Schrödinger functional coupling with improved gauge actions in SU(3) gauge theory

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for CP-PACS collaboration:

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Outline

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  - L_{max}/r_0 **(low energy scale ratio)**
- **Conclusion**

Introduction

- **Motivation**

  $\boxed{\textbf{High energy experiments}}$ $\overline{\mathrm{MS}}$**-scheme**

$$\alpha_{\overline{\mathrm{MS}}}(m_Z) = 0.1172(20)$$

Ref. K. Hagiwara et al. PRD 66 (2002) PDG Collaboration

- **Problem**
  - **Non-perturbative evolution of running coupling**

$$\Uparrow$$

$$\boxed{\textbf{Step scaling function(SSF), Schrödinger Functional (SF)}}$$

$\sim$ **ALPHA Collaboration**

Strategy

$\boxed{\textbf{Output}}$	**PT matching**	**SF scheme**
$\alpha_{\overline{\mathrm{MS}}}$	$\longleftarrow$	$\alpha_{\mathrm{SF}}(q = 2/L)$
		$\uparrow$
		$\alpha_{\mathrm{SF}}(q = 1/L)$
		$\bullet$
energy $\uparrow$		**NP evolution**
		$\bullet$
		$\alpha_{\mathrm{SF}}(q = 2/L_{\mathrm{max}})$
$\boxed{\textbf{Input}}$		$\uparrow$
Hadronic scheme	$\longrightarrow$	$\alpha_{\mathrm{SF}}(q = 1/L_{\mathrm{max}})$
m_π, f_π, r_0	**matching** L_{max}/r_0	

Quench results

No dynamical quark effect

$$\begin{aligned} \Lambda_{\overline{\rm MS}} &= 251(21)\mathbf{MeV} \\ \alpha_{\overline{\rm MS}}(90\mathbf{GeV}) &= 0.0818(10) \end{aligned}$$

Ref. ALPHA Collab., P. Weisz talk at Tsukuba July 2001

$N_f = 3$ project

- $N_f = 3$ QCD simulation

  CP-PACS/JLQCD Collaborations

$\boxed{\text{Ultimate goal}}$ **: evaluation of $\alpha_{\overline{MS}}$ from $N_f = 3$ QCD simulation**

- **Why do we choose Iwasaki action ?**

  Ref. JLQCD Collab., Nucl. Phys. B (Proc. Suppl.) 106, 263 (2002)

plaquette action + $O(a)$ improved Wilson fermion	Iwasaki action + $O(a)$ improved Wilson fermion
$\downarrow$	$\downarrow$
strong lattice artifacts associated with phase transition	such lattice artifacts are absent

$$\Downarrow$$

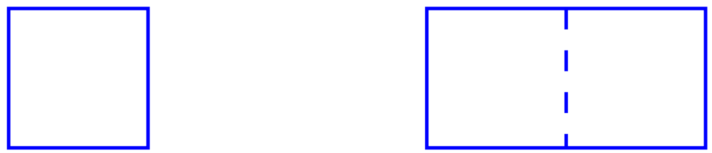
We decided to adopt the Iwasaki action as gauge part.

Purpose of this poster

- **Universality check**
- **Cut off effect**

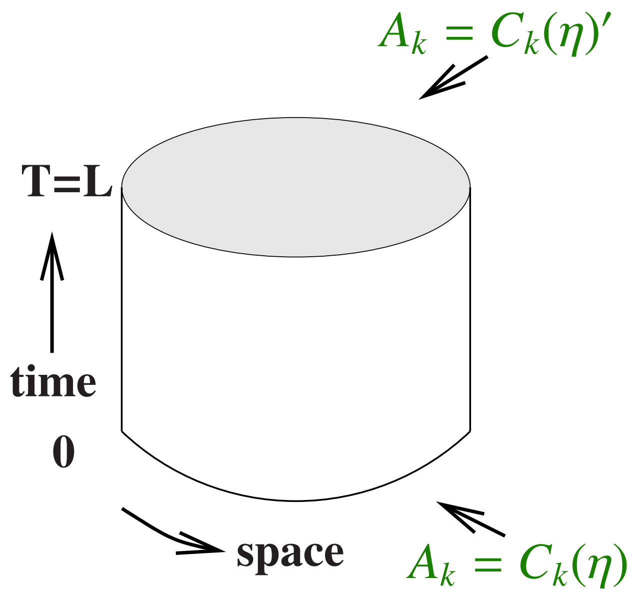
for SSF and low energy scale ratio
employing improved gauge actions
in SU(3) gauge theory.

$\sim$ **toward** $N_f = 2, 3$ **simulations**



SF set up and SF coupling

- **SF set up**



- **Definition of SF coupling**

$$\bar{g}^2_{\rm SF}(L) = k/\Gamma'|_{\eta=\nu=0} = k/\left\langle \frac{\partial S}{\partial \eta} \right\rangle\Big|_{\eta=\nu=0},$$

$$e^{-\Gamma} = \mathcal{Z} = \int D[U]e^{-S[U]},$$

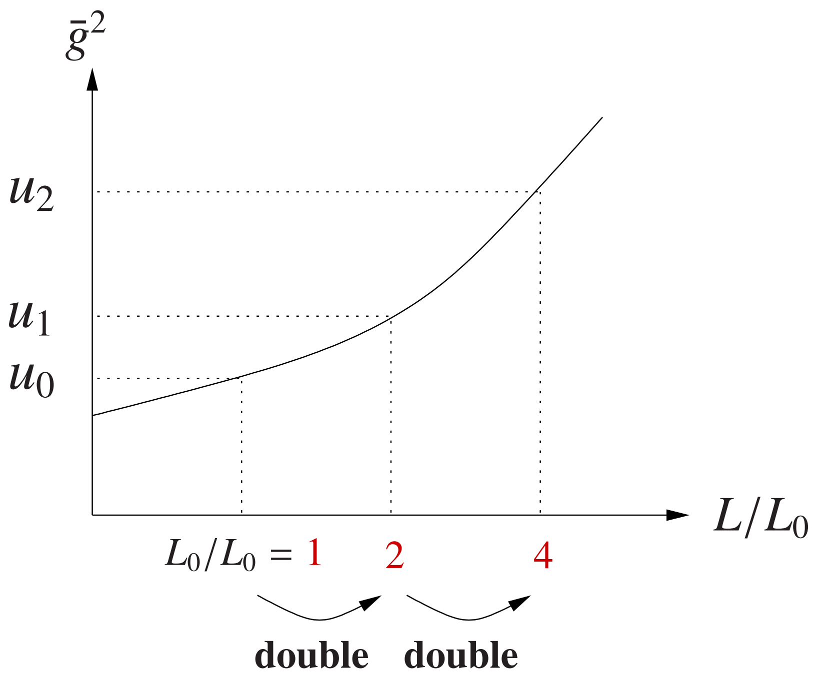
where $S[U] = S_{\rm imp}[U]$: improved gauge action.

Non-perturbative evolution of running coupling

- **Step scaling function (SSF)**
  - **Continuum SSF**

$$\sigma(2,u) = \bar{g}^2(2L)\Big|_{\bar{g}^2(L)=u}$$

$$\begin{aligned} u_0 &= \bar{g}^2(L_0) \\ u_1 &= \sigma(2,u_0) \\ u_2 &= \sigma(2,u_1) \\ &\vdots \end{aligned}$$



  - **Lattice SSF**

$\Sigma(2,u,a/L)$: **SSF calculated on the lattice**

$\boxed{\textbf{Continuum limit}}$

$$\sigma(2,u) = \lim_{a/L\to 0} \Sigma(2,u,a/L)$$

- **Low energy scale**

$$u_i = \bar{g}^2(L_i) \qquad i = 0, 1, 2, \ldots, n$$

$$L_i = 2^{i-n} L_n = 2^{i-n} L_{\rm max} \quad (L_n = L_{\rm max})$$

  - **Def of $L_{\rm max}$**

$$\bar{g}^2_{\rm SF}(L_{\rm max}) = 3.480$$

  - **Continuum limit of low energy scale ratio**

$$L_{\rm max}/r_0 = \lim_{a/L_{\rm max}\to 0} \underbrace{(L_{\rm max}/a)}_{\textbf{SF scheme}} \times \underbrace{(a/r_0)}_{\textbf{Hadronic scheme}}$$

$$r_0 = 0.5\ \textbf{fm : Sommer scale}$$

O(a) improvement

- **Physical quantity on the lattice** $\qquad$ a**: lattice spacing**

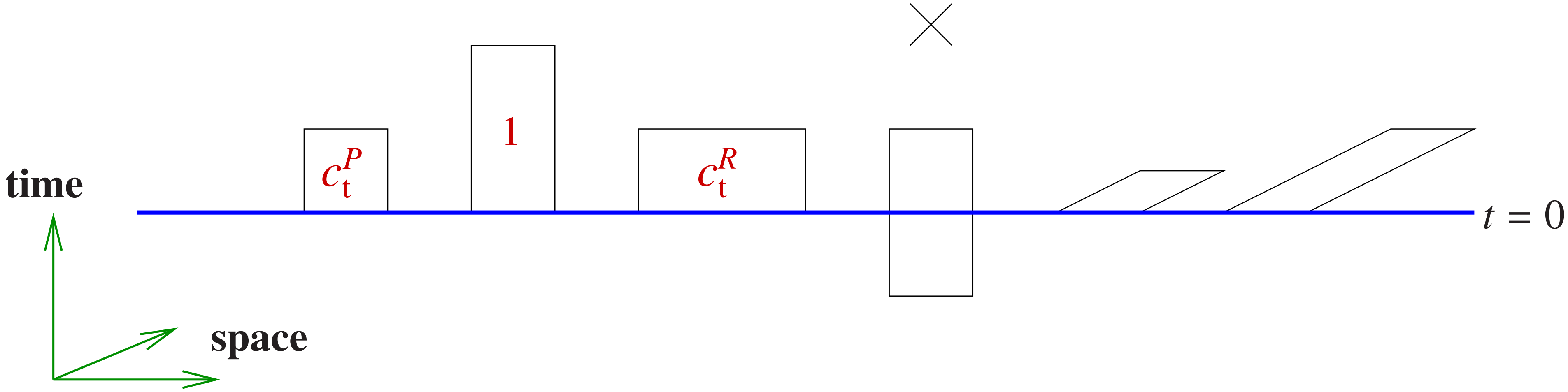
$$O_{\mathrm{lattice}}(a) = \underbrace{O_{\mathrm{lattice}}(0)}_{\text{continuum value}} \left[1 + \underbrace{C_1 a + O(a^2)}_{\text{lattice artifact}}\right]$$

- **Renormalization and improvement**

Renormalization	Improvement
relevant, marginal operator	irrelevant operator
coefficient of counterterm	improvement coefficient
renormalization condition	improvement condition

O(a) boundary improvement coefficients

- **O(a) boundary improvement coefficients**



$$c_t^P(g_0^2) = 1 + c_t^{P(1)} g_0^2 + O(g_0^4),$$
$$c_t^R(g_0^2) = 3/2 + c_t^{R(1)} g_0^2 + O(g_0^4),$$

tree-level O(a) improvement OK

Ref. S. Aoki et al., NPB 540 (1999) 501 $\longrightarrow$ tree level

S. Takeda et al., PRD 68 (2003) 014505 $\longrightarrow$ 1-loop level

Improvement condition

$$c_0 c_{\mathrm{t}}^{P(1)} + 4 c_1 c_{\mathrm{t}}^{R(1)} = A_1/2$$

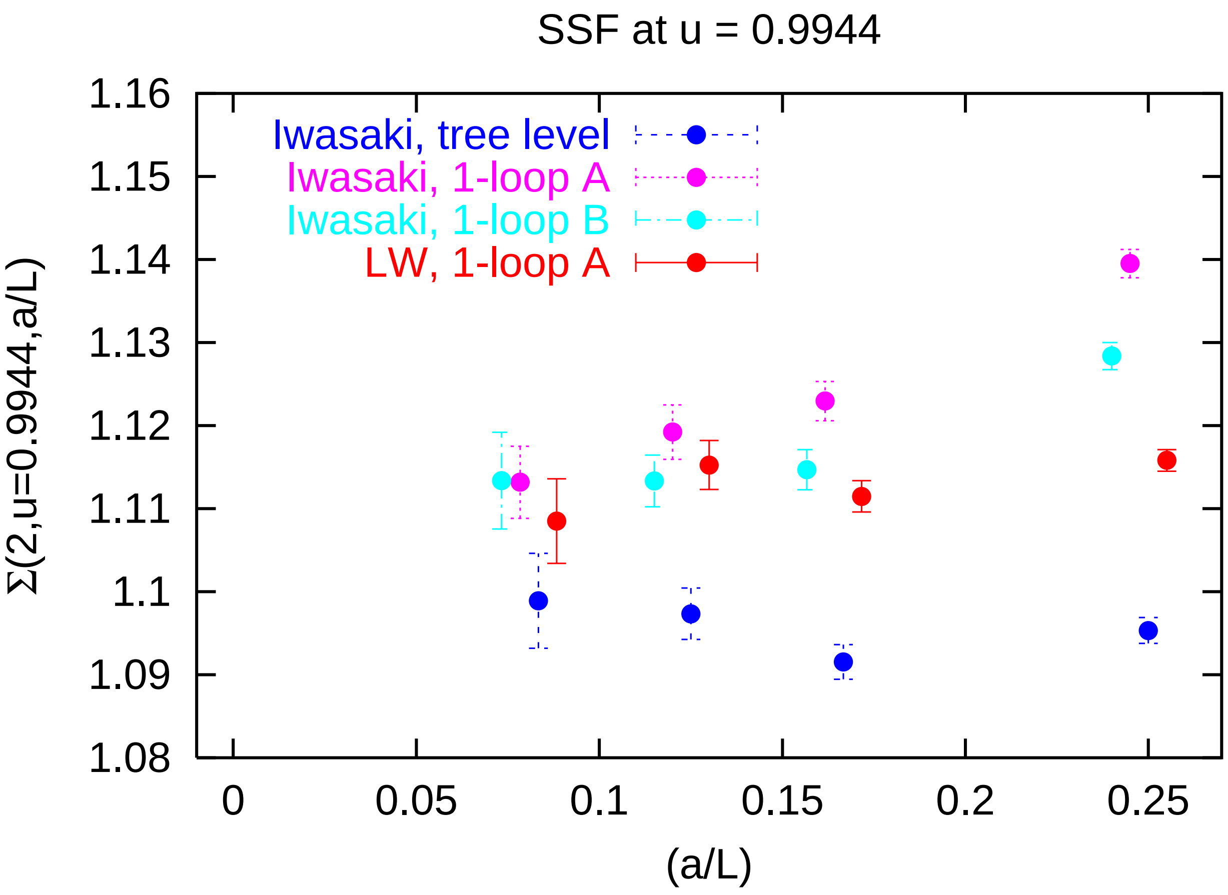
A_1**: coefficient of** $O(a/L)$ **term in 1-loop correction of SF coupling (known from perturbative analysis)**

$$\mathrm{condition\,A} \quad : \quad c_{\mathrm{t}}^{R(1)} = 2 c_{\mathrm{t}}^{P(1)}$$
$$\mathrm{condition\,B} \quad : \quad c_{\mathrm{t}}^{R(1)} = 0$$

action	$c_{\mathrm{t}}^{P(1)}$ for A	$c_{\mathrm{t}}^{P(1)}$ for B	A_1
Iwasaki	0.1518	0.04161	0.3036
LW	−0.00297		−0.00594

Simulation results

- **SSF at $u = 0.9944$ (weak coupling)**



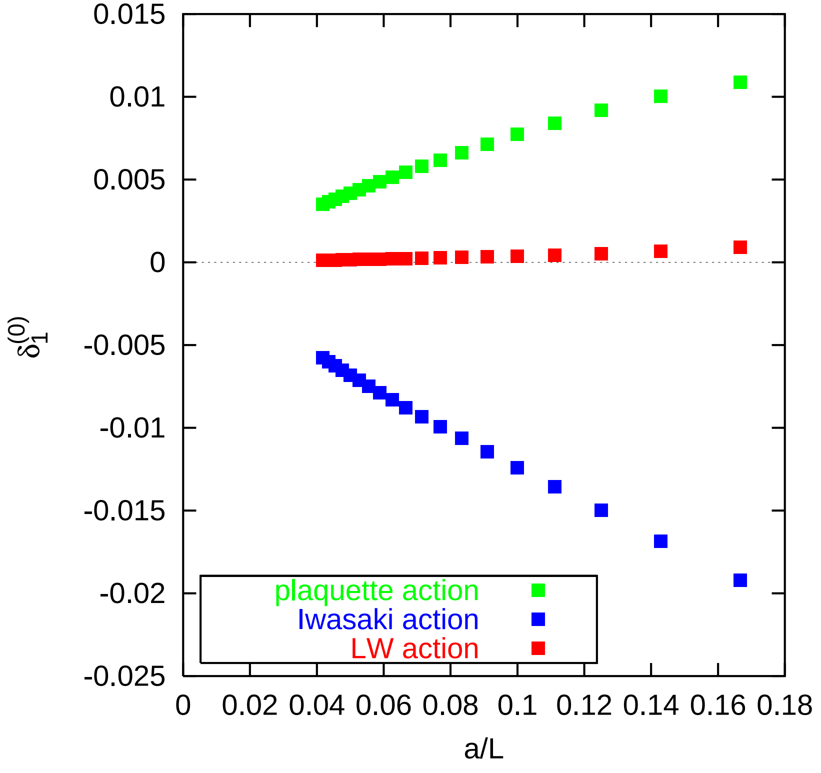
Ref. M. Luscher et al., NPB 413 (1994) 481

Relative deviation

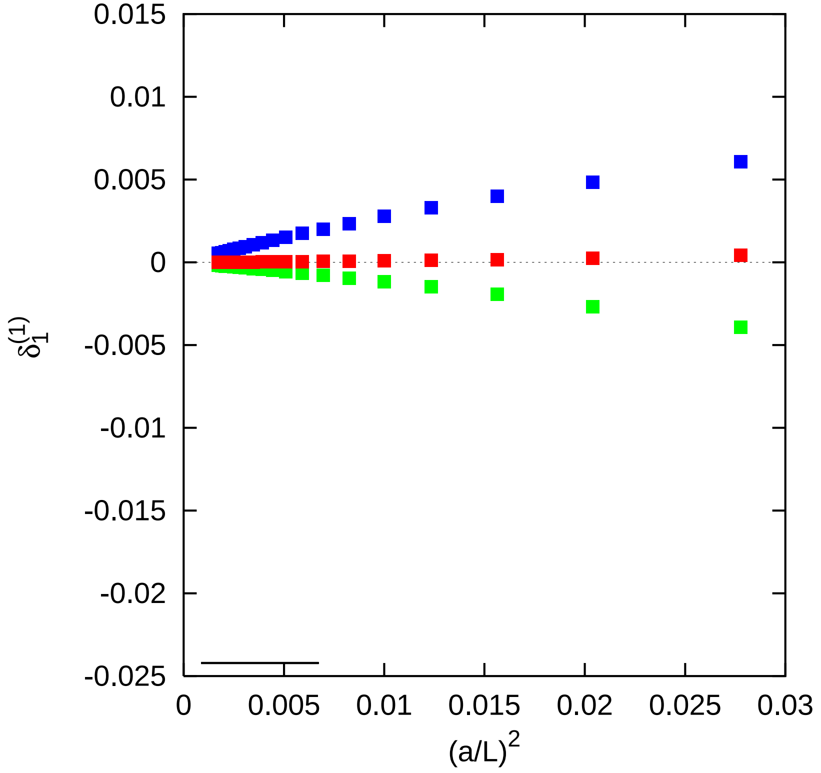
$$\delta(2,u,a/L) = \frac{\Sigma(2,u,a/L) - \sigma(2,u)}{\sigma(2,u)} = \delta_1^{(k)}(2,a/L)u + O(u^2),$$

$k = 0, 1$: **tree level, 1-loop O(a) improvement case**

with tree-level boundary term



with 1-loop condition A boundary term



- **Perturbative improvement**

$$
\begin{aligned}
\Sigma_1^{(k)}(2,u,a/L) &= \frac{\Sigma^{(k)}(2,u,a/L)}{1+\delta_1{}^{(k)}(2,a/L)u}, \\
\Sigma^{(k)}(2,u,a/L) &: \quad \textbf{raw data} \\
\delta_1{}^{(k)}(2,a/L) &: \quad \textbf{1-loop deviation}
\end{aligned}
$$

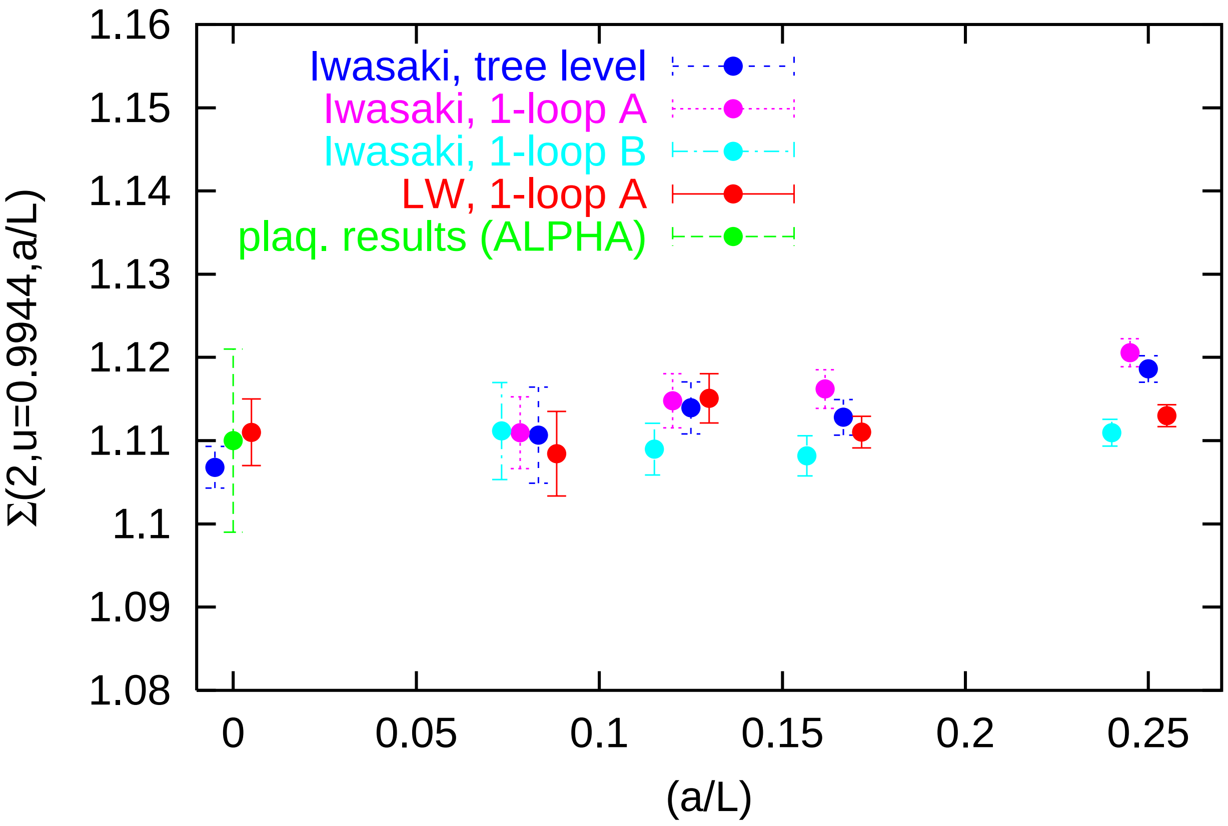
$$\Downarrow$$

$$\boxed{\textbf{no 1-loop order lattice artifact}}$$

ALPHA Collaboration, NPB 437 (1995) 447

SSF at $u = 0.9944$ (weak coupling)

perturbative improvement is implemented



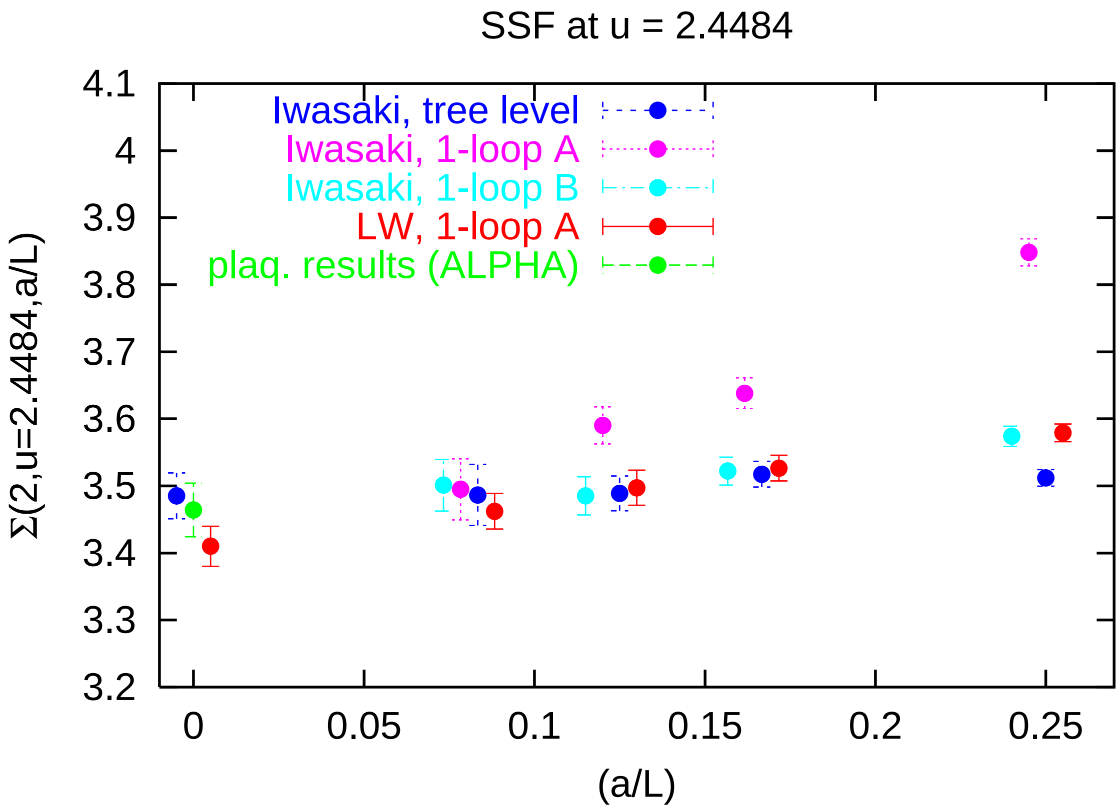
Iwasaki action $\longrightarrow$ **joint fit (all linear)**

LW action $\longrightarrow$ **linear fit**

plaquette action $\longrightarrow$ **Ref. ALPHA Collaboration, NPB 544 (1999) 669**

SSF at $u = 2.4484$ (strong coupling)

perturbative improvement is implemented

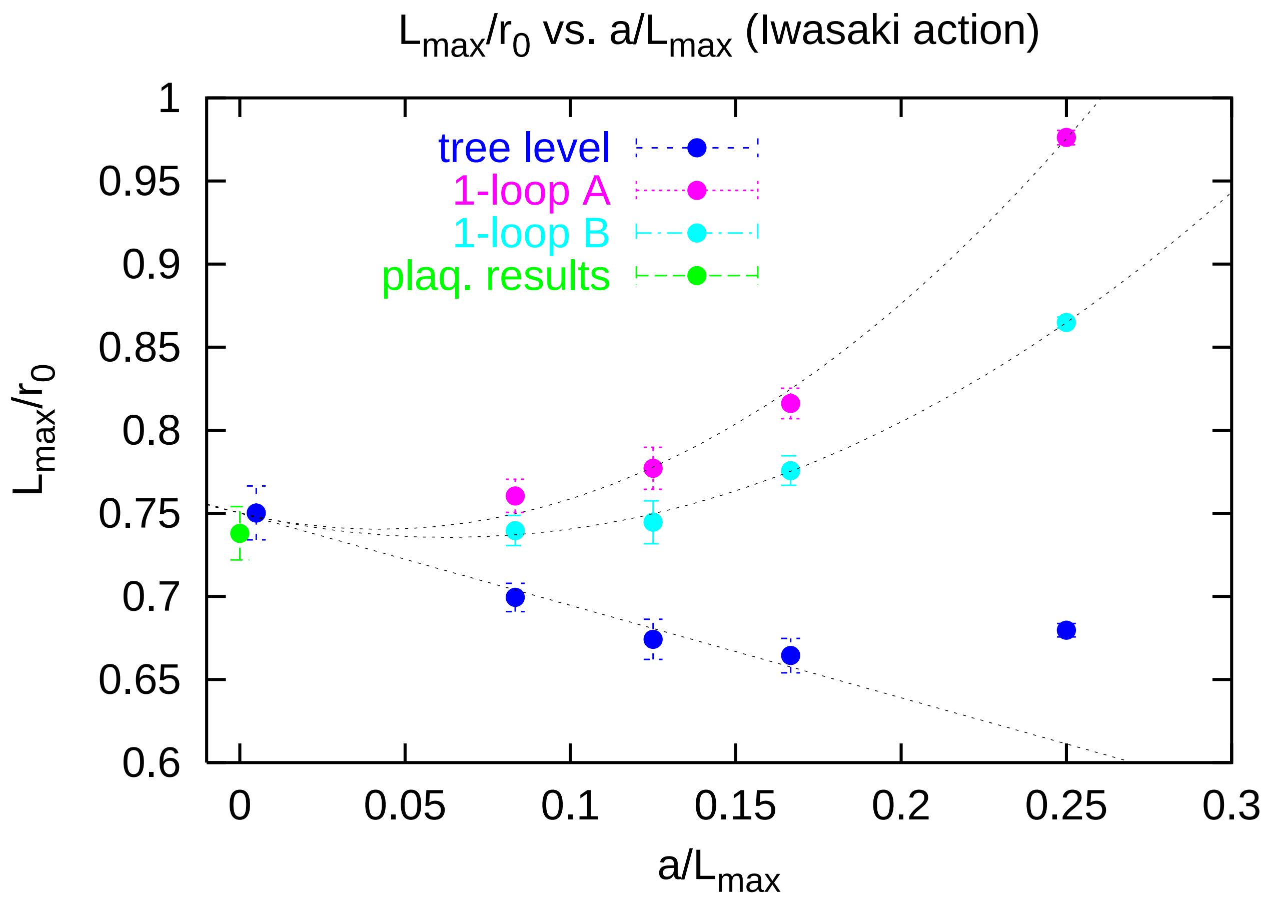


Iwasaki action $\longrightarrow$ **joint fit**

(tree:linear, 1-loop:linear + quadratic)

LW action $\longrightarrow$ **linear fit**

Low energy scale ratio $L_{\rm max}/r_0$



Iwasaki action $\longrightarrow$ **joint fit**

(tree:linear, 1-loop:linear + quadratic)

plaquette action $\longrightarrow$ **Ref. S. Necco et al., NPB 622 (2002) 328**

Conclusion

- **SSF and low energy scale ratio**
  $\Longrightarrow$ universality OK
- **Cut off dependence**
  For the Iwasaki action, perturbative improvement is very efficient for reducing the lattice artifact of SSF especially with O(a) boundary improvement coefficients at tree and 1-loop condition B.

We are going to proceed to $N_{\rm f} = 2, 3$ cases