◆ 研究会「素粒子物理学の進展」

New Methods to Approximate Neutrino Oscillation Probability and CP Violation in Matter

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at YITP

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Collaborated with
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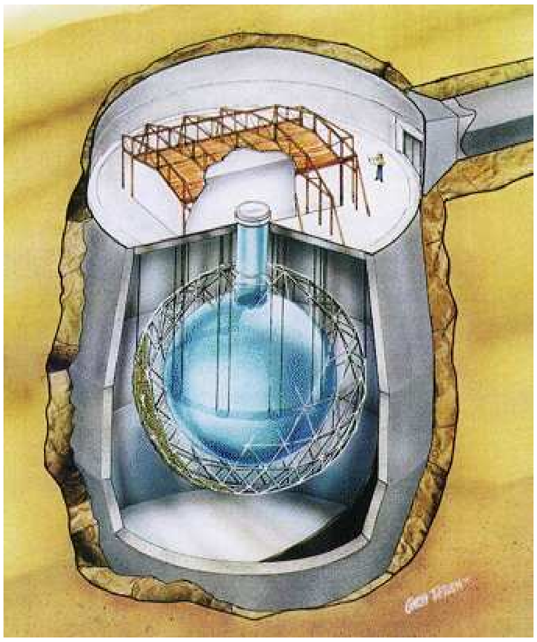
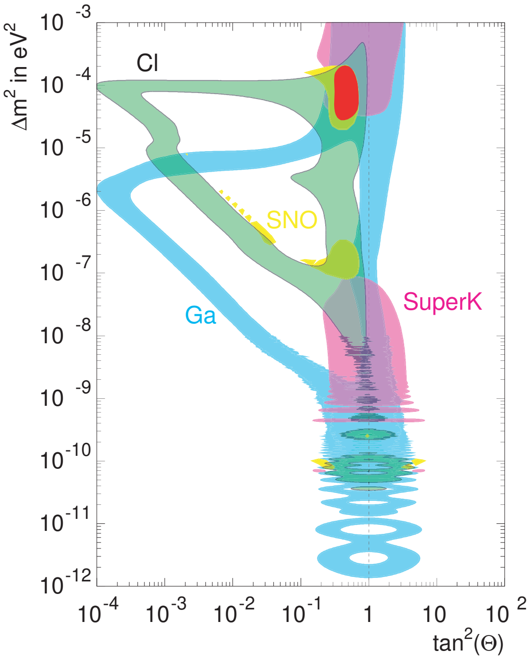
Reference

1. A.T. , K. Kimura and H. Yokomakura
   to be appeared in Phys. Lett. B
2. H. Yokomakura, K. Kimura and A. T.
   Phys. Lett. B544 (2002) 286
3. K. Kimura, A. T. and H. Yokomakura
   Phys. Rev. D66 (2002) 073005
4. K. Kimura, A. T. and H. Yokomakura
   Phys. Lett. B537 (2002) 86

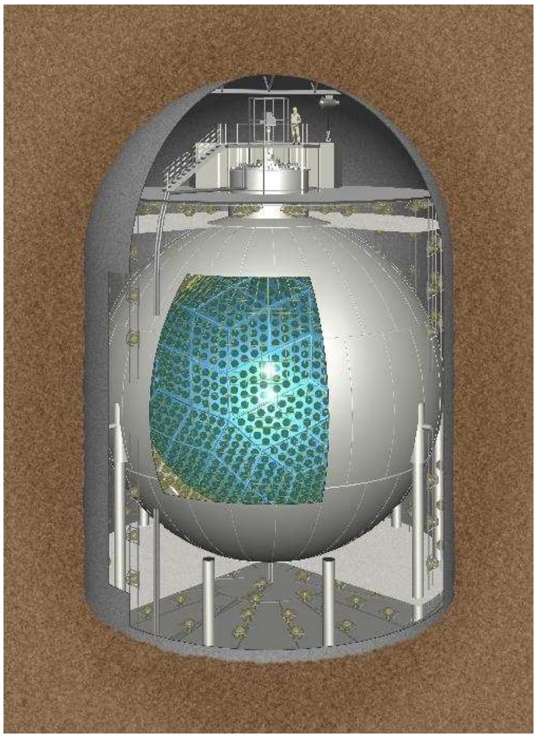
Recent Results



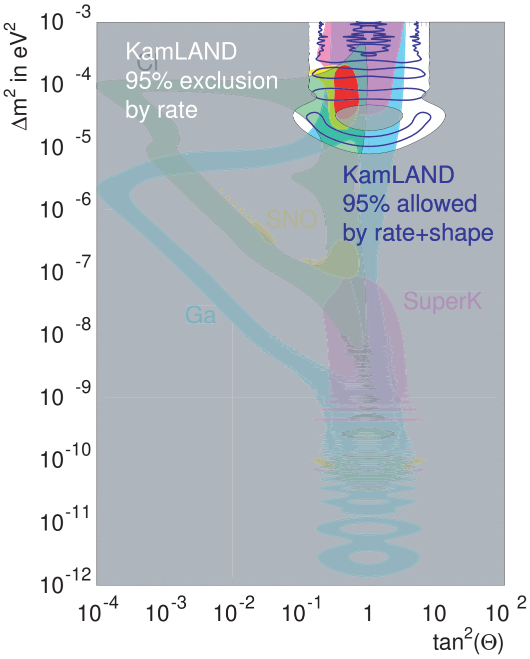
SNO Detector



Allowed Region



KamLAND Detector



Allowed Region

Remark

The solution of solar neutrino problem is Large Mixing Angle MSW Solution

Why LMA MSW solution is so impotant?

The mass hierarchy ratio

$$\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}}$$

is important beacuse it determines whether CP violation can be observed or not.

1. Small Mixing Angle Solution $\alpha = 10^{-3}$
2. Low Mass Solution $\alpha = 10^{-4}$
3. Vacuum Oscillation $\alpha = 10^{-7}$
4. Large Mixing Angle Solution $\alpha = 0.05$

Remark

There is no room for exploring CP violation except for Large Mixing Angle MSW solution.

Neutrino Physics in Future

Present

- Atmospheric Neutrino

$$\theta_{23} \sim \frac{\pi}{4}, \qquad |\Delta m^2_{31}| \sim 3 \times 10^{-3}\mathrm{eV}^2$$

- Solar Neutrino

$$\theta_{12} \lesssim \frac{\pi}{4}, \qquad \Delta m^2_{21} \sim 6 \times 10^{-5}\mathrm{eV}^2$$

- CHOOZ

$$\theta_{13} \leq 0.2$$

$\Downarrow$ Neutrino Beam with High Intensity

Future

- Super-beam or Neutrino Factory

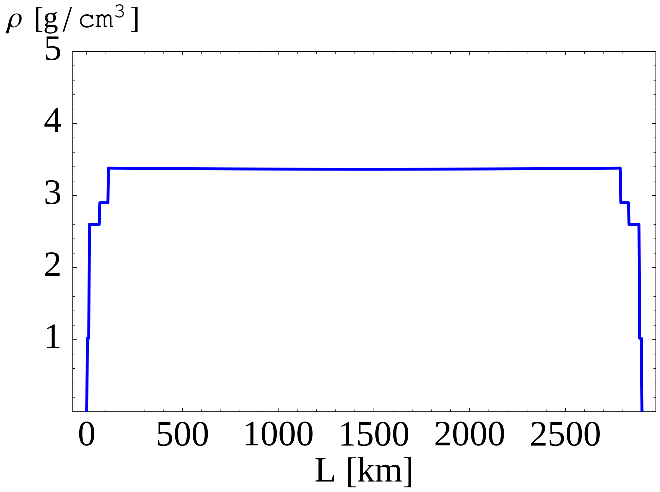
$$\theta_{13}, \qquad \mathrm{sgn}(\Delta m^2_{31}), \qquad \delta$$

Remark

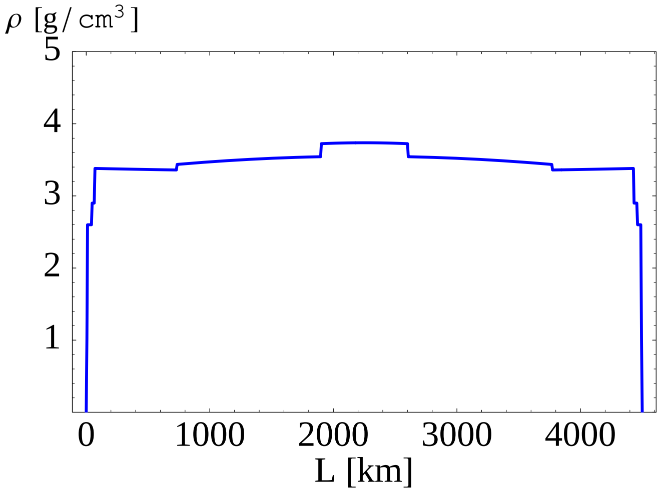
1. Long Baseline Experiments will be planned.
2. Matter effects would be important.

Density Profile along the Neutrino Path

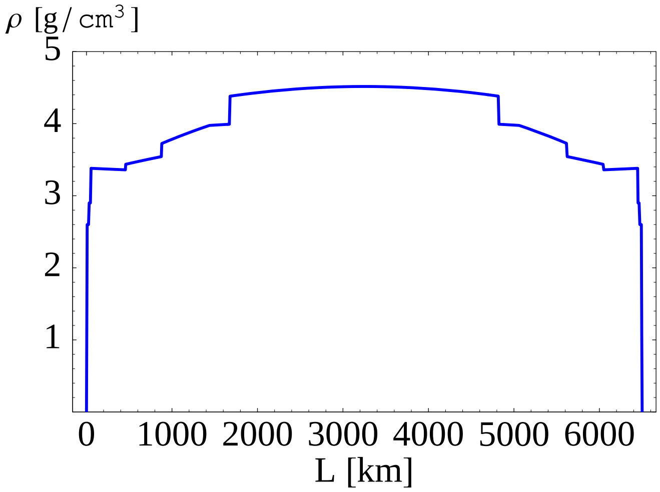
(1) $L = 2900$ km



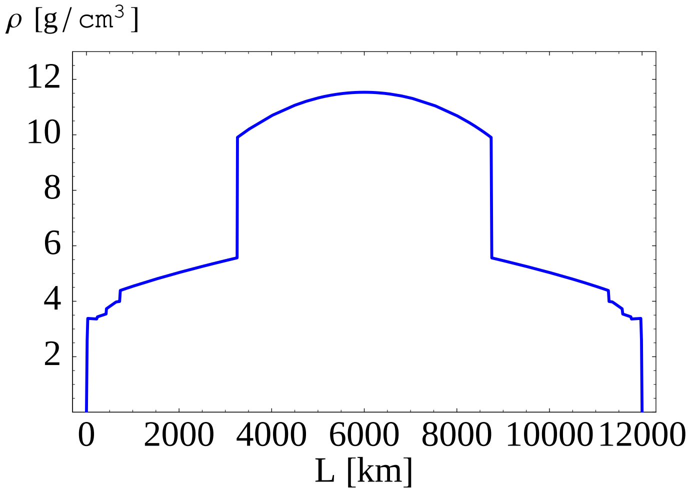
(2) $L = 4500$ km



(3) $L = 6500$ km



(4) $L = 12800$ km

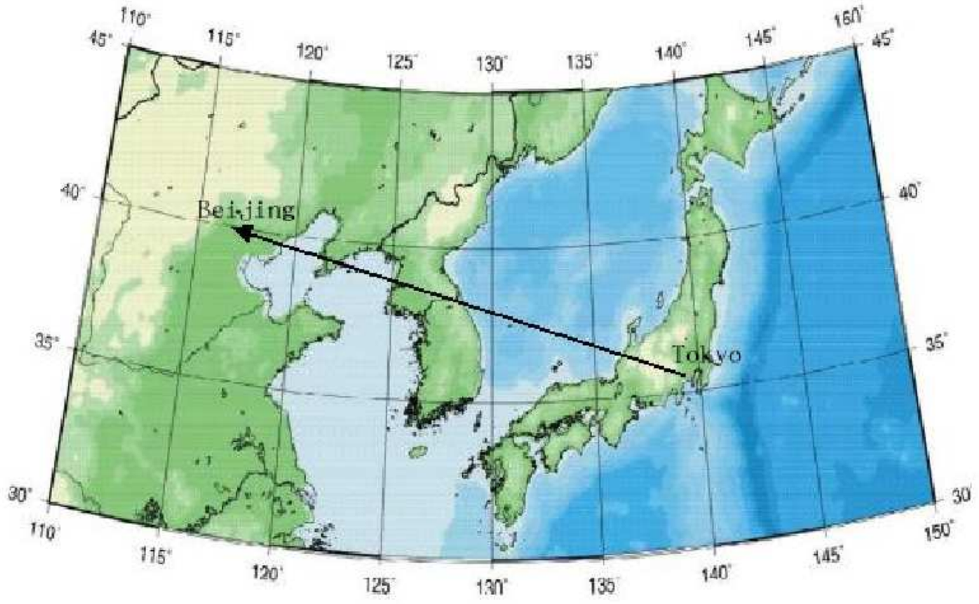


Remark

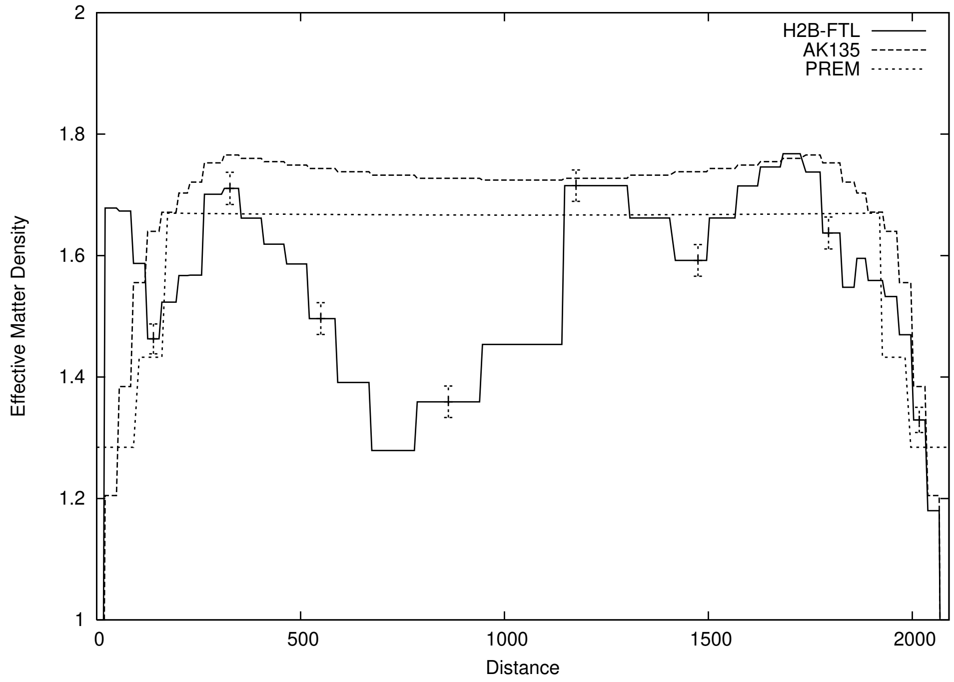
1. Matter density is almost constant in $L < 5000$ km
2. Matter profile effect is important in $L > 5000$ km

Density Profile from J-PARC to Beiging

J-PARC to Beiging



Density



Remark

It seems that constant approximation is not so good for particular baseline.

Hint from Solar Neutrino Problem

Solar neutrino problem is solved standard three steps.

1. Kuo-Pantaleone's Formula

$$P^{(3)}(\nu_e \to \nu_e) = c_{13}^4 P^{(2)}(\nu_e \to \nu_e) + s_{13}^4$$

   This reduces three generation to two generation.

2. Parke's Formula

$$P^{(2)}(\nu_e \to \nu_e) = \frac{1}{2} + \left(\frac{1}{2} - P^{\text{jump}}\right) \cos\theta_0 \cos\theta$$

3. Landou Zener's Formula

$$P^{\text{jump}} = \exp\left(-\frac{\Delta m^2}{2E|(dn_e/dr)/r|_{\text{res}}} \frac{\sin^2 2\theta}{\cos 2\theta}\right)$$

   n_e is number density of electron

Strategy

We would like to extend Kuo-Pantaleone's Formula for solar neutrino to it for LBL experiments.

Perturbation due to two small parameters

Two small parameters

1. $\alpha = \Delta m_{21}^2/\Delta m_{31}^2 \simeq 0.03$
2. $s_{13} = \sin\theta_{13} \leq 0.16$

CP dependence of $P(\nu_e \to \nu_\mu)$ on α and s_{13}

$$P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C$$

$$\lim_{\alpha\to 0} A = 0 \qquad \lim_{s_{13}\to 0} A = 0$$
$$\lim_{\alpha\to 0} B = 0 \qquad \lim_{s_{13}\to 0} B = 0$$

Double suppression of CP violating effects

$$A = O(\alpha s_{13}) \qquad B = O(\alpha s_{13})$$

These means that dependence of $P(\nu_e \to \nu_\mu)$ on CP phase disappears in the limit $\alpha \to 0$ or $s_{13} \to 0$.
It seems that in order to calculate A or B first order calculation is needed.

Main result

We propose a more simple method!

Remarkable Features of Effective Masses

Hamiltonian

$$H = U\mathrm{diag}(0, \Delta_{21}, \Delta_{31})U^\dagger + \mathrm{diag}(a, 0, 0)$$

Standard Parametrization

$$U = O_{23}\Gamma_\delta O_{13}\Gamma_\delta^\dagger O_{12}$$

Eigenvalue equation

$$\det(\lambda - H) = 0$$

Effective Masses

$$\lambda_1 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2 - 3t}\left[u + \sqrt{3(1-u^2)}\right]$$
$$\lambda_2 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2 - 3t}\left[u - \sqrt{3(1-u^2)}\right]$$
$$\lambda_3 = \frac{1}{3}s + \frac{2}{3}u\sqrt{s^2 - 3t}$$

where

$$\begin{aligned} s &= \Delta_{21} + \Delta_{31} + a \\ t &= \Delta_{21}\Delta_{31} + a[\Delta_{21}(1 - s_{12}^2 c_{13}^2) + \Delta_{31}(1 - s_{13}^2)] \\ u &= \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{2s^3 - 9st + 27a\Delta_{21}\Delta_{31}c_{12}^2 c_{13}^2}{2(s^2-3t)^{3/2}}\right)\right] \end{aligned}$$

V. Barger et. al., Phys. Rev. D22,2718 (1980)

Remarkable features

Effective masses do not depend on 2-3 mixing and CP phase.

Decomposition of Hamiltonian

Standard Parametrization

$$U = O_{23}\Gamma_\delta O_{13}\Gamma_\delta^\dagger O_{12}$$

Commutation relations

$$[O_{23}, V(t)] = \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \frac{1}{2E}\begin{pmatrix} a(t) & & \\ & 0 & \\ & & 0 \end{pmatrix}\right] = 0,$$

$$[\Gamma_\delta, V(t)] = \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix}, \frac{1}{2E}\begin{pmatrix} a(t) & & \\ & 0 & \\ & & 0 \end{pmatrix}\right] = 0$$

Decomposition of Hamiltonian

$$\begin{aligned} H &= U\mathrm{diag}(0, \Delta_{21}, \Delta_{31})U^\dagger + \mathrm{diag}[a(t), 0, 0] \\ &= O_{23}\Gamma_\delta \\ &\quad \underbrace{\left(O_{13}O_{12}\mathrm{diag}(0, \Delta_{21}, \Delta_{31})O_{12}^T O_{13}^T + \mathrm{diag}[a(t), 0, 0]\right)}_{\substack{\| \\ H'(t)}} \Gamma_\delta^\dagger O_{23}^T \end{aligned}$$

Therefore

$$H(t) = [O_{23}\Gamma_\delta]H'(t)[O_{23}\Gamma_\delta]^\dagger$$

In constant matter

$$\det(\lambda - H) = \det(\lambda - H')$$

Remark

Hamiltonian can be decomposed in arbitrary matter.

CP Dependence in Arbitrary Matter

Schrödinger equation and Time Evolution Operator

$$i\frac{\partial \nu}{\partial t} = H(t)\nu, \quad \nu(t) = S(t)\nu(0)$$

Transformation of base

$$\nu' = [O_{23}\Gamma_\delta]^\dagger \nu$$

$$i\frac{\partial \nu'}{\partial t} = H'(t)\nu', \quad \nu'(t) = S'(t)\nu'(0)$$

$$\boxed{S(t) = [O_{23}\Gamma_\delta]S'(t)[O_{23}\Gamma_\delta]^\dagger}$$

$$P(\nu_e \to \nu_\mu) \;=\; |S_{\mu e}|^2 = |S'_{\mu e}c_{23} + S'_{\tau e}s_{23}e^{i\delta}|^2$$

$$\boxed{P(\nu_e \to \nu_\mu) = A\cos\delta + B\sin\delta + C}$$

Remark

The CP dependence of $P(\nu_e \to \nu_\mu)$ in arbitrary matter is simple.

Order Counting of $S'_{\mu e}$ and $S'_{\tau e}$

$$
\begin{aligned}
A &= 2\mathrm{Re}[S'^{*}_{\mu e} S'_{\tau e}] c_{23} s_{23} = O(\alpha s_{13}) \\
B &= -2\mathrm{Im}[S'^{*}_{\mu e} S'_{\tau e}] c_{23} s_{23} = O(\alpha s_{13})
\end{aligned}
$$

Order counting of $S'_{\mu e}$ and $S'_{\tau e}$

$$
\begin{aligned}
H' &= O_{13} O_{12} \mathrm{diag}(0, \Delta_{21}, \Delta_{31}) O_{12}^T O_{13}^T \\
&+ \mathrm{diag}(a(t), 0, 0)
\end{aligned}
$$

$$
\begin{aligned}
H^{\ell} &= \lim_{s_{13}\to 0} H' \\
&= \begin{pmatrix} \Delta_{21} s_{12}^2 + a(t) & \Delta_{21} s_{12} c_{12} & 0 \\ \Delta_{21} s_{12} c_{12} & \Delta_{21} c_{12}^2 & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix}
\end{aligned}
$$

$$
\lim_{s_{13}\to 0} S'_{\tau e} = 0 \qquad S'_{\tau e} = O(s_{13})
$$

$$
\begin{aligned}
H^{h} &= \lim_{\alpha\to 0} H' \\
&= \begin{pmatrix} \Delta_{31} s_{13}^2 + a(t) & 0 & \Delta_{31} s_{13} c_{13} \\ 0 & 0 & 0 \\ \Delta_{31} s_{13} c_{13} & 0 & \Delta_{31} c_{13}^2 \end{pmatrix}
\end{aligned}
$$

$$
\lim_{\alpha\to 0} S'_{\mu e} = 0 \qquad S'_{\mu e} = O(\alpha)
$$

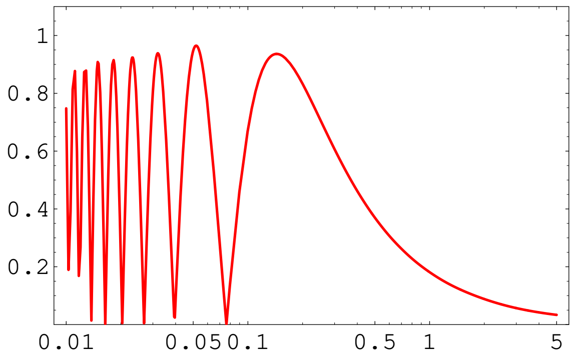
Remark

$S'_{\mu e}$ is $O(\alpha)$ and $S'_{\tau e}$ is $O(s_{13})$.

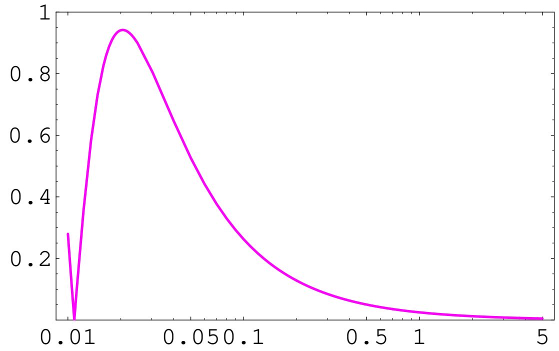
Dependence of $S'_{\mu e}$ in α and s_{13}

We investigate the dependence for $S'_{\mu e}$ of small parameters α and s_{13} at $L = 3000$ km

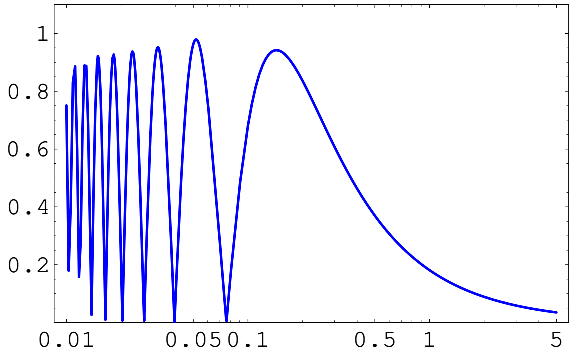
(1) $s_{13} = 0.16, \alpha = 0.03$



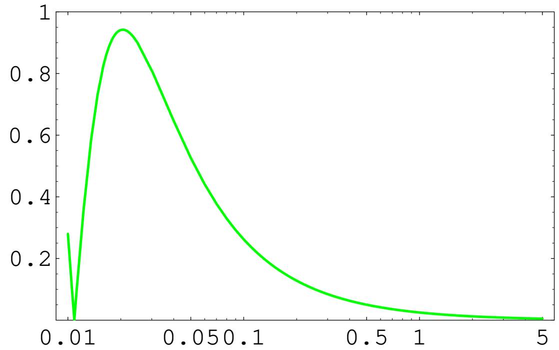
(2) $s_{13} = 0.16, \alpha = 10^{-3}$



(3) $s_{13} = 10^{-2}, \alpha = 0.03$



(4) $s_{13} = 10^{-2}, \alpha = 10^{-3}$



A figure (1) is almost same as figure (3) and (2) is almost same as (4).

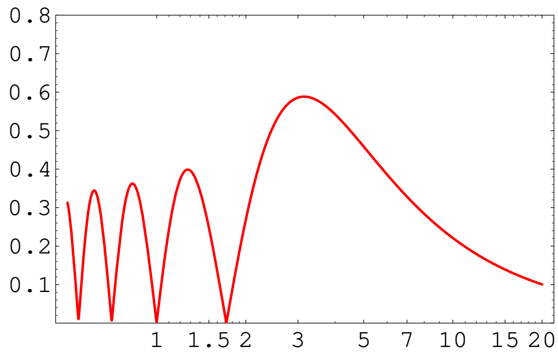
Remark

$S'_{\mu e}$ strongly depends on α and hardly depends on s_{13}.

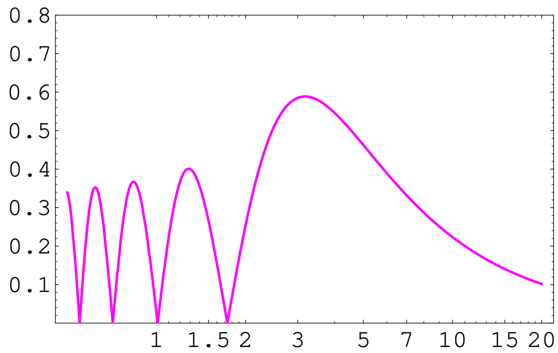
Dependence of $S'_{\tau e}$ in α and s_{13}

We investigate the dependence for $S'_{\tau e}$ of small parameters α and s_{13} at $L = 3000$ km

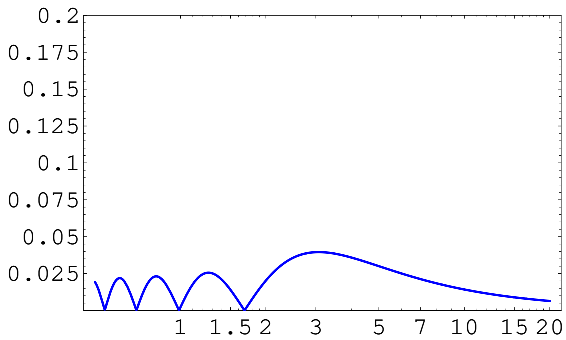
(1) $s_{13} = 0.16, \alpha = 0.03$



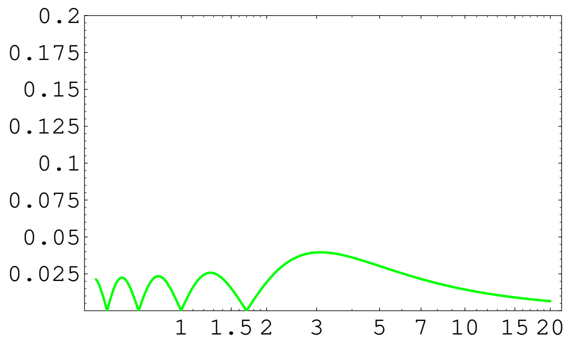
(2) $s_{13} = 0.16, \alpha = 10^{-3}$



(3) $s_{13} = 10^{-2}, \alpha = 0.03$



(4) $s_{13} = 10^{-2}, \alpha = 10^{-3}$



A figure (1) is almost same as figure (2) and (3) is almost same as (4).

Remark

$S'_{\tau e}$ strongly depends on s_{13} and hardly depends on α

Main Result

Two remarkable features

1. $S'_{\mu e} = O(\alpha)$, $\quad S'_{\tau e} = O(s_{13})$
2. $S'_{\mu e}$ and $S'_{\tau e}$ hardly depend on s_{13} and α, respectively.

- Uaual Method
  Calculate $S'_{\mu e}$ and $S'_{\tau e}$ in first order perturbation
- Our New Method

$$\begin{aligned} S'_{\mu e} &= \lim_{\theta_{13}\to 0} S'_{\mu e} + O(\alpha s_{13}) + O(\alpha^2 s_{13}) + \cdots \\ &\simeq \lim_{\theta_{13}\to 0} S'_{\mu e} = S^{\ell}_{\mu e} \end{aligned}$$

$$\begin{aligned} S'_{\tau e} &= \lim_{\alpha\to 0} S'_{\tau e} + O(\alpha s_{13}) + O(\alpha^2 s_{13}) + \cdots \\ &\simeq \lim_{\alpha\to 0} S'_{\tau e} = S^{h}_{\tau e} \end{aligned}$$

  Calculate $S'_{\mu e}$ and $S'_{\tau e}$ without first order perturbation

Main Result

$$\begin{aligned} A &\simeq 2\mathrm{Re}[S^{\ell *}_{\mu e} S^{h}_{\tau e}] c_{23} s_{23}, \\ B &\simeq -2\mathrm{Im}[S^{\ell *}_{\mu e} S^{h}_{\tau e}] c_{23} s_{23}, \\ C &\simeq |S^{\ell}_{\mu e}|^2 c_{23}^2 + |S^{h}_{\tau e}|^2 s_{23}^2, \end{aligned}$$

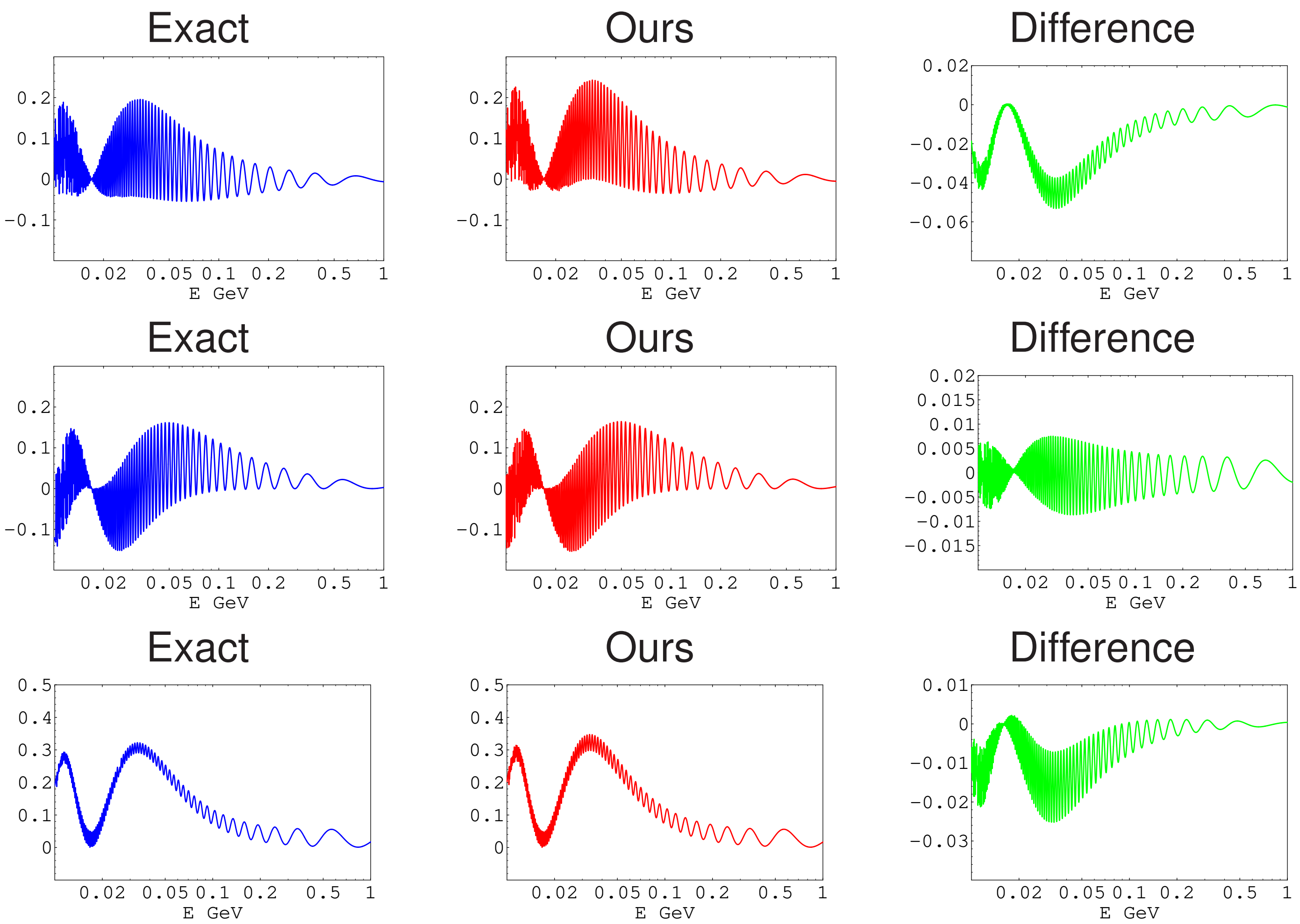
Remark

We can calculate A and B without first order perturbation.

Comparision with Numerical Calculation

Comparision of our result with numerical calculation in

$$L = 730 \text{ km}$$
$$0.01(\text{GeV}) \leq E \leq 1(\text{GeV})$$

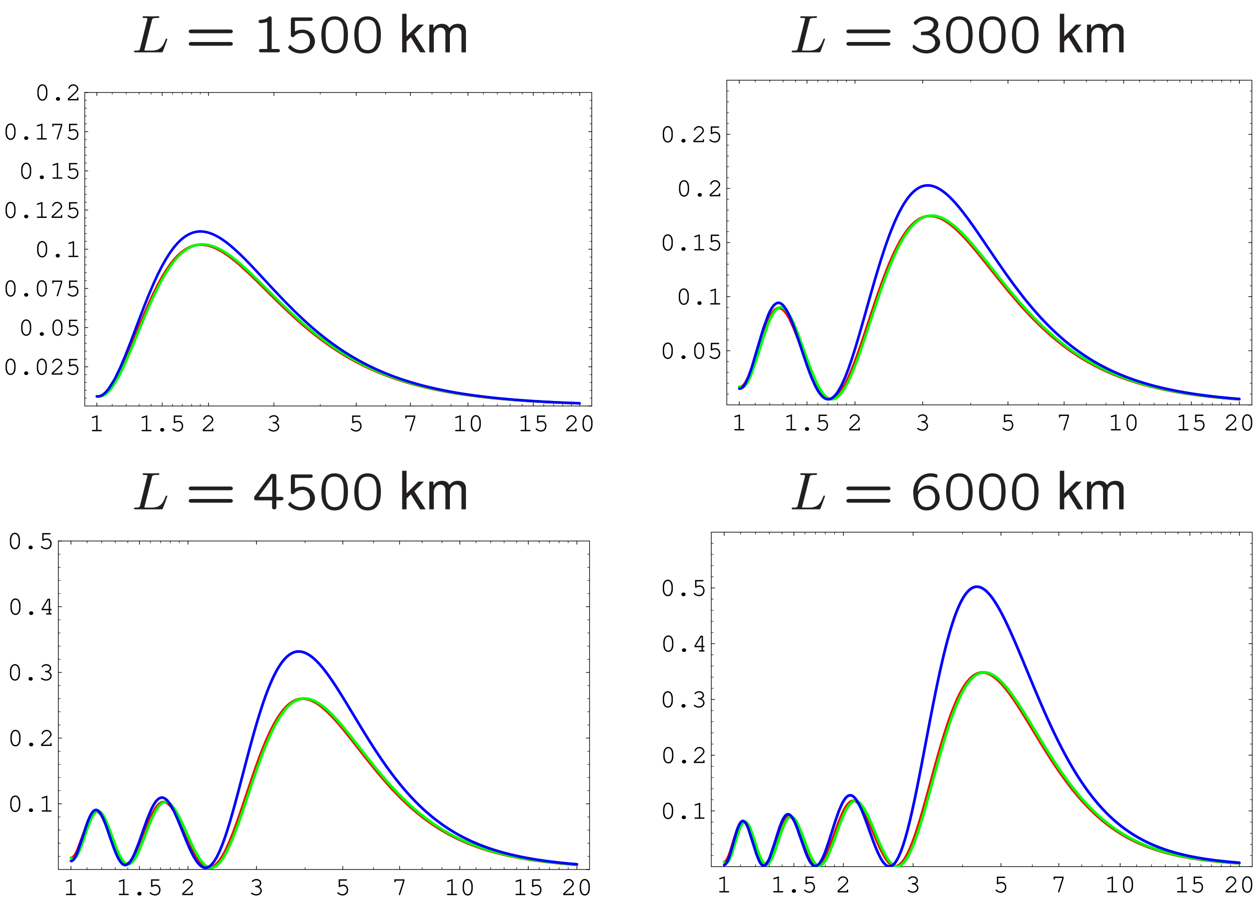


Remark

1. Very good agreement with exact solusion.
2. Applicable to wide energy region.

Comparision with Formula by Usual Method

Comparision of our result with formula by usual method



The longer baseline become, the larger the difference between usual method and numerical calculation become.

Remark

Our Formula is good than formula by usual method in the MSW resion.

Summary

We have calculated $P(\nu_e \to \nu_\mu)$ in arbitrary matter.

1. We have proposed a simple method that cab be derived an approximate formula in arbitrary matter

2. We have obtained the approximate formula without first order perturbation.

$$
\begin{aligned}
A &\simeq 2\mathrm{Re}[S^{l*}_{\mu e} S^{h}_{\tau e}] c_{23} s_{23}, \\
B &\simeq -2\mathrm{Im}[S^{l*}_{\mu e} S^{h}_{\tau e}] c_{23} s_{23}, \\
C &\simeq |S^{\ell}_{\mu e}|^2 c^2_{23} + |S^{h}_{\tau e}|^2 s^2_{23},
\end{aligned}
$$

3. We have shown that our formula has wide applicable energy region.

4. We have compaired the formula derived by usual method. we have shown that our formula has a good precision even in the MSW region.