbf{q}} + \mathcal{L}_{\mathbf{g}} + \theta_{QCD}\frac{ig^2}{32\pi^2}G_{\mu\nu}\widetilde{G}_{\mu\nu}$$

$$\widetilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}$$

where

$G_{\mu\nu}$: gluon field strength

$\mathcal{L}_{\mathbf{g}}$: gauge part, $\mathcal{L}_{\mathbf{q}}$: quark part

Nucleon form factor

There are three nucleon form factors between electromagnetic current and nucleon state.

$$\langle N(p_{out})|J_\mu^{EM}(q)|N(p_{in})\rangle = \bar{u}(p_{out})\Gamma_\mu u(p_{in})$$

$$\Gamma_\mu = e\gamma_\mu F_1(q^2) + \frac{ieq_\nu}{2m_N}\sigma_{\mu\nu}F_2(q^2)$$

$$+\frac{eq_\nu}{2m_N}\gamma_5\sigma_{\mu\nu}F_3(q^2), \quad q = p_{out} - p_{in}$$

$$J_\mu^{EM}(x) = e^u\bar{\psi}(x)\gamma_\mu\psi(x) + e^d\bar{\psi}(x)\gamma_\mu\psi(x)$$

Each form factor can be defined as

- $e\,F_1(0)$: Electric charge
- $\frac{e}{2m_N}F_2(0)$: Magnetic dipole moment
- $\frac{e}{2m_N}F_3(0)$: Electric dipole moment

Also the combination between $F_1,\ F_2$ can be defined electric and magnetic form factor

$$G_e = F_1 + \frac{\mathbf{q}^2}{(2m_N)^2}F_2, \quad G_m = F_1 + F_2$$

CP symmetry for matrix element

- F_1, F_2 : even, F_3 : odd

Because θ term in $\mathcal{L}_{QCD}$ is CP odd, if $\theta \neq 0$, EDM can be obtained non-zero value on non-trivial (instanton) configuration.

Previous research

- S. Aoki, A. Gochsch, PRD 63 (1989) 1125
- S. Aoki, A. Gochsch, A. V. Manohar,
  S. R. Sharpe, PRD65 (1990) 1092
- S. Aoki, T. Hatsuda, PRD45 (1992) 2427
- D. Guadagnoli, V. Lubicz, G. Martinelli, S. Simula,
  JHEP 0304 (2003) 019

But these were not determined explicit d_n value on lattice simulation.

Strategy

- Topological charge is corresponding to chrial symmetry for the fermion, so on the lattice the fermion need to have mostly chiral symmetry. $\rightarrow$ Domain-wall, Overlap
- If NEDM can detect CP symmetry of topological charge distribution, we can meaure its response in the signal.
- We calculate NEDM in the first order of θ term.

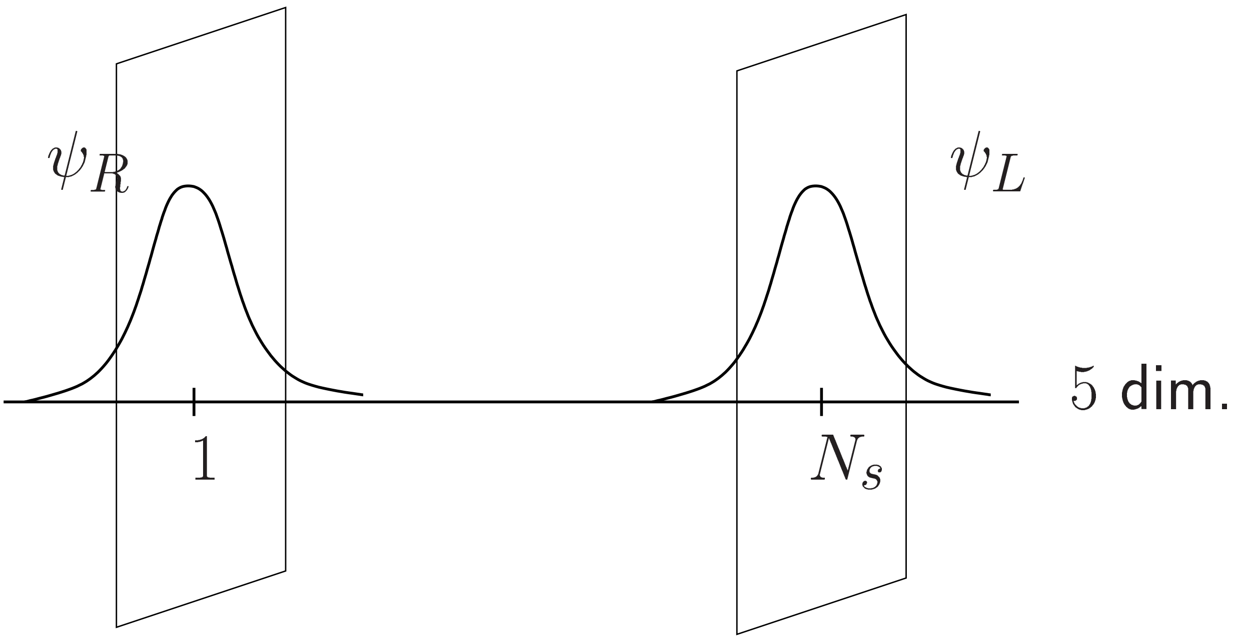
Domain-wall fermion

Dirac fermion

$$\begin{aligned} q(x) &= P_L\psi_1(x) + P_R\psi_{N_s}(x), \\ \bar{q}(x) &= \bar{\psi}_1(x)P_R + \bar{\psi}_{N_s}(x)P_R \end{aligned}$$

parameter

- N_s : size of 5 dimension
- m_f : quark mass,
- M : domain-wall height



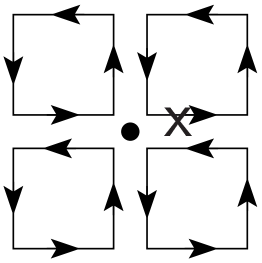
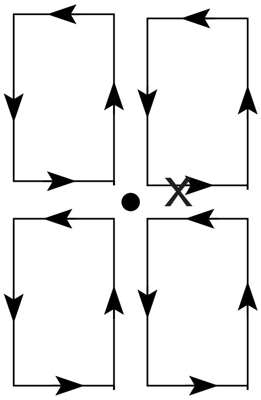
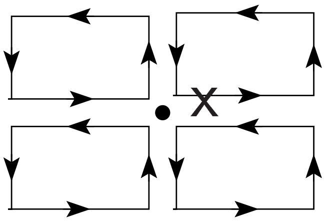
Topological charge

- Bosinic definition

$$
\begin{aligned}
Q_B &= \frac{1}{32\pi^2} \varepsilon_{\mu\nu\alpha\beta} \,\mathsf{Tr}\,(G_{\mu\nu} G_{\alpha\beta}) \\
&= c_0 Q^P + c_1 Q^R
\end{aligned}
$$

where

$$
\begin{aligned}
Q^P &= \frac{1}{32\pi^2} \varepsilon_{\mu\nu\alpha\beta} \,\mathsf{Tr}\,(F^P_{\mu\nu} F^P_{\alpha\beta}) \\
Q^R &= \frac{2}{32\pi^2} \varepsilon_{\mu\nu\alpha\beta} \,\mathsf{Tr}\,(F^R_{\mu\nu} F^R_{\alpha\beta}) \\
c_0 &= 5/3, \quad c_1 = -1/12
\end{aligned}
$$

$F^P_{\mu\nu} =$  , $F^R_{\mu\nu} =$  $+$ 

In bare configuration, Q_B is not integer, but after cooling one can become mostly integer.

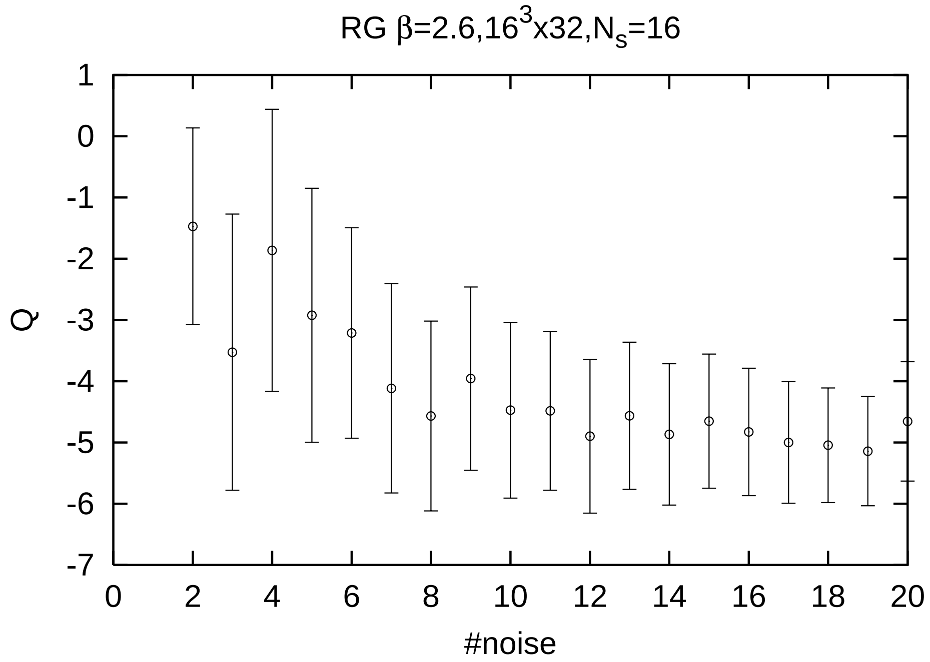
- Fermionic definition
  - Trace method

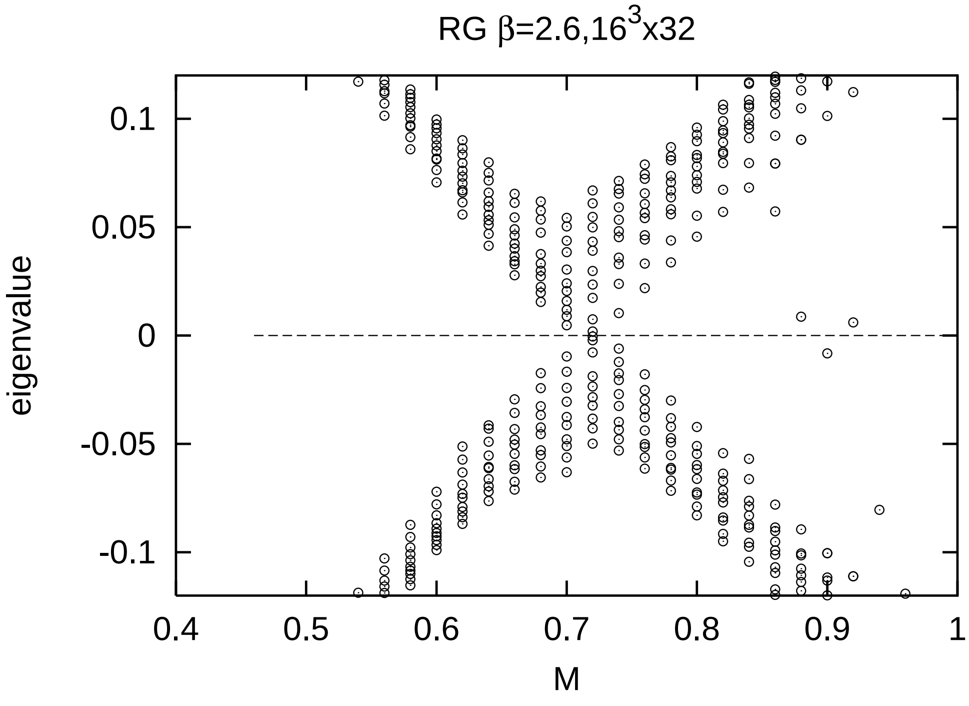
$$
\begin{aligned}
Q_F &= \sum_x \langle {\psi'}_{N_s/2}(x) \gamma_5 \bar{\psi}'_{N_s/2}(x) \rangle \\
&= \mathrm{Tr}\left(\gamma_5 D^{-1}_{N_s/2, N_s/2}(x,x)\right)
\end{aligned}
$$

    In order to calculate space-time trace, we use U(1) noise method $\longrightarrow$ very costly !!
  - Spectral flow

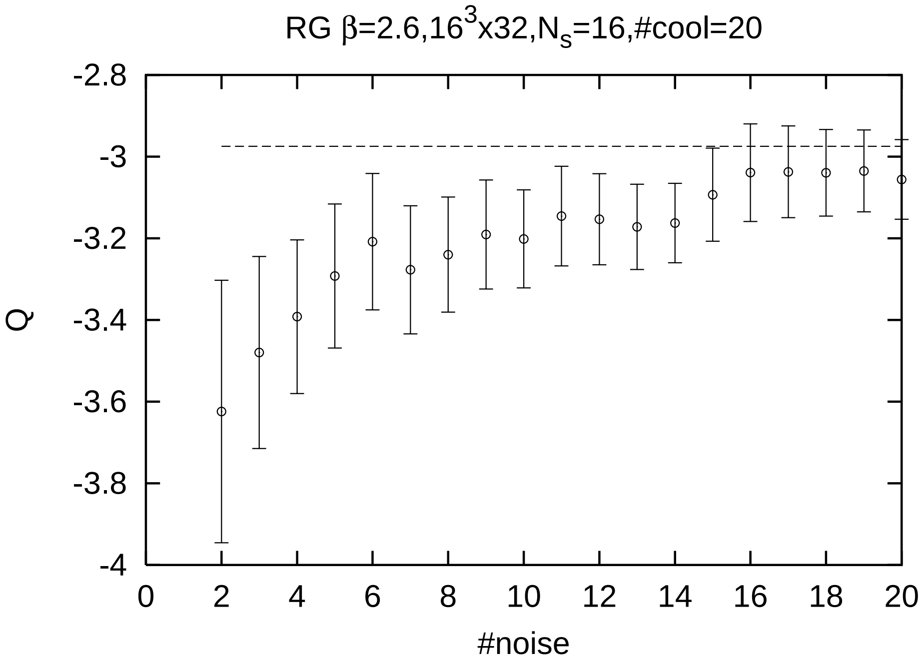
    Calculation of Hermitian Wilson operator H_w: count cross point

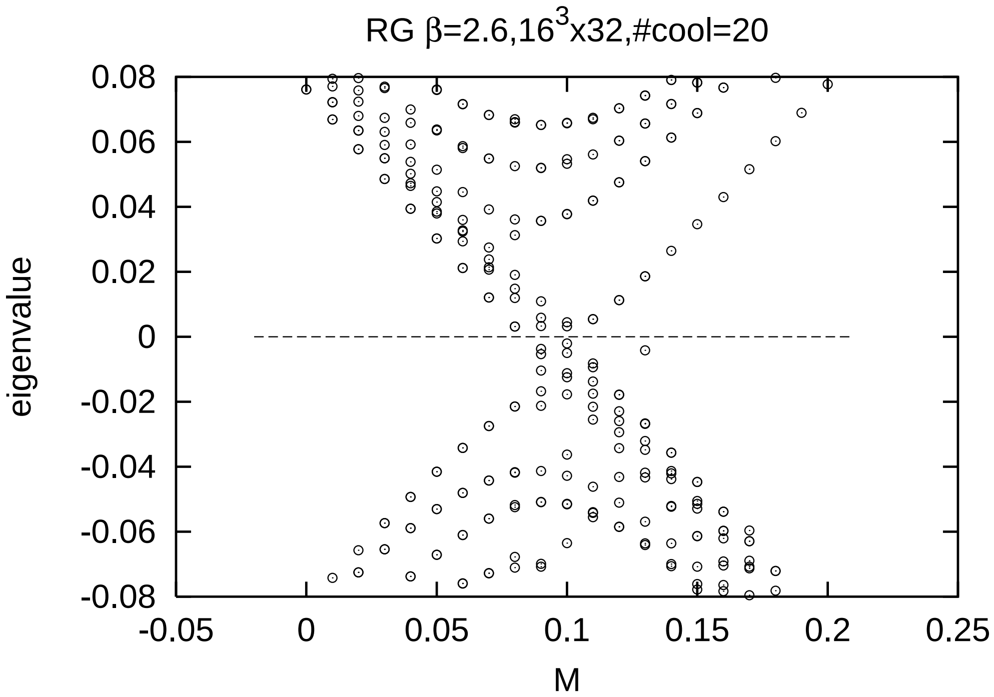
    $\longrightarrow$ It is difficult to count, becouse of no clear cross point.





Before cooling





After # cooling = 20

Lattice formulation of NEDM

Definitions

$$G^{NJ_\mu N}(-\mathbf{q},\mathbf{q},t,\tau;\Gamma_i)$$
$$=\sum_{\mathbf{x},\mathbf{y}}\langle\chi_\alpha(\mathbf{y},t)|J_\mu(\mathbf{x},\tau)|\bar{\chi}_{\alpha'}\rangle(\Gamma_\lambda)_{\alpha\alpha'}e^{-i\mathbf{y}\mathbf{q}}e^{i\mathbf{x}\mathbf{q}}$$

$$G^{NJ_\mu QN}(-\mathbf{q},\mathbf{q},t,\tau;\Gamma_i)$$
$$=\sum_{\mathbf{x},\mathbf{y}}\langle\chi_\alpha(\mathbf{y},t)|J_\mu(\mathbf{x},\tau)\,Q\,|\bar{\chi}_{\alpha'}\rangle(\Gamma_\lambda)_{\alpha\alpha'}e^{-i\mathbf{y}\mathbf{q}}e^{i\mathbf{x}\mathbf{q}}$$

$$G^{NN}(\mathbf{q},t)=\sum_{\mathbf{x}}\langle\chi_\alpha(\mathbf{x},t)|\chi_{\alpha'}(0)\rangle(\Gamma^+)_{\alpha\alpha'}e^{-i\mathbf{x}\mathbf{q}}$$

$$R=\frac{G^{NN}(\mathbf{q},t)G^{NN}(\mathbf{q},\tau)G^{NN}(0,t-\tau)}{G^{NN}(\mathbf{0},t)G^{NN}(\mathbf{0},\tau)G^{NN}(\mathbf{q},t-\tau)}$$

Interpolating operator of Nucleon :

$$\chi_\alpha=\varepsilon^{abc}(d^{t\,a}C\gamma_5u^b)d^c_\alpha,\quad C:\text{charge conjugation}$$

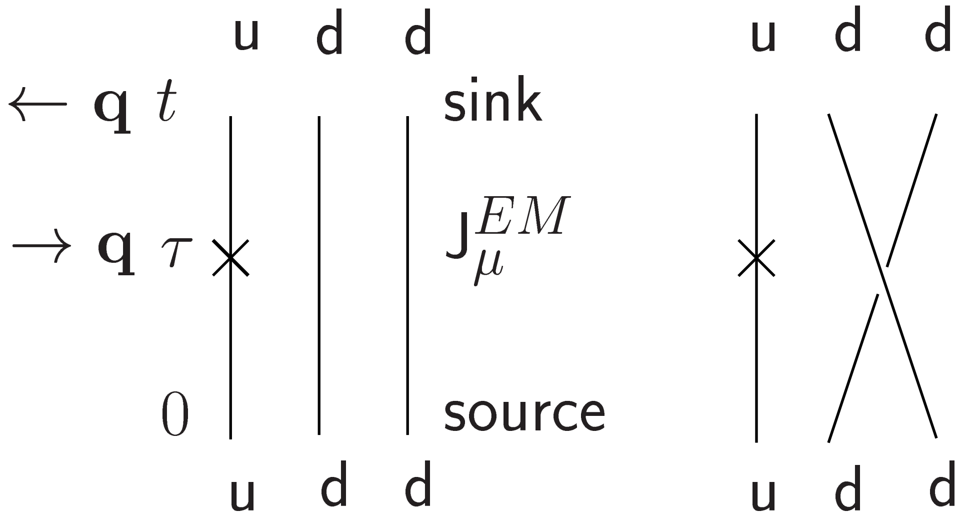
Projection matrix :

$$\Gamma^+=\frac{\gamma_4+1}{2},\quad \Gamma^{(\mathrm{i})}_\lambda=\frac{i}{2}\gamma_\lambda\gamma_5\Gamma^+,\quad \Gamma^{(\mathrm{ii})}_\lambda=\frac{i}{2}\gamma_\lambda\Gamma^+$$

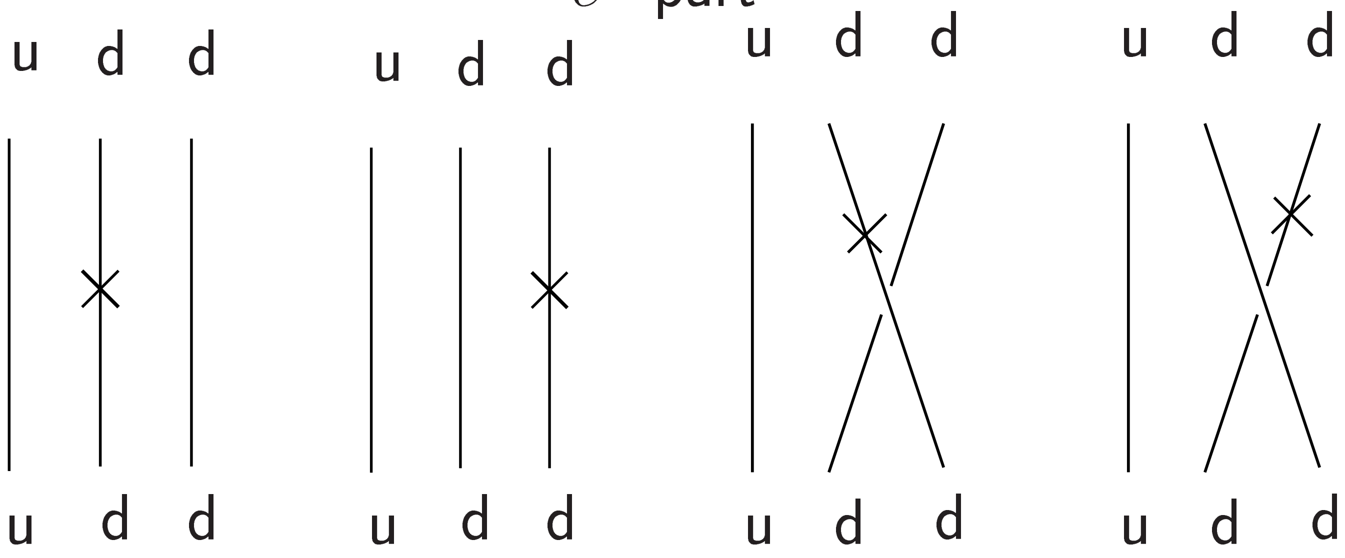
We calcurate three term :

$$G^{NN}(\mathbf{q},t),\ G^{NN}(\mathbf{q},t-\tau),\ G^{NJ_\mu N}(-\mathbf{q},\mathbf{q},t,\tau;\Gamma_i)$$

Diagrams for $\langle N J_\mu N \rangle$



e^u part



e^d part

Proton can be obtained by exchanging $e^u \leftrightarrow e^d$.

In practice the 3-point function is calculated by the source method.

- Electric form factor (# conf=708)

$$G_e(q^2) \sim \frac{G^{Nj_4N}(\mathbf{0},\mathbf{q},t,\tau;\frac{1}{2}\Gamma_4)}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

- Magnetic form factor (# conf=90)

$$\varepsilon_{ijk}\frac{q_k}{2m_N}G_m(q^2) \sim \frac{G^{Nj_iN}(\mathbf{0},\mathbf{q},t,\tau;\Gamma_j^{(\mathrm{i})})}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

- Electric dipole moment

$$\frac{q_i}{2m_N}F_3(q^2) \sim \frac{G^{Nj_4QN}(\mathbf{0},\mathbf{q},t,\tau;\Gamma_i^{(\mathrm{i})})}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

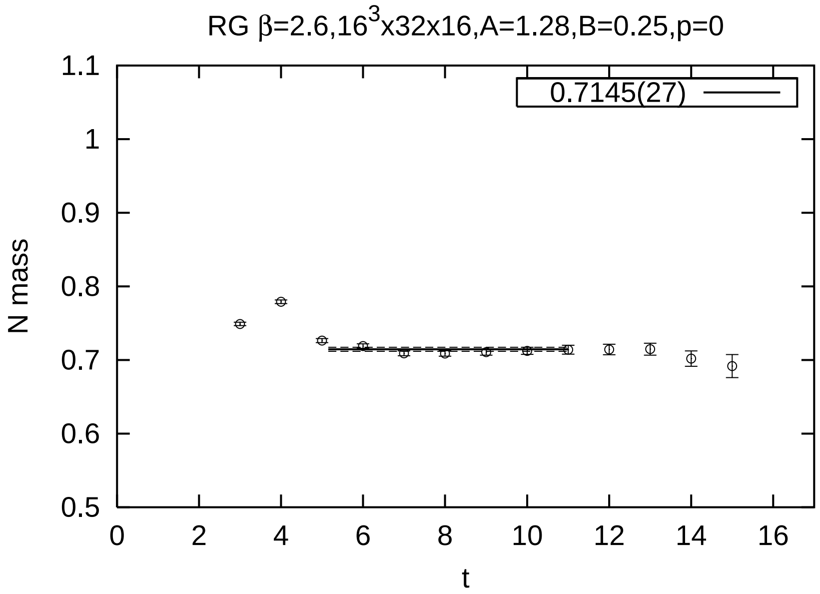
$$\varepsilon_{ijk}\frac{q_k}{2m_N}F_3(q^2) \sim \frac{G^{Nj_iQN}(\mathbf{0},\mathbf{q},t,\tau;\Gamma_j^{(\mathrm{ii})})}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

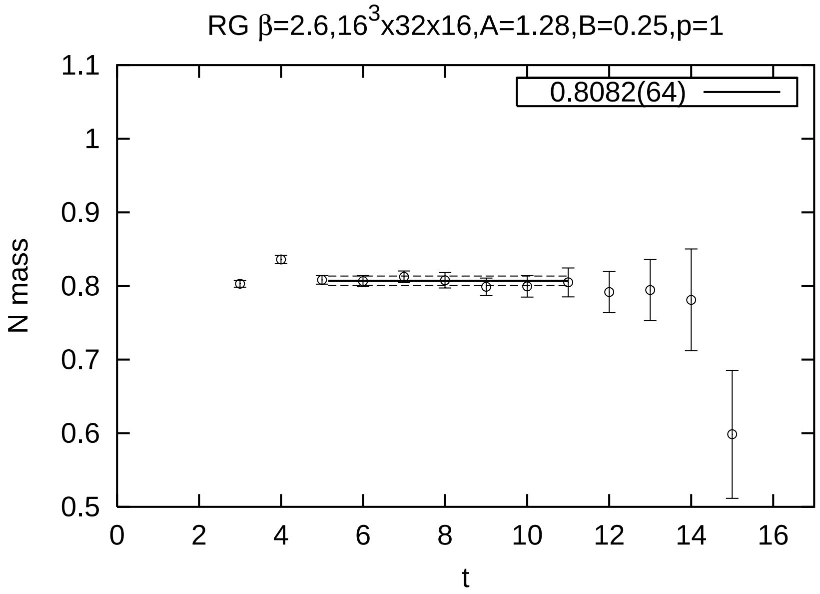
Numerical simulation

Parameters

- Gauge action :
  Renormalization Group (RG) improved action
  Coupling constant $\beta = 6/g^2$=2.6
- Lattice spacing : $a^{-1} \simeq$ 2 GeV
- Lattice size : $16^3 \times 32$, Physical size $V \simeq 1.6$ fm
- Periodic boundary condition
- Quark mass : $m_f = 0.03\,a^{-1} \simeq$ 0.06 GeV
- DW parameter : N_s=16, DW height M = 1.8
- $m_{PS}/m_V \simeq$ 0.62
- Operator incerted time : τ = 6 (fixed)
- momentum : p=$\pi/8a$=0.3927 $a^{-1} \simeq$ 0.7854 GeV

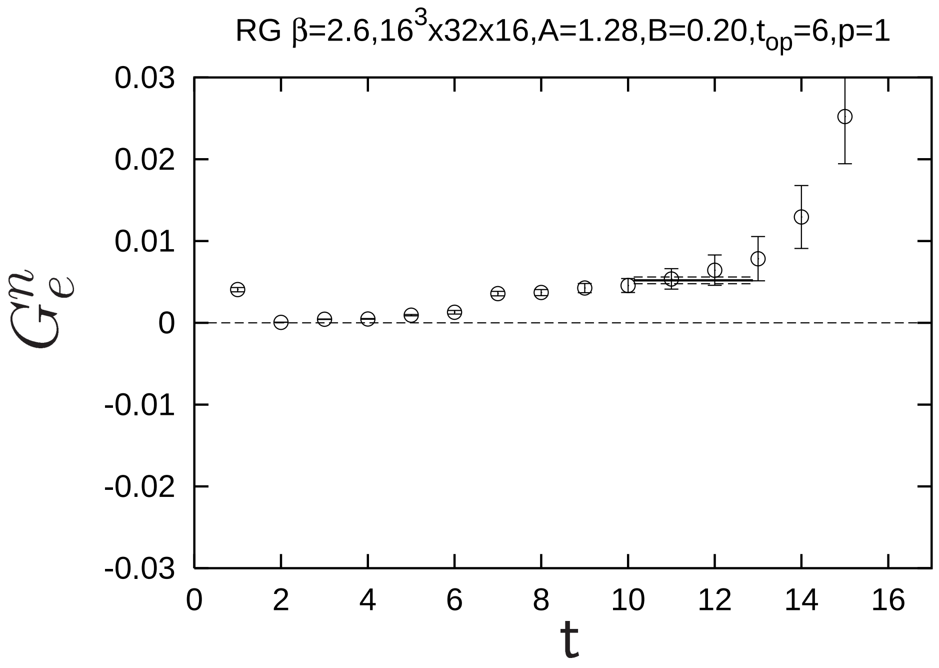
Effective mass plot for Nucleon

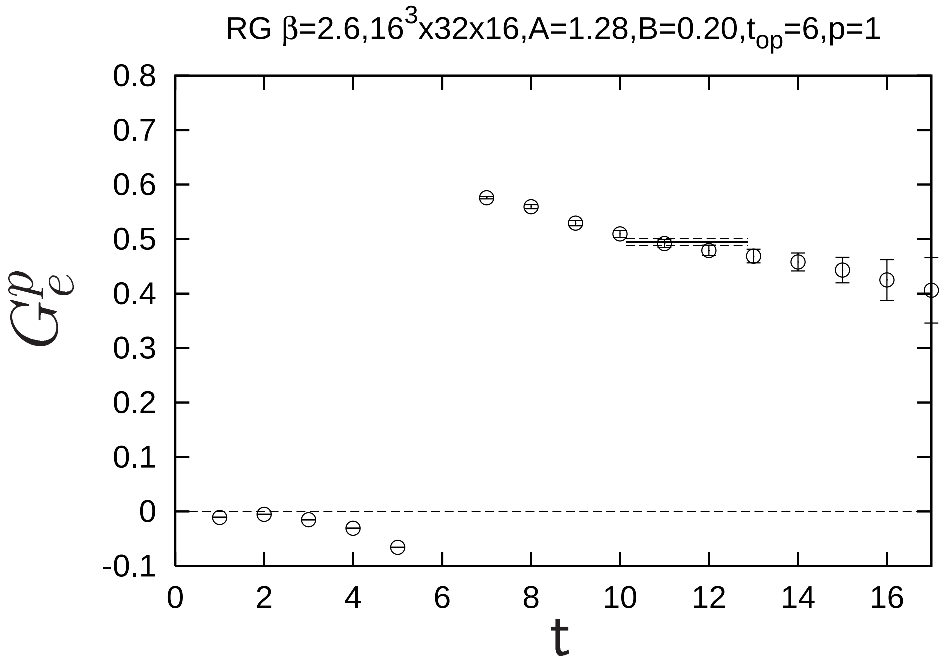




We can see the signal from $t = 5$.

Electric form factor (CP even)





Magnetic form factor (CP even)

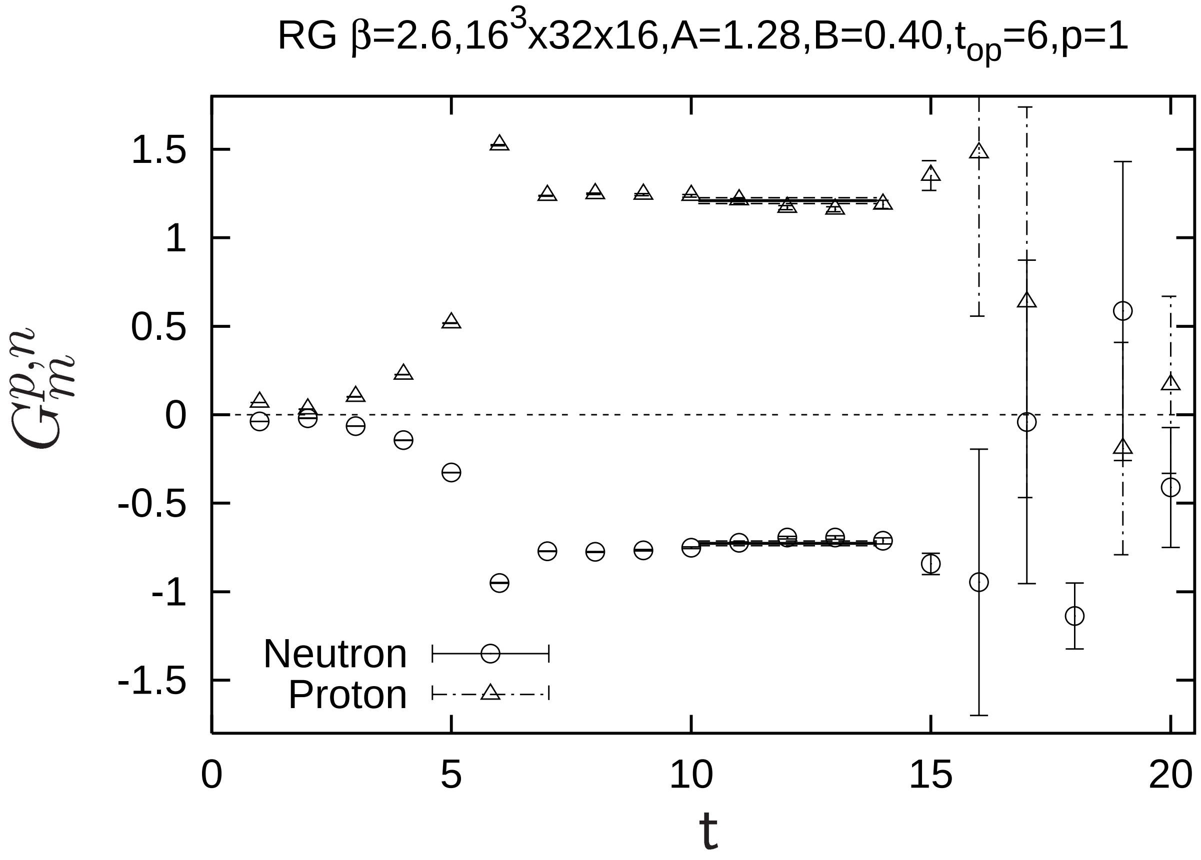


Table of G_e, G_m results

Comparison with other groupe

	fermion	$m_\pi a$	$m_N a$
Our data	DW	0.3094(11)	0.7145(27)
Wilcox et al. [1]	Wilson	0.369(9)	0.74(4)
UKQCD [2]	clover	0.3012(10)	0.7096(46)

Electric form factor

	# conf	G_e^n	G_e^p
Our data	$\mathbf{709}$	0.0052(4)	0.4946(67)
Wilcox et al.	$\mathbf{12}$	0.033(17)	0.481(34)
UKQCD	$O(\mathbf{700})$	-	0.474(17)

Magnetic form factor

	# conf	G_m^n	G_m^p
Our data	$\mathbf{80}$	-0.7269(136)	1.2102(169)
Wilcox et al.	$\mathbf{12}$	-0.748(69)	1.17(11)
UKQCD	$O(\mathbf{700})$	-0.73(4)	1.38(9)

[1] Walter Wilcox, Terrence Draper, Keh-Fei Liu, Phys.Rev. D46 (1992) 1109-1122

[2] M. Göckeler, *et al.* (QCDSF Collabolation), hep-lat/0303019

Electric dipole moment

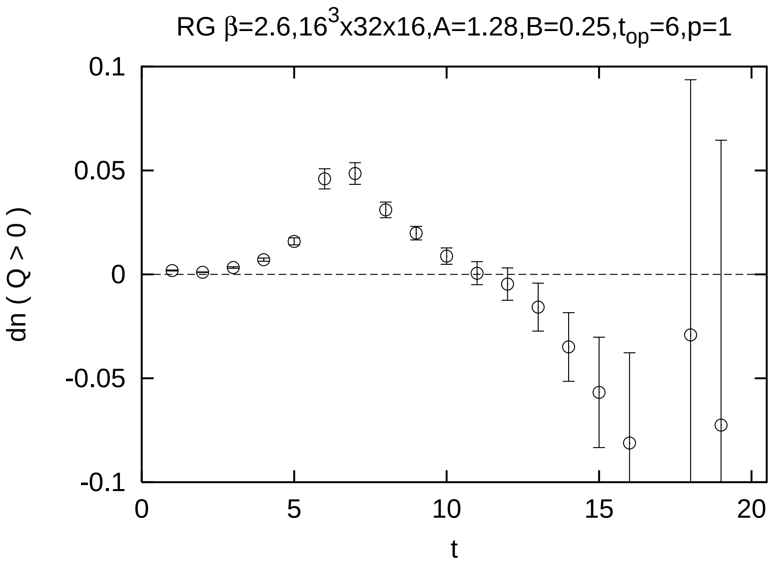
Method (I)

$$\frac{G^{Nj_4QN}(\mathbf{0},\mathbf{q},t,\tau;\Gamma_i^{(\mathrm{i})})}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

$\#$ conf $=$ 709

Response for topological charge distribution

- Neutron



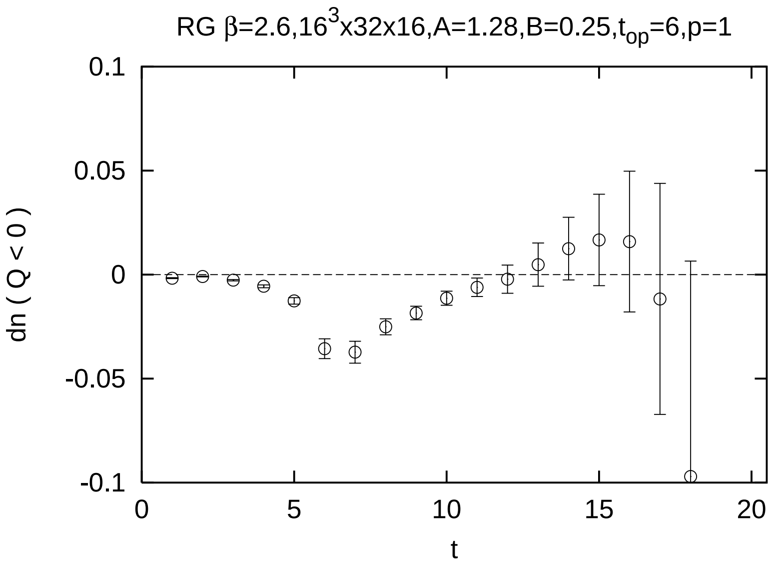
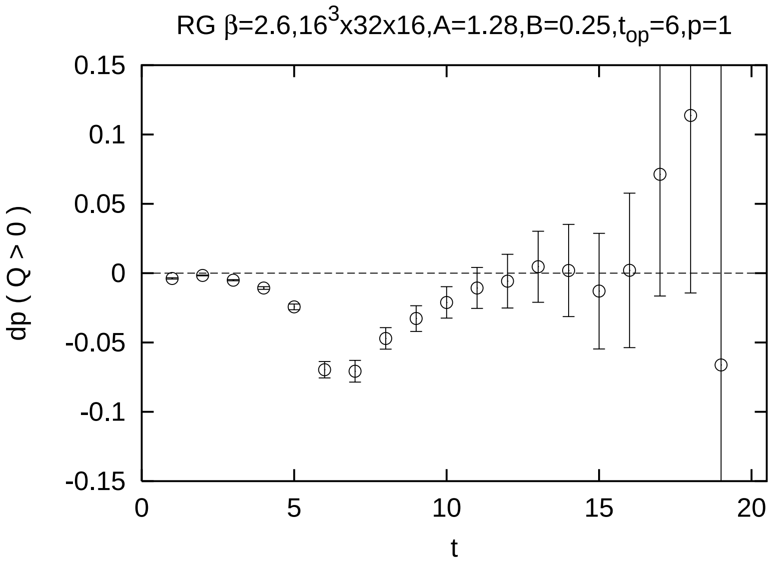
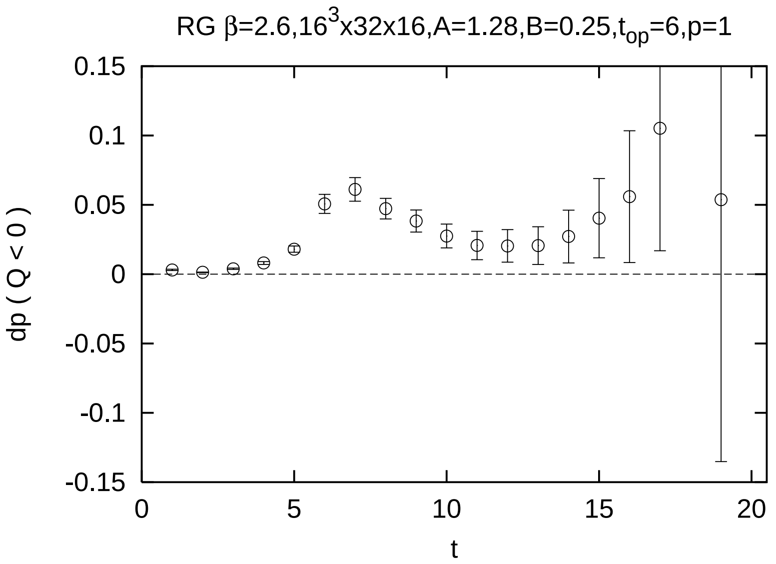


Figure shows the good response from topological charge distribution.

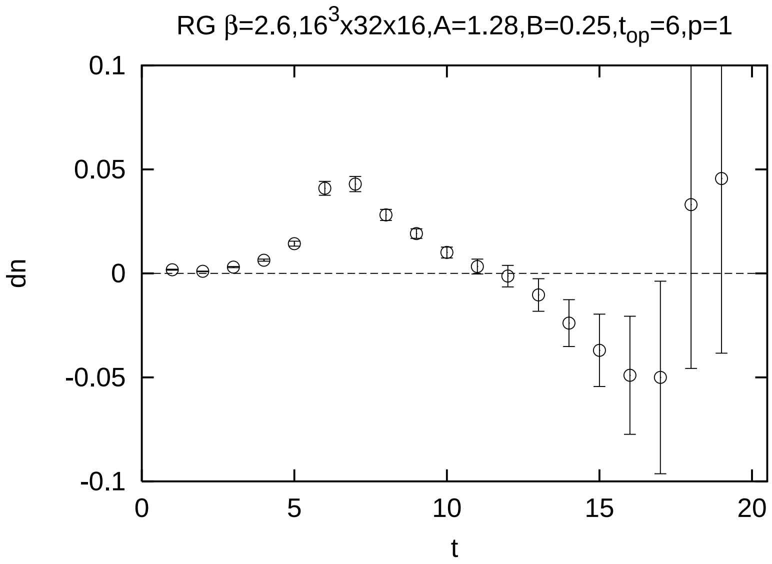
• Proton


RG β=2.6,16^3x32x16,A=1.28,B=0.25,t_op=6,p=1
dp (Q > 0)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
5
10
15
20
t

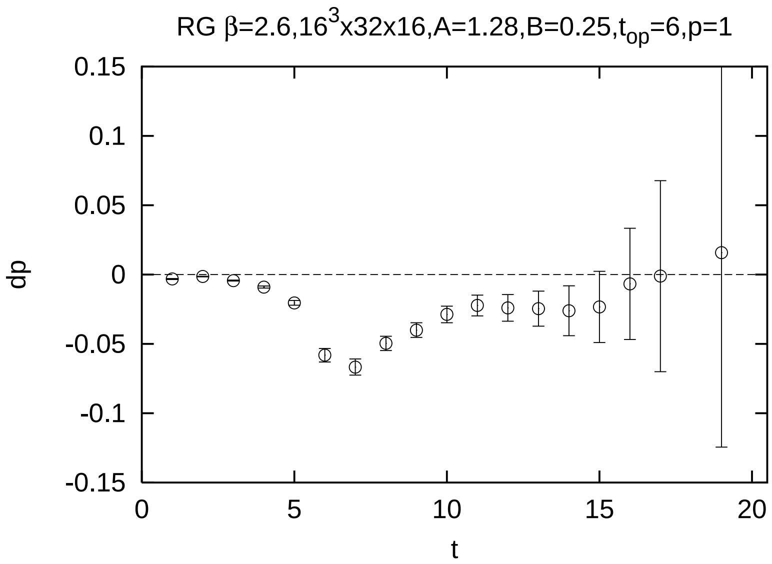

RG β=2.6,16^3x32x16,A=1.28,B=0.25,t_op=6,p=1
dp (Q < 0)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
0
5
10
15
20
t

Electric dipole moment

- Neutron



- Proton



Neutron : can not see the signal !

Proton : plateau in t$=10 \sim 13$

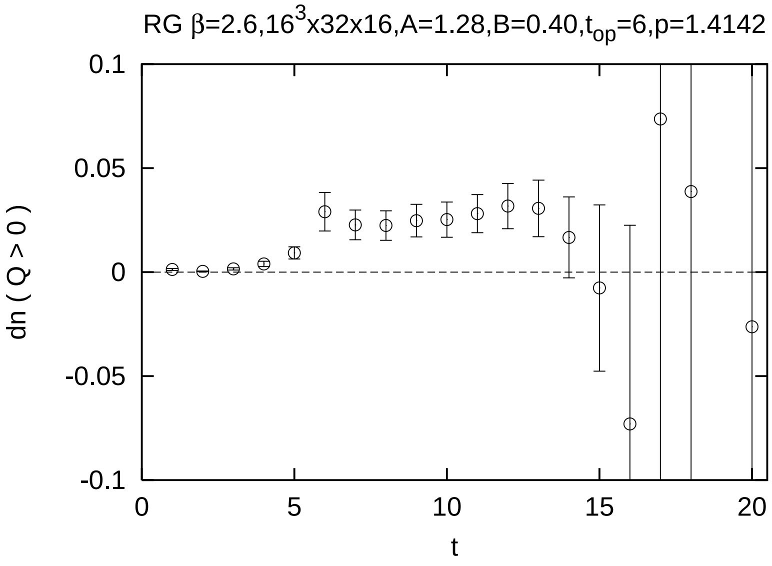
Method (II) (Preliminary)

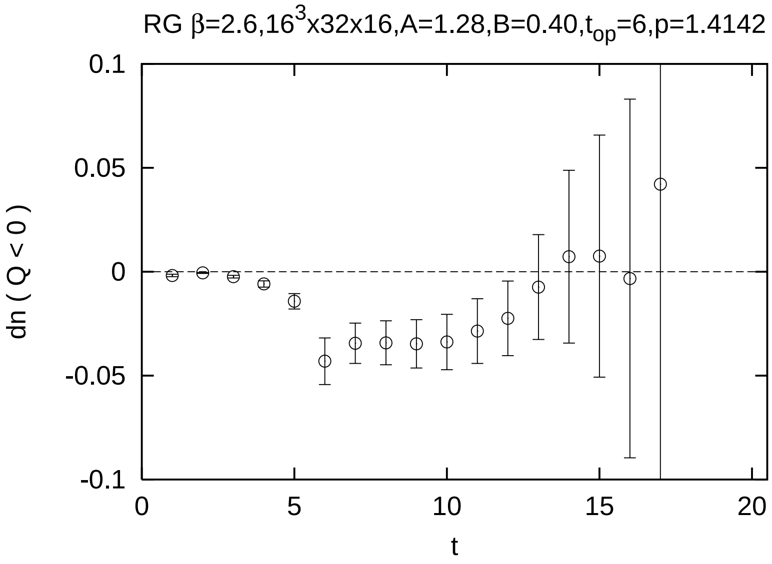
$$\frac{G^{Nj_iQN}(\mathbf{0},\mathbf{q},t,\tau;\Gamma_j^{(\mathrm{ii})})}{G^{NN}(\mathbf{q},t)}R^{1/2}$$

$\#$ conf $= 80$

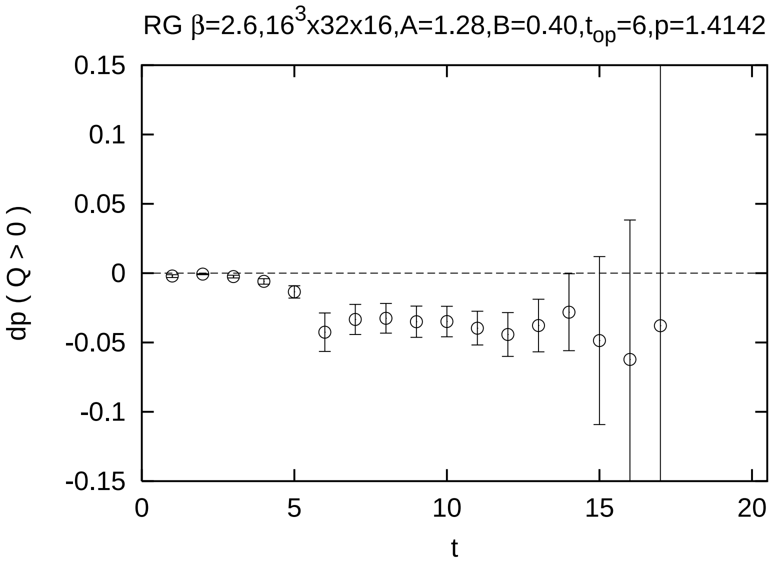
Response for topological charge distribution

- Neutron

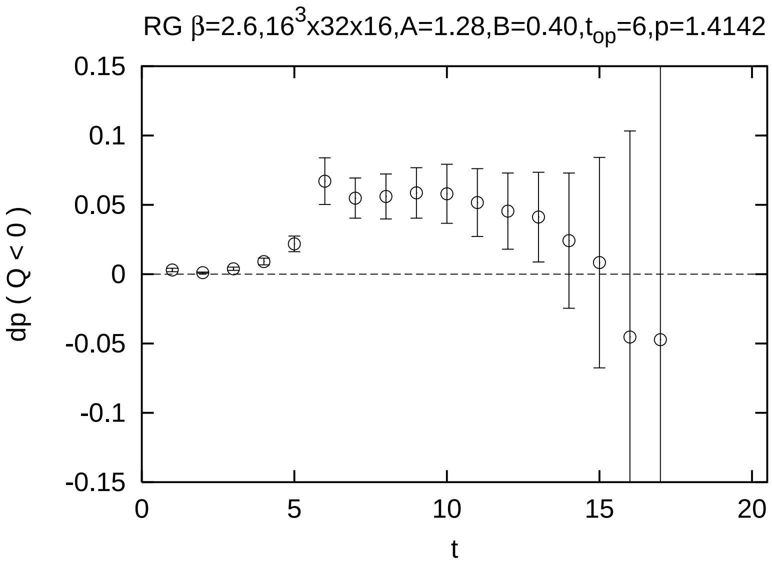




- Proton



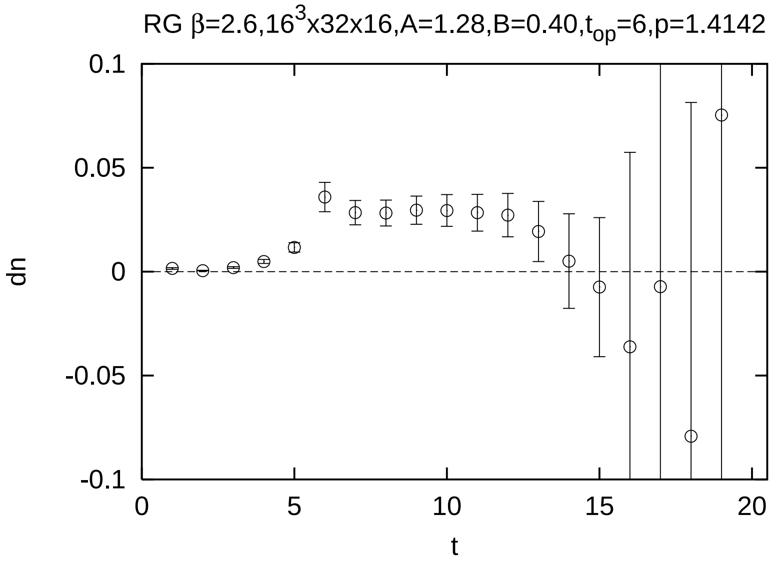
RG β=2.6,16^3x32x16,A=1.28,B=0.40,t_op=6,p=1.4142
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
dp (Q > 0)
0
5
10
15
20
t



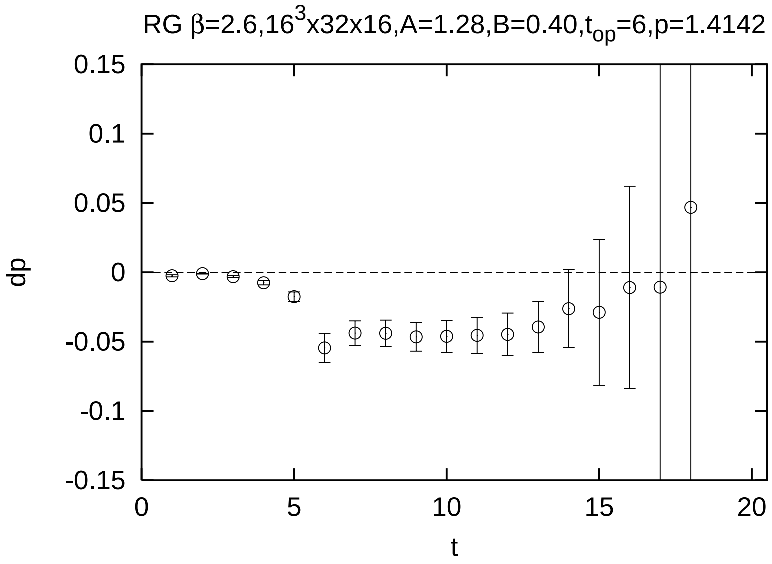
RG β=2.6,16^3x32x16,A=1.28,B=0.40,t_op=6,p=1.4142
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
dp (Q < 0)
0
5
10
15
20
t

Electric dipole moment

- Neutron



- Proton



Neutron : plateau in t=$10 \sim 13$
Proton : plateau in t=$10 \sim 13$

Summary and Furture work

- Response for topological charge
  N $\rightarrow$ OK, P $\rightarrow$ OK
- Signal
  N $\rightarrow$ Method (I) : non-plateau (statistical ?)
  Method (II) : plateau in t $= 10 \sim 13$
  P $\rightarrow$ We will obtain the signal both
  Method (I) and (II) in t$=10 \sim 13$
- momentum dependence, chiral limit, overlap, $\cdots$