June 29, 2004
YITP workshop

Democratic mass matrices by strong unification and the lepton mixing angles

Tatsuo Kobayashi (Kyoto)
Hiromasa Shirano (Kanazawa)
H. T. (Kanazawa)

work in progress

CONTENTS:

I. Mass hierarchy and flavor symmetries

(1) Flavor structures

- Hierarchical masses of quarks and leptons :
  ($\lambda \sim \lambda_c \sim 0.22$)

$$\begin{array}{ccccccccccc} m_u & : & m_c & : & m_t & \sim & \lambda^{6-8} & : & \lambda^4 & : & 1 \\ m_d & : & m_s & : & m_b & \sim & \lambda^4 & : & \lambda^2 & : & 1 \\ m_e & : & m_\mu & : & m_\tau & \sim & \lambda^{4-5} & : & \lambda^2 & : & 1 \end{array}$$

- Mixing angles :
  – Small mixing angles in the CKM matrix

$$V_{\mathsf{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ -\lambda & 1 & \lambda^2 \\ \lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$

  – Bi-large mixing angles in the MNS matrix

$$V_{\mathsf{NMS}} \sim \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & U_{e3} \\ \sin\theta_{12}/\sqrt{2} & -\cos\theta_{12}/\sqrt{2} & 1/\sqrt{2} \\ \sin\theta_{12}/\sqrt{2} & \cos\theta_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2 2\theta_{12} \sim 0.84, \quad \sin^2 2\theta_{23} \sim 1.0, \quad |U_{e3}| < 0.2$$

thanks to the neutrino oscillation observations

- Neutrino masses :

$$\Delta m^2_{\mathsf{sol}} \sim 6.9 \times 10^{-5}\ \mathsf{eV}^2$$
$$\Delta m^2_{\mathsf{atm}} \sim 2.6 \times 10^{-3}\ \mathsf{eV}^2$$

- SU(5) GUT :

$$W = Y^u_{ij}\ \mathbf{10}_i \mathbf{10}_j H(\mathbf{5}) + Y^d_{ij}\ \mathbf{10}_i \mathbf{5}^*{}_j H(\mathbf{5}^*) + Y^\nu_{ij}\ \mathbf{5}^*{}_i \mathbf{1}_j H(\mathbf{5})$$

Note : $Y^e = (Y^d)^T \quad \Rightarrow \quad m_b/m_\tau$

Where does the remarkable difference in mixing among quarks and leptons come from?

(2) Froggatt-Nielsen mechanism

- $U(1)_{\mathsf{FN}}$ charge assignment :

F-N charge	1st gen.	2nd gen.	3rd gen.
$q(10_i)$	3 (or 4)	2	0
$q(5^*_i)$	$a+1$ (or a)	a	a

Higher dimensional operators

$$W = \left(\frac{\chi}{\Lambda}\right)^{q(10_i)+q(10_j)} \mathbf{10}_i \mathbf{10}_j H(\mathbf{5}) + \cdots \qquad \text{for } q(\chi) = -1$$

Spontaneous breaking of $U(1)_{\mathsf{FN}}$ $\Rightarrow$ $\lambda = \langle\chi\rangle/\Lambda$

- Form of the Yukawa coupling matrices :

$$Y^u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y^d \sim \lambda^a \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix},$$

$$M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \text{ (SA) or } \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \text{ (A)}$$

– There are O(1) uncertainties to all elements.
– F-N charges for the right handed neutrinos do not affect the neutrino mass matrix.

- Features :
  - Mass hierarchy : very good
  - CKM mixing angles : very good
  - MNS mixing angles : OK but accidental
    - ∗ Large mixings arise from neutrino mass matrix.
    - ∗ (SA) : $U_{e3} \sim \lambda$ may be OK. Large θ_{12} must be accidental.
    - ∗ (A) : Small U_{e3} must be accidental.
    - ∗ Large θ_{23} also must be somewhat accidental.
  - F-N charge assignment may follow from E_6 unification.

(3) Democratic matrices by S_3 flavor symmetries

- S_3 flavor symmetries :

  Each of the SM-fields $(Q_i, u_i, d_i, L_i, e_i)$ is assumed to make a $\mathbf{3}$ of distinct S_3 flavor symmetry.

$$Y^u = \frac{y_0}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv y_0 J$$

$$J = A\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^T, \qquad A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}$$

$\Rightarrow$ hierarchy of 3rd generation masses from others

- Quark sector :

Fritzsch, NPB 155 (1979) 189
Koide, PRD 28 (1983) 252; 39 (1989) 1391

Introduce small flavor symmetry breakings, $\epsilon_a \ll \delta_a \ll 1$:

$$M_q \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_q & 0 & 0 \\ 0 & \epsilon_q & 0 \\ 0 & 0 & \delta_q \end{pmatrix} \qquad (q = u, d)$$

$$= A \begin{pmatrix} 0 & -\sqrt{1/3}\epsilon_q & -\sqrt{2/3}\epsilon_q \\ -\sqrt{1/3}\epsilon_q & (2/3)\delta_q & -(\sqrt{2}/3)\delta_q \\ -\sqrt{2/3}\epsilon_q & -(\sqrt{2}/3)\delta_q & 1+(1/3)\delta_q \end{pmatrix} A^T$$

– Mass hierarchy : very good

$$m_1^q : m_2^q : m_3^q \sim \epsilon_q^2/\delta_q : \delta_q : 1 \qquad \begin{cases} \epsilon_u \sim 10^{-4}, & \delta_u \sim 10^{-2} \\ \epsilon_d \sim 10^{-2}, & \delta_d \sim 10^{-1} \end{cases}$$

– CKM mixing angles : very good

$$V_{\text{CKM}} \sim \begin{pmatrix} 1 & O(\epsilon_d/\delta_d) & O(\epsilon_d) \\ O(\epsilon_d/\delta_d) & 1 & O(\delta_d) \\ O(\epsilon_d) & O(\delta_d) & 1 \end{pmatrix}$$

- Lepton sector :

Fritzsch, Xing, PLB 372 (1996) 265;
Tanimoto, PLB 483 (2000) 417;
Fritzsch, Xing, hep-ph/0406206

$$W = Y_{ij}^e \; L_i \; e_j \; H^d + \frac{\kappa_{ij}}{2M^R} \; L_i \; L_j \; H^u \; H^u$$

Mass matrices :

$$M_e \;\propto\; \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -\epsilon_e & 0 & 0 \\ 0 & \epsilon_e & 0 \\ 0 & 0 & \delta_e \end{pmatrix} \qquad \begin{cases} \epsilon_e \sim 10^{-2} \\ \delta_e \sim 10^{-1} \end{cases}$$

$$M_\nu \;\propto\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon_\nu & 0 \\ 0 & 0 & \delta_\nu \end{pmatrix}$$

– Not only J but also I terms are allowed by S_3 symmetry.

– If $r = 0$, which has no symmetry ground,

$$\sin^2 2\theta_{12} \sim 1, \qquad \sin^2 2\theta_{23} \sim 0.94.$$

Namely a nearly diagonal neutrino mass matrix is favorable.

– Phenomenologically, $r \sim \epsilon_\nu \ll 1$.
Anyway fine-tuning is required.

- See-saw :

$$W = Y_{ij}^\nu \; L_i \; \nu_j \; H^u + \frac{1}{2} M_{ij}^R \; \nu_i \; \nu_j$$

– ν_i must belong to $\mathbf{3}$ of the identical S_3 group for L_i.

$$Y^\nu \sim y_0^\nu \; (I + r \; J) + \Delta Y^\nu, \quad M^R \sim M_0^R \; (I + r' \; J) + \Delta M^R$$

– Y^ν and also M^R must be nearly diagonal, which is not explained by the flavor symmetry.

- SU(5) GUT :

Fukugita, Tanimoto, Yanagida, PRD57 (1998) 4429

$$W = Y_{ij}^u \; \mathbf{10}_i \mathbf{10}_j H^u(\mathbf{5}) + Y_{ij}^d \; \mathbf{10}_i \mathbf{5^*}_j H^d(\mathbf{5^*}) + \cdots$$

– S_3 symmetry allows $\qquad Y^u = y_0^u (J + r'' I)$

– r'' must be suppressed to $O(10^{-5})$ by fine tuning!

- Problems in democratic approach :
  - S_3 flavor symmetry does not explain the nearly diagonal neutrino mass.

    Tanimoto, Watari, Yanagida, PLB 461 (1999) 345
  - GUT requires the same kind of fine tuning.
  - No grounds for the small breaking parameters, which are assumed to be diagonal.

(4) Origin of flavor structures

A. Flavor symmetries

- Abelian : Froggatt-Nielsen U(1)
- Non-abelian : U(2), SU(3)
- Discrete groups : S_3, A_4, ...

General features :

- Introduce "flavons" charged under the flavor symmetry.
- Consider generic higher dimensional operators allowed by the flavor symmetries. The coefficients are not specified and should be O(1).
- Assume VEVs of the "flavons". Then the effective Yukawa operators are generated with small parameters depending on the VEVs and dimensionality of the symmetric operators.
- There always remain O(1) ambiguity in the Yukawa couplings.

B. Strong dynamics

- Quasi infrared fixed point
  $\Rightarrow$ Top Yukawa coupling
- Power-law running behavior in extra dimensions
- Large anomalous dimensions by strong interactions
  Ex : Nelson-Strassler models
  $\Rightarrow$ Froggatt-Nielsen type of mass matrices

C. Geometries in brane worlds

- Distance between the branes
- Profiles of the wave functions and their overlapping

II. Democratic Yukawa couplings by strong unification

(1) P-R fixed point of Yukawa couplings

Pendelton, Ross, PLB 98 (1981) 291

Case of a single Yukawa coupling

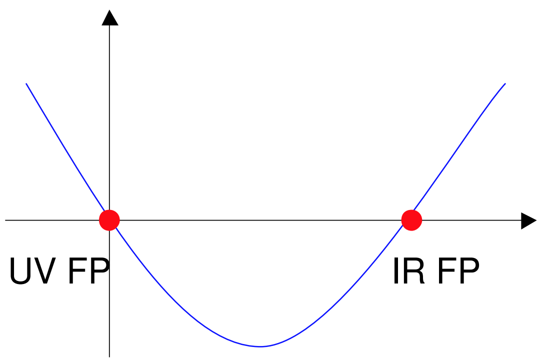
- RG equations for $\alpha_g = g^2/8\pi^2$, $\alpha_y = |Y|^2/8\pi^2$:

$$\mu \frac{d\alpha_y}{d\mu} = (a\alpha_y - c\alpha_g)\alpha_y, \qquad \mu \frac{d\alpha_g}{d\mu} = -b{\alpha_g}^2$$

- Infrared fixed point for $x = \alpha_y/\alpha_g$:

$$\mu \frac{dx}{d\mu} = [ax - (c-b)]\,\alpha_g x \quad \Rightarrow \qquad x^* = (c-b)/a$$

IR attractive fixed point



- Convergence behavior:

$$\mu \frac{d\Delta x}{d\mu} = (c-b)\alpha_g \Delta x$$

– Strong convergence is realized for large α_g and $c - b$.
– In asymptotically non-free gauge theories, Yukawa couplings may converge strongly.

$\Leftarrow$
∗ Strong gauge coupling at UV
∗ Large $c - b$, since $b < 0$

Bando, Sato, Yoshioka, PTP 98 (1997) 169; PTP 100 (1998) 797.

- Extra dimension :

Bando, Kobayashi, Noguchi, Yoshioka,
PLB 480 (2000) 187; PRD 63 (2001) 113017

RG equations for dimensionless couplings in $d = 4 + \delta$,

$$\mu \frac{d\alpha_y}{d\mu} = \delta\alpha_y + (a\alpha_y - c\alpha_g)\alpha_y,$$
$$\mu \frac{d\alpha_g}{d\mu} = \delta\alpha_g - b{\alpha_g}^2$$

– Same RG equation for $x = \alpha_y/\alpha_g$ and, therefore, the same fixed point.
– Couplings in the four dimensional F.T. show power-law running.
– In the case of $b < 0$, the gauge coupling may satisfy a UV fixed point, which appears strong (${\alpha_g}^* = O(1)$) in general.
– Strong convergence may be realized even for the "aymptotically free" case.

$$\Delta x(\mu) \sim \left(\frac{\mu}{\Lambda}\right)^{(c-b){\alpha_g}^*} \Delta x(\Lambda)$$

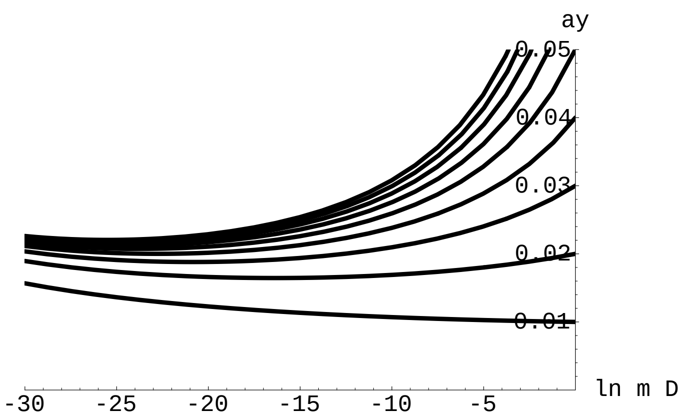
(2) Quasi infrared fixed point

Hill, PRD 24 (1981) 691

- Large Yukawa couplings

– Large Top Yukawa at high energy scale is viable.
– Yukawa couplings in the democratic form may be large also. Then $\tan\beta$ must be large.

RG flows of Top Yukawa coupling in the MSSM :



(3) Fixed point of the democratic type

Abel, King, PLB 435 (1998) 73

- Democratic Higgs :
  Introduce a Higgs field for every Yukawa coupling

$$W = Y_{ij}\, Q_i\, q_j\, H_{ij} \qquad (i,j=1,2,3)$$

- Beta function for $\alpha_{y_{ij}} = |Y_{ij}|^2/8\pi^2$:

$$\mu \frac{d\alpha_{y_{ij}}}{d\mu} = \delta\alpha_{y_{ij}} \; + \; \left(\gamma_{Q_i} + \gamma_{q_j} + \gamma_{H_{ij}}\right)\alpha_{y_{ij}}$$

where γ are anomalous dimensions :

$$\begin{aligned} \gamma_{Q_i} &= \left[a_Q(\alpha_{y_{i1}} + \alpha_{y_{i2}} + \alpha_{y_{i3}}) - c_Q\alpha_g\right], \\ \gamma_{u_i} &= \left[a_q(\alpha_{y_{1i}} + \alpha_{y_{2i}} + \alpha_{y_{3i}}) - c_q\alpha_g\right], \\ \gamma_{H_{ij}} &= \left[3a_H\alpha_{y_{ij}} - c_H\alpha_g\right] \end{aligned}$$

Note : All anomalous dimensions are diagonal (no mixing).

- IR fixed point for $x_{ij} = \alpha_{y_{ij}}/\alpha_g$:

$$x_{ij}^* = x^* = \frac{c-b}{3a}$$

where $a = a_Q + a_q + a_H$, $c = c_Q + c_q + c_H$.

- Linear perturbation around the fixed point :
  $\Delta x_{ij} = x_{ij} - x^*$

$$\mu\frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* \begin{pmatrix} a' & a_Q & a_Q & a_u & 0 & 0 & a_u & 0 & 0 \\ a_Q & a' & a_Q & 0 & a_u & 0 & 0 & a_u & 0 \\ a_Q & a_Q & a' & 0 & 0 & a_u & 0 & 0 & a_u \\ a_u & 0 & 0 & a' & a_Q & a_Q & a_u & 0 & 0 \\ 0 & a_u & 0 & a_Q & a' & a_Q & 0 & a_u & 0 \\ 0 & 0 & a_u & a_Q & a_Q & a' & 0 & 0 & a_u \\ a_u & 0 & 0 & a_u & 0 & 0 & a' & a_Q & a_Q \\ 0 & a_u & 0 & 0 & a_u & 0 & a_Q & a' & a_Q \\ 0 & 0 & a_u & 0 & 0 & a_u & a_Q & a_Q & a' \end{pmatrix} \Delta x_{ij}$$

where $a' = a_Q + a_q + 3a_H$

The eigenvalues are all positive ;

$$3a_H, 3a_H, 3a_H, 3a_H, 3(a_Q + a_H), 3(a_Q + a_H),$$
$$3(a_u + a_H), 3(a_u + a_H), 3a$$

therefore IR attractive

- **Alternatives with flavor symmetries**

An example with three Higgs : $Q_i, q_i, H_i (i = 1, 2, 3)$

$$W = \sum_{i,j (\mathrm{mod}\ 3)} Y_{ij}\, Q_i\, q_j\, H_{3-i-j}$$

Also we impose a discrete symmetry ;

$$Q_i \rightarrow Q_{(i+1)}, \quad q_i \rightarrow q_{(i-1)}, \quad H_i \rightarrow H_i,$$

then this model has an IR attractive fixed point.

III. Higgs sector

- Superpotential for the democratic Higgs

$$W = M\sum_i (H(\mathbf{5})_{i+1,j} - H(\mathbf{5})_{i,j})(H(\mathbf{5}^*)_{i+1,j} - H(\mathbf{5}^*)_{i,j}) + M\sum_j (H(\mathbf{5})_{i,j+1} - H(\mathbf{5})_{i,j})(H(\mathbf{5}^*)_{i,j+1} - H(\mathbf{5}^*)_{i,j})$$

$\Rightarrow$ Scalar potential :

$$V \propto \sum_i |H(\mathbf{5})_{i+1,j} - H(\mathbf{5})_{i,j}|^2 + \sum_j |H(\mathbf{5})_{i,j+1} - H(\mathbf{5})_{i,j}|^2 + \cdots$$

Note : This potential satisfies a shift symmetry :

$$H(\mathbf{5})_{i,j} \to C, \qquad H(\mathbf{5}^*)_{i,j} \to \bar{C}$$

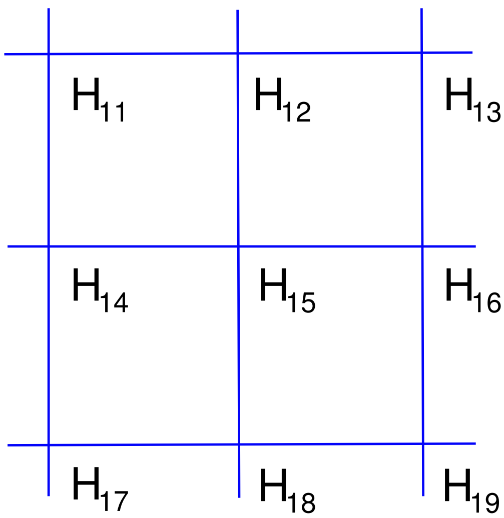
- Higgs fields in the MSSM

Shift symmetry $\Rightarrow$
- There is a single massless mode H.
- Other modes acquire mass of order M, which gives decoupling scale.

$$H_{ij} = \frac{1}{3}H + \cdots \qquad \text{for all } (i,j)$$

$\Rightarrow$ Democratic Yukawa couplings in the MSSM

- Dimensional deconstruction



IV. Large mixings in the lepton sector

(1) Origin of large mixing in the lepton sector

- SU(5) GUT :

$$W = Y^u_{ij}\ \mathbf{10}_i \mathbf{10}_j H(\mathbf{5})_{ij} + Y^d_{ij}\ \mathbf{10}_i \mathbf{5^*}_j H(\mathbf{5^*})_{ij} + Y^\nu_{ij}\ \mathbf{5^*}_i \mathbf{1}_j H(\mathbf{5})_{ij}$$

If Y^ν is not made democratic at the decoupling scale, then large mixing may appear in the lepton sector.

- Yukawa interactions with vector-like fields :

$$W = \kappa_i\ \mathbf{1}_i\ \Phi_i\ \bar{\Phi}_i\ +\ \cdots$$

$\Rightarrow$

- – The right-handed neutrinos may obtain large positive anomalous dimensions.
- – Y^ν is suppressed and the non-trivial IR fixed point vanishes.

- Toy model with one generation :

$$W = Y\ \mathbf{5}^*\ \mathbf{1}\ H(\mathbf{5}) + \kappa\ \mathbf{1}\ \Phi\ \bar{\Phi}$$

RG eqns for $\alpha_g = g^2/8\pi^2$, $\alpha_y = Y^2/8\pi^2$, $\alpha_\kappa = \kappa^2/8\pi^2$:

$$\begin{aligned}
\frac{d\alpha_g}{d\ln\mu} &= -b{\alpha_g}^2, \\
\frac{d\alpha_y}{d\ln\mu} &= \left[7\alpha_y + R\alpha_\kappa - 4C_2(\mathbf{5})\alpha_g\right]\alpha_y, \\
\frac{d\alpha_\kappa}{d\ln\mu} &= \left[5\alpha_y + (R+2)\alpha_\kappa - 4C_2(\mathbf{R})\alpha_g\right]\alpha_\kappa
\end{aligned}$$

RG eqns for $x_y = \alpha_y/\alpha_g$, $x_\kappa = \alpha_\kappa/\alpha_g$:

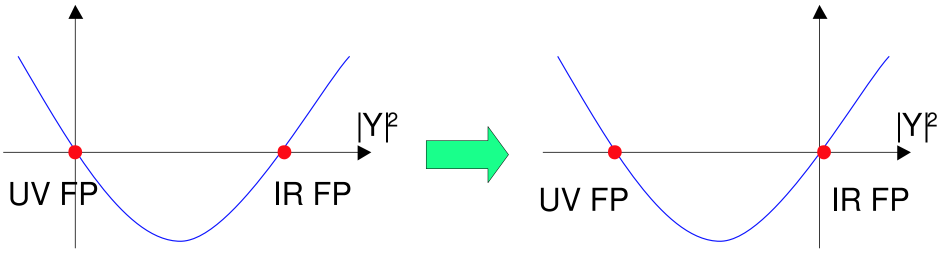
$$\begin{aligned}
\frac{dx_y}{d\ln\mu} &= \left[7x_y + Rx_\kappa - 4C_2(\mathbf{5}) + b\right]\alpha_g x_y, \\
\frac{dx_\kappa}{d\ln\mu} &= \left[5x_y + (R+2)x_\kappa - 4C_2(\mathbf{R}) + b\right]\alpha_g x_\kappa
\end{aligned}$$

IR attractive fixed points :

For large $\mathbf{R}$ (or many $(\Phi\bar{\Phi})$),

$$x_y^* = 0, \qquad x_\kappa^* = \frac{1}{R+2}\left[4C_2(\mathbf{R}) - b\right]$$

Y is suppressed with some power of the scale!



(2) Neutrino mass matrix :

- See-saw :

$$W = Y_{ij}^\nu\ \mathbf{5}^*{}_i \mathbf{1}_j H(\mathbf{5})_{ij} + \frac{1}{2} M_{ij}^R\ \mathbf{1}_i\ \mathbf{1}_j + \kappa_i\ \mathbf{1}_i\ \Phi_i\ \bar{\Phi}_i\ +\ \cdots$$

$\Rightarrow$

  - Majorana mass for the right-handed neutrinos are also suppressed.
  - Enhancement of anomalous dimensions of the singlets does NOT affect the neutrino masses significantly.

What is the difference?

- Alignment of the neutrino mass matrix :

$$\begin{aligned} \frac{d}{d\ln\mu}\ln\left(\frac{\alpha_{y_{ik}^\nu}}{\alpha_{y_{jk}^\nu}}\right) &= \left(\gamma_{\mathbf{5}_i^*} + \gamma_{\mathbf{1}_k} + \gamma_{H(\mathbf{5})_{ik}}\right) - \left(\gamma_{\mathbf{5}_j^*} + \gamma_{\mathbf{1}_k} + \gamma_{H(\mathbf{5})_{jk}}\right) \\ &= a_{\mathbf{5}^*}\left(\sum_k \alpha_{y_{ik}^\nu} - \alpha_{y_{jk}^\nu}\right) + 3a_H\left(\alpha_{y_{ik}^\nu} - \alpha_{y_{jk}^\nu}\right) \\ &\ll 1 \qquad \text{(because } Y_{ij}^\nu \text{ are suppressed.)} \end{aligned}$$

Neutrino mass matrix is NOT aligned!!

V. Towards realistic mass matrices

(1) Quark and charged lepton mass matrices :

Hierarchy between 1st and 2nd generation masses requires two small parameters of different scales.

- Initial couplings
  - Suppose one of the initial couplings, say Y_{33}, is comparatively smaller than others.
  - While other couplings approach to the IR fixed point, Y_{33} does not grow up immediately.
- RG evolution of the Yukawa matrix $(t = \ln \mu)$

$$\frac{d}{dt}[\Delta x_{13} - \Delta x_{23}] = \alpha_g x^*(a' - a_q)[\Delta x_{13} - \Delta x_{23}]$$
$$\frac{d}{dt}[\Delta x_{31} - \Delta x_{32}] = \alpha_g x^*(a' - a_Q)[\Delta x_{31} - \Delta x_{32}]$$

$\Rightarrow \quad \Delta x_{13} \sim \Delta x_{23},\ \Delta x_{31} \sim \Delta x_{32}$

$$\frac{d}{dt}[\Delta x_{11} - \Delta x_{21}] = \frac{d}{dt}[\Delta x_{12} - \Delta x_{22}] = \alpha_g x^* a_Q [\Delta x_{13} - \Delta x_{23}]$$
$$\frac{d}{dt}[\Delta x_{11} - \Delta x_{12}] = \frac{d}{dt}[\Delta x_{21} - \Delta x_{22}] = \alpha_g x^* a_q [\Delta x_{31} - \Delta x_{32}]$$

$\Rightarrow \quad \Delta x_{ij}\ (i,j = 1,2)$ are all equal.

Consequently we obtain the Yukawa couplings at low energy scale (decoupling scale) as

$$Y = \frac{y_0}{3}\begin{pmatrix} 1+O(\epsilon) & 1+O(\epsilon) & 1+\delta' \\ 1+O(\epsilon) & 1+O(\epsilon) & 1+\delta' \\ 1+\delta'' & 1+\delta'' & 1+\delta \end{pmatrix}$$

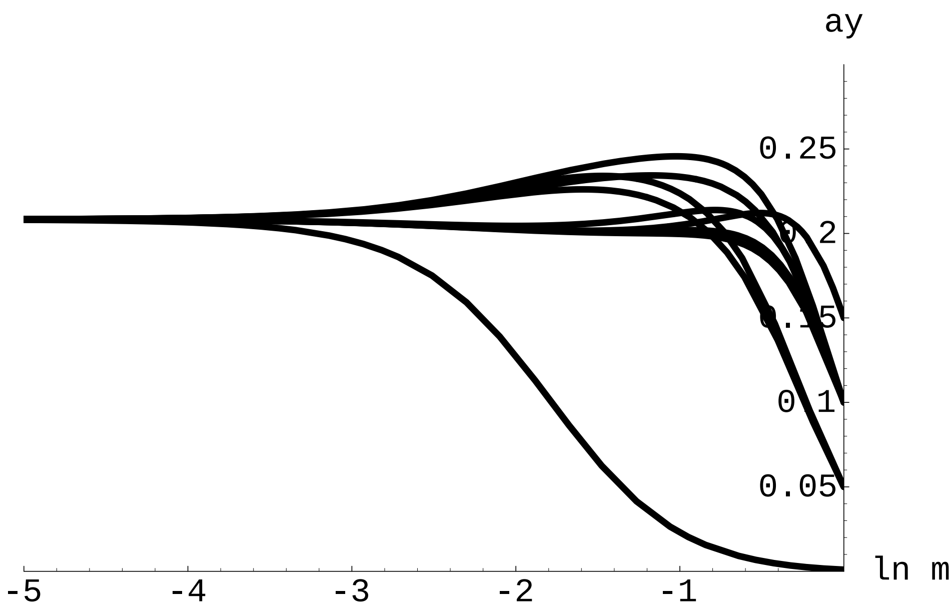
where $|\epsilon| \ll |\delta| \sim |\delta'| \sim |\delta''| \ll 1$

- Form of the mass matrices

$$A^T Y A = y_0 \begin{pmatrix} O(\epsilon) & O(\epsilon) & O(\delta) \\ O(\epsilon) & O(\delta) & O(\delta) \\ O(\delta) & O(\delta) & 3+O(\delta) \end{pmatrix}$$

Viable quark masses and mixings may be given by parameters of $O(\epsilon)$ and $O(\delta)$.

Illustrative example of the RG flows :



- Non-democratic perturbation :

  Assume a small interaction only for H_{33}, *e.g.*

$$\lambda H_{33}\ \Psi\ \Sigma \qquad\qquad (\lambda \ll 1)$$

  Linear perturbation around the fixed point

$$\mu\frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* \left[(\mathcal{M}\Delta x)_{ij} + \delta_\lambda \delta_{i3}\delta j3\right]$$

$\Rightarrow$ Same form of the mass matrices

(2) Neutrino masses and mixings :

- In general, neutrino mass matrix may display anarchy depending on the initial couplings.
- If the initial value of Y^ν is close to diagonal, then the solar and atomospheric neutrino mixings as well as small $|U_{e3}|$ may be explained.
- Only if Y^ν_{33} happens to somewhat larger others, then the atomospheric neutrino mixings and small $|U_{e3}|$ are realized.

VI. Summary and discussions

(1) Summary :

A. Strong gauge interaction may lead to democratic form of Yukawa couplings, if Higgs sector is extended.

B. Difference in quark mixing and lepton mixing may have dynamical origin. We showed that additional Yukawa interaction of the right-handed neutrino can realize such a difference.

C. It is also shown that some dispersion among the initial couplings are enhanced by renoramlization effect and eventually may appear to be hierarchical mass matrices at low energy.

(2) Further considerations :

- Complex phases of the Yukawa couplings.
- Explicit models and realistic parameters.
- Features of the models:
  large $\tan\beta$, small Dirac neutrino Yukawa Y^{ν} and so on.
- Alignment of the soft SUSY breaking parameters.