4

THE HIGGS BOSON

AND

HIGGS BOSONS

Minimal Higgs Boson

SU(2) doublet 4成分 $\begin{cases} 1 \text{ physical } H \\ 3 \text{ "unphysical" } (W^{\pm}, Z) \end{cases}$

Higgs = "真空の vibration"

→ 量子数は真空と同じ (無色透明)

$J = 0$

$P = +$

$C = +$

$Q = 0$

⋮

Higgs coupling は 真空期待値 v を $v + H$ として得られる

(Higgs self coupling → もっと複雑)

Gauge boson, fermion の質量はすべて SU(2)×U(1) の破れからくるので, Higgs との coupling は

mass term で $m \rightarrow m\left(1 + \frac{H}{v}\right)$

のおきかえでわかる。

(second "universality" of weak interactions)

v は $v = \frac{2m_W}{g}$ で与えられる. $\left(v^2 = \frac{1}{\sqrt{2}G_F} \right)$

$$\boxed{m \rightarrow m\left(1 + \frac{g}{2m_W} H\right)}$$

Higgs couplings

With gauge bosons

$$\mathcal{L}_{mass} = m_W^2\, W_\mu^\dagger W_\mu \;\rightarrow\; m_W^2 \left(1+\frac{g}{2m_W}H\right)^2 W_\mu^\dagger W_\mu$$

$$= (\text{mass}) + g m_W\, H\, W_\mu^\dagger W_\mu + \frac{1}{4} g^2 H^2 W_\mu^\dagger W_\mu$$

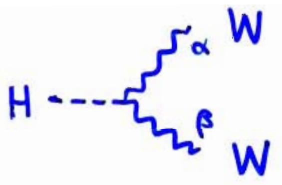
& $$\frac{1}{2} m_Z^2 Z_\mu^2 \;\rightarrow\; \frac{1}{2} m_Z^2 \left(1+\frac{g_Z}{2m_Z}H\right)^2 Z_\mu^2$$

$$= (\text{mass}) + \frac{1}{2} g_Z m_Z\, H Z_\mu^2 + \frac{1}{8} g_Z^2 H^2 Z_\mu^2$$

$$\frac{g}{m_W} = \frac{g/\cos\theta_W}{m_W/\cos\theta_W} = \frac{g_Z}{m_Z}$$

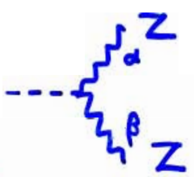
non-standard form of gauge coupling

FROM



$i g m_W\, g_{\alpha\beta}$

"diagonal"



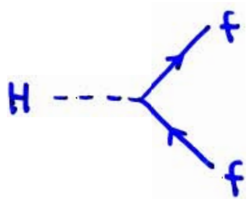
$i g_Z m_Z\, g_{\alpha\beta}$

- No $HZ\gamma$, $H\gamma\gamma$, Hgg at tree order

With fermions

$$\mathcal{L}_{mass} = -m_f \bar{f} f \;\rightarrow\; -m_f \left(1+\frac{g}{2m_W}H\right)\bar{f}f$$

$$= (\text{mass}) - \frac{g m_f}{2m_W} H \bar{f} f$$



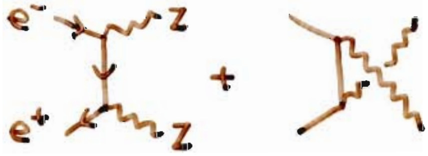
$-i\,\frac{g m_f}{2m_W}$

flavor-conserving scalar coupling (0^{++})

Something needed in J=0 sector

Assume $SU(2) \times U(1)$ without Higgs (Glashow model),
W, Z masses put "by hand"

→ calculate
e.g. $e^+e^- \to ZZ$

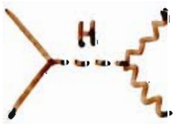


$$\frac{d\sigma}{d\cos\theta} \sim \frac{\pi\alpha_Z^2}{16}\frac{m_e^2}{m_Z^4} \quad (s \ggg m_Z^2) \qquad \alpha_Z = \frac{\alpha}{\sin^2\theta_W \cos^2\theta_W}$$

This comes from J=0 partial wave and eventually violates unitarity at high energies.

⇒ Need something to cure the J=0 part

Adding the standard Higgs cancels the ill behavior entirely.



Higgs-fermion coupling must proportional to the fermion mass!

another example:
$W^+W^- \to W^+W^-$

$$\Rightarrow \quad \mathcal{M} \sim E^2, \qquad \frac{d\sigma}{d\cos\theta} \sim s$$

add

$$\Rightarrow \quad \mathcal{M} \sim \text{const.}, \qquad \frac{d\sigma}{d\cos\theta} \sim \frac{1}{s}$$

"Perturbative" upper bound on m_H

If $\lambda \gtrsim 1$ perturbation expansion is meaningless.

A bound $\lambda \lesssim 1$ translates into an upper bound on m_H through the relation $\lambda = \frac{1}{\sqrt{2}} G_F m_H^2$

There is some arbitrariness here since the normalization of λ is a convention (I could define 6λ as λ), and one can argue that $\frac{\lambda}{4\pi}$ may be more appropriate than λ.

To quantify the bound, Lee + Quigg + Thacker required that the lowest-order (tree) amplitudes for W_L, Z_L, and H scattering should obey partial-wave unitarity at very high energy.

- $\mathcal{M}(W_L^+ W_L^- \to W_L^+ W_L^-) \underset{s \gg m_H^2 \gg m_W^2}{\sim} -2\sqrt{2}\, G_F m_H^2$

  $T^{(J=0)} = -\dfrac{G_F m_H^2}{2\sqrt{2}\pi}$ ($J=0$ partial wave amplitude)

  $|T^{(J=0)}| \leq 2$ gives $m_H^2 \leq \dfrac{4\sqrt{2}\pi}{G_F}$ corresponds to $\frac{\lambda}{4\pi} \leq 1$

- Slight improvement attained by considering $W_L^+W_L^-, Z_LZ_L, HH$ coupled channel

$$m_H^2 \leq \frac{8\sqrt{2}\pi}{3G_F} = (1.01\,\text{TeV})^2$$

Is the upper limit meaningful? What happens for large λ?

If $\frac{\lambda}{4\pi} > 1$ the Higgs sector becomes strongly interacting.

($\rightarrow$ nonlinear σ model)

$\Gamma_H \sim m_H$ (in perturbation theory)

$\lambda - m_H$ relation questionable:

The picture is based on perturbation expansion (free Higgs field + interaction) For $\lambda \gg 1$ need a different picture.

Physical Higgs particle (if exists) should be a kind of composite particle, far from the elementary boson in the perturbative picture.

A rich spectrum of bound states?

Lattice Monte Carlo suggests an upper bound $m_H \lesssim 600$ GeV ??

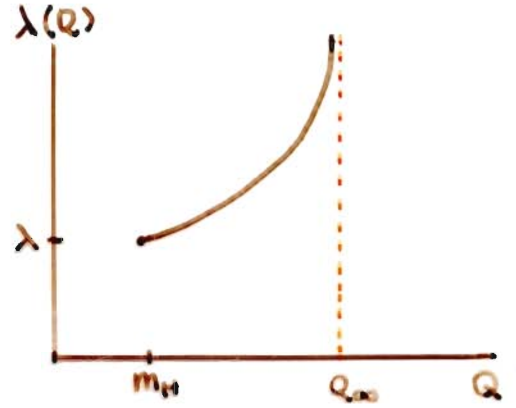
No definite statement possible

"Prejudice-dependent" upper bound on m_H

The Higgs self coupling is asymptotically non-free.

The lowest-order renormalization group eq. gives (for $m_t \to 0$!)

$$\lambda(Q) = \frac{\lambda}{1 - \frac{3\lambda}{4\pi^2}\log\frac{Q^2}{m_H^2}} \quad ; \quad \lambda = \frac{m_H^2}{2v^2} = \frac{1}{\sqrt{2}} G_F m_H^2$$



$\lambda(Q)$ diverges at

$$Q_\infty = m_H \exp\left(\frac{2\sqrt{2}\pi^2}{3 G_F m_H^2}\right)$$

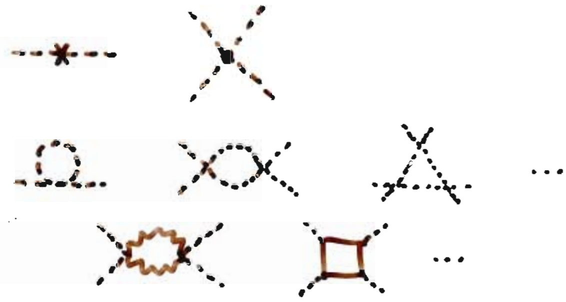
m_H (GeV)	Q_∞ (GeV)	
10	∞	
100	4×10^{36}	
		← M_{Planck}
150	4×10^{17}	
		← M_{GUT}
200	9×10^{10}	
300	2×10^{6}	
500	1.2×10^{4}	
1000	2200	

Requiring $\lambda(Q)$ be finite or perturbatively controllable for $Q \leq$ (any mass scale you like) gives a bound for m_H

Effective potential

Tree $\quad V_0(\varphi) = \mu^2\varphi^2 + \lambda\varphi^4$

One-loop $\quad V_1(\varphi) = \xi\varphi^4 \ln\frac{\varphi^2}{M^2}$



$(\mu^2 \sim 0)$

$\xi = O(\lambda^2, g^4, f^4)$

M : くりこみ点

One-loop "correction" can be important when $\lambda \sim g^4$

$$V(\varphi) = \mu^2\varphi^2 + \lambda\varphi^4 + \xi\varphi^4 \ln\frac{\varphi^2}{M^2}$$

$$\frac{dV}{d\varphi} = 4\varphi\left[\tfrac{1}{2}\mu^2 + (\lambda + \tfrac{1}{2}\xi)\varphi^2 + \xi\varphi^2 \ln\frac{\varphi^2}{M^2}\right]$$

$$\frac{d^2V}{d\varphi^2} = 2\mu^2 + (12\lambda + 14\xi)\varphi^2 + 12\xi\varphi^2 \ln\frac{\varphi^2}{M^2}$$

$$= \frac{3}{\varphi}\frac{dV}{d\varphi} - 4\mu^2 + 8\xi\varphi^2$$

$$\left.\frac{dV}{d\varphi}\right|_{\varphi=v} = 0 \qquad \left.\frac{d^2V}{d\varphi^2}\right|_{\varphi=0} = 2\mu^2$$

$$m_H^2 = \left.\frac{d^2V}{d\varphi^2}\right|_{\varphi=v} = -4\mu^2 + 8\xi v^2$$ > 0 even for $\mu^2 > 0$ (if small)

$\mu^2 = 0 \quad m_H^2 = 8\xi v^2$
(Coleman - Weinberg)
"radiative" sym. br.

$$V(\varphi) = \frac{1}{4}\varphi\frac{dV}{d\varphi} + \frac{1}{2}\varphi^2(\mu^2 - \xi\varphi^2)$$

$$V(\varphi = v) = \frac{1}{2}v^2(\mu^2 - \xi v^2) = -\frac{1}{8}v^2(m_H^2 - 4\xi v^2)$$

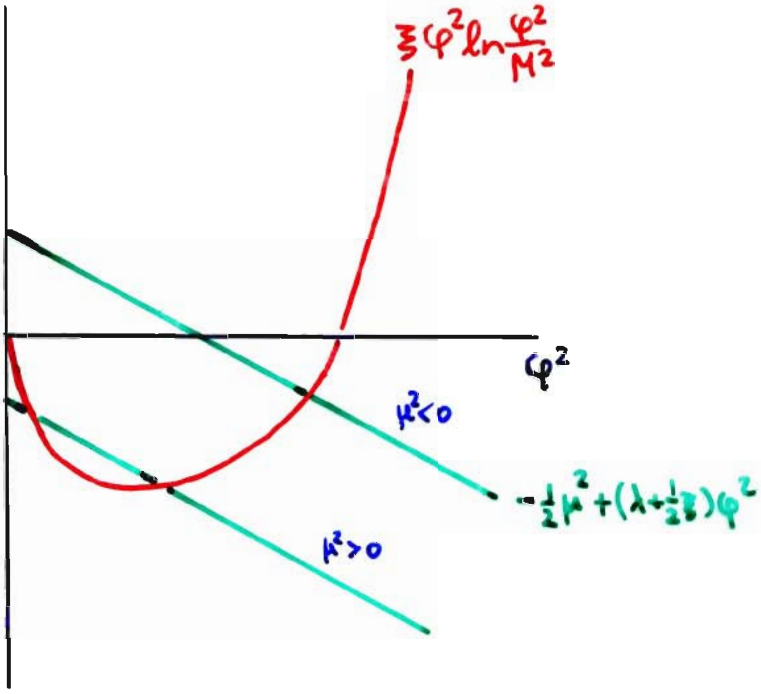
$$V(\varphi = v) < V(0) = 0 \quad \Rightarrow \quad m_H^2 > 4\xi v^2$$

(Linde - Weinberg bound)

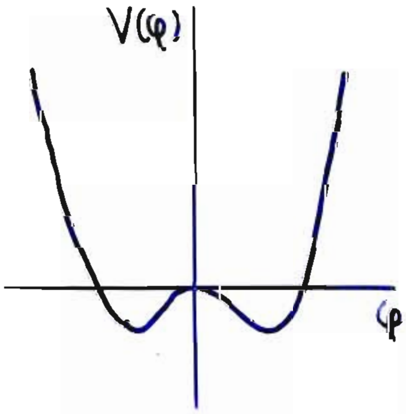
S.M. no heavy fermion $\to \xi = \frac{3}{64}(2\alpha_W^2 + \alpha_Z^2)$

negative contrib. to ξ

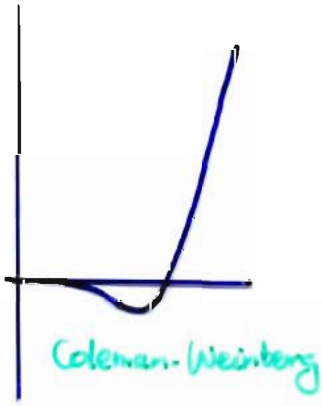
$$m_H^2 > \frac{3\alpha^2}{16\sqrt{2}G_F}\frac{1}{\sin^4\theta_W}\left(2 + \frac{1}{\cos^4\theta_W}\right) = (6.5\,\text{GeV})^2 \quad (\sin^2\theta_W = 0.23)$$



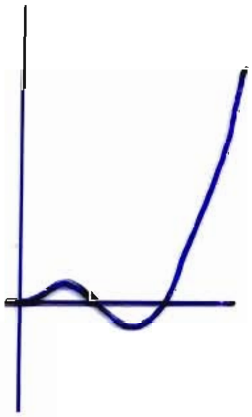
$\mu^2<0$
$m_H^2>8\xi v^2$



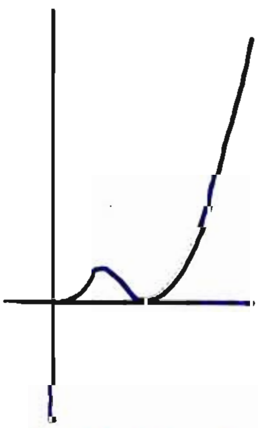
$\mu^2=0$
$m_H^2=8\xi v^2$



$0<\mu^2<2\xi v^2$
$4\xi v^2<m_H^2<8\xi v^2$

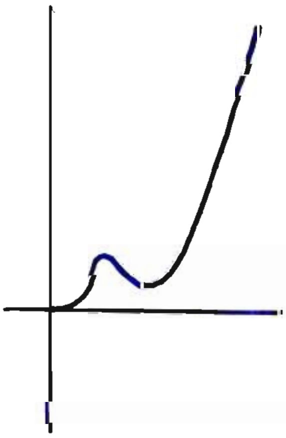


$\mu^2=2\xi v^2$
$m_H^2=4\xi v^2$

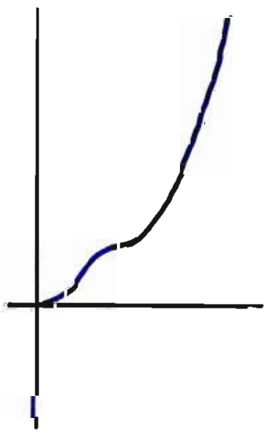


Linde-Weinberg

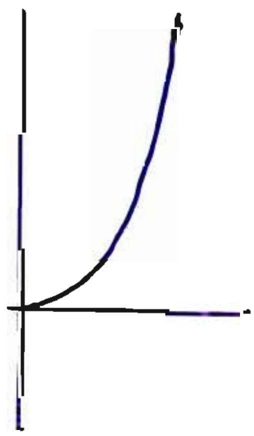
$2\xi v^2<\mu^2<4\xi v^2$
$(\ m_H^2<4\xi v^2)$



$\mu^2=4\xi v^2$
$(m_H^2=0)$



$\mu^2>4\xi v^2$



Higgs decay modes and rates

$H \to f\bar{f}$

$$\mathcal{M} = -\frac{g m_f}{2 m_W} \bar{u}(p) v(\bar{p})$$

$$\sum_{\text{spin}} |\mathcal{M}|^2 = \frac{g^2 m_f^2}{4 m_W^2} \underbrace{\mathrm{Tr}(\not{p}+m_f)(\not{\bar{p}}-m_f)}_{= 4 p\cdot\bar{p} - 4m_f^2 \,=\, 2(m_H^2 - 4m_f^2) \,=\, 2 m_H^2 \beta_f^2}$$

$m_H^2 = (p+\bar{p})^2 = 2p\cdot\bar{p} + 2m_f^2$

$$= \frac{g^2 m_f^2 m_H^2}{2 m_W^2} \beta_f^2$$

$\beta_f = \sqrt{1 - \frac{4m_f^2}{m_H^2}}$

$$\Gamma(H \to f\bar{f}) = \frac{1}{2m_H} d\Phi_2 \sum |\mathcal{M}|^2$$

$$= \frac{\beta_f}{16\pi m_H} \cdot \frac{g^2 m_f^2 m_H^2}{2 m_W^2} \beta_f^2$$

$$= \frac{\alpha m_f^2 m_H}{8 m_W^2 \sin^2\theta_W} \beta_f^3$$

$$= \frac{G_F m_f^2 m_H}{4\sqrt{2}\pi} \beta_f^3$$

(quark なら 3倍 !)

P wave $f\bar{f}$

$0^{++} = {}^3P_0$

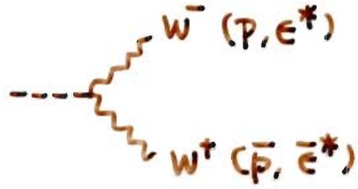
$\Gamma \sim \beta_f^3$

$\Gamma \propto m_f^2$

e.g. $\frac{\Gamma(H \to \mu^+\mu^-)}{\Gamma(H \to e^+e^-)} \sim \frac{m_\mu^2}{m_e^2}$

Heaviest (accessible) fermion has largest BR

$H \to W^+W^-$



$$\boxed{\mathcal{M} = g m_W\, \epsilon^* \cdot \bar{\epsilon}^*}$$

$$p = (E\ 0\ 0\ p) \qquad \bar{p} = (E\ 0\ 0\ -p)$$

$$E = \frac{m_H}{2} \qquad p = \frac{m_H}{2}\beta_W \qquad \beta_W = \sqrt{1 - \frac{4m_W^2}{m_H^2}}$$

$$\epsilon_{T1} = (0\ 1\ 0\ 0) \qquad \bar{\epsilon}_{T1} = (0\ -1\ 0\ 0)$$

$$\epsilon_{T2} = (0\ 0\ 1\ 0) \qquad \bar{\epsilon}_{T2} = (0\ 0\ -1\ 0)$$

$$\epsilon_L = \frac{1}{m_W}(p\ 0\ 0\ E) \qquad \bar{\epsilon}_L = \frac{1}{m_W}(p\ 0\ 0\ -E)$$

$$\epsilon_{Ti}^* \cdot \bar{\epsilon}_{Tj}^* = \delta_{ij}$$

$$\epsilon_{Ti}^* \cdot \bar{\epsilon}_L^* = 0$$

$$\epsilon_L^* \cdot \bar{\epsilon}_{Ti}^* = 0$$

$$\epsilon_L^* \cdot \bar{\epsilon}_L^* = \frac{1}{m_W^2}(p^2 + E^2) = \frac{m_H^2}{2m_W^2} - 1$$

$$\Gamma(H \to W_T^+ W_T^-) = \frac{\beta_W}{16\pi m_H} \sum_{T_1, T_2} |\mathcal{M}(H \to W_T^+ W_T^-)|^2 = \frac{\alpha m_W^2 \beta_W}{2 m_H \sin^2\theta_W}$$

$$\Gamma(H \to W_L^+ W_L^-) = \frac{\alpha m_H^3 \beta_W}{16 m_W^2 \sin^2\theta_W} \left(1 - \frac{2m_W^2}{m_H^2}\right)^2$$

$$\Gamma(H \to W^+ W^-) = \Gamma(H \to W_T^+ W_T^-) + \Gamma(H \to W_L^+ W_L^-)$$

$$= \frac{\alpha m_H^3 \beta_W}{16 m_W^2 \sin^2\theta_W} \left(1 - \frac{4m_W^2}{m_H^2} + \frac{12 m_W^4}{m_H^4}\right)$$

$$= \frac{G_F m_H^3}{8\sqrt{2}\pi} \beta_W \left(\qquad \text{''} \qquad\right)$$

$H \to ZZ$

Essentially the same as $H \to W^+W^-$ (mass difference)

but, identical final particles $\Rightarrow$ phase space $\times \frac{1}{2}$

$$\Gamma(H \to ZZ) = \frac{G_F m_H^3}{16\sqrt{2}\pi} \beta_Z \left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12 m_Z^4}{m_H^4}\right)$$

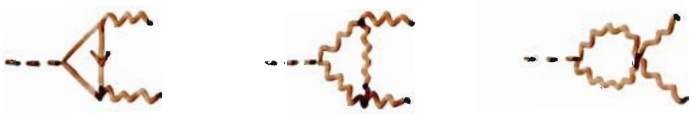
$m_H \gg m_W, m_Z$ limit で

$$\Gamma(H \to ZZ) = \frac{1}{2} \Gamma(H \to W^+W^-)$$

$H \to \gamma\gamma,\ Z\gamma,\ gg$

Only via fermion, W loop

$\downarrow$ not for gluons



Small BR in general

$\alpha_W = \alpha / \sin^2\theta_W$

まとめ

$$\Gamma(H \to f\bar{f}) \sim \alpha_W m_H \left(\frac{m_f}{m_W}\right)^2 \sim (\text{Yukawa})^2 m_H$$

$$\Gamma(H \to W^+W^-, ZZ) \sim \alpha_W m_H \left(\frac{m_H}{m_W}\right)^2 \not\propto (\text{gauge})^2 m_H$$

$$\sim \lambda\, m_H$$

$\uparrow$ Higgs self coupling

$m_H \gg m_W$ では $\Gamma(H \to W_L^+ W_L^-) \gg \Gamma(H \to W_T^+ W_T^-)$

W_L の自由度は scalar から来ている.

→ 事実 high energy では scalar のごとくふるまう

(Equivalence theorem)

Equivalence theorem : simple example

Theorem (Cornwall + Levin + Tiktopoulos / Lee + Quigg + Thacker / Chanowitz + Gaillard)

$W_L \sim \phi$ (unphysical scalar) @ $E \gg m_W$

$H \to \phi^+\phi^-$ を計算してみよう!

Interaction $\quad V(\varphi) = \mu^2 \varphi^\dagger\varphi + \lambda(\varphi^\dagger\varphi)^2 \qquad \varphi^\dagger\varphi = \frac{1}{2}\sum_{i=0}^{3}\varphi_i^2$

$$= \cdots + \frac{1}{4}\lambda\left[(v+H)^2 + \varphi_a^2\right]^2 \qquad a = 1,2,3$$

$$= \frac{g\, m_H^2}{2m_W} H\left(\phi^+\phi^- + \frac{1}{2}\chi^2\right) + \cdots$$

Amplitude $\quad \mathcal{M}(H \to \phi^+\phi^-) = (-i)(-i)\dfrac{g m_H^2}{2m_W} \qquad \mathcal{L} = \cdots - V$

$$\Gamma(H \to \phi^+\phi^-) = \frac{\beta_\phi}{16\pi m_H}|\mathcal{M}|^2$$

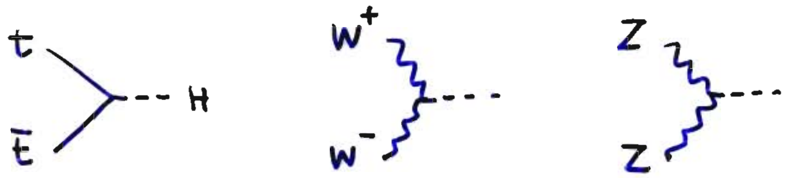
$$= \frac{\alpha\, m_H^3}{16\, m_W^2 \sin^2\theta_W}\beta_\phi = \frac{G_F m_H^3}{8\sqrt{2}\pi}\beta_\phi$$

$$\Gamma(H \to W_L^+ W_L^-) = \frac{G_F m_H^3}{8\sqrt{2}\pi}\beta_W\left(1 - \frac{2m_W^2}{m_H^2}\right)^2$$

$m_H \gg m_W \quad \Rightarrow \quad \Gamma(H \to W_L^+W_L^-) = \Gamma(H \to \phi^+\phi^-) + O\left(\dfrac{m_W^2}{m_H^2}\right)$

How to produce Higgs boson

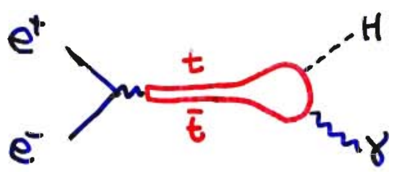
Principle : Reverse the (dominant) decay process



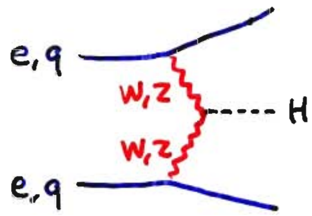
Higgs couples preferentially to heavy particles
→ heavy particle in the initial state required

Obstacle : Stable particles (beams and targets) are light and very weakly couple to the Higgs

Solution : Heavy particle intermediate state

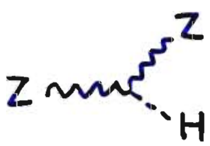


Toponium → Hγ

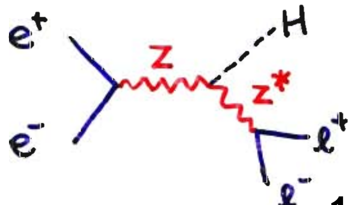


$e^+e^- \to \nu\bar{\nu}H$, e^+e^-H
$p\bar{p} \to (WW \to)\ H + \text{any}$

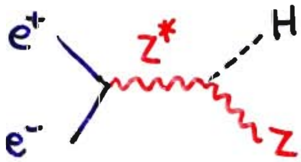
Alternative principle : Reverse part of the decay process

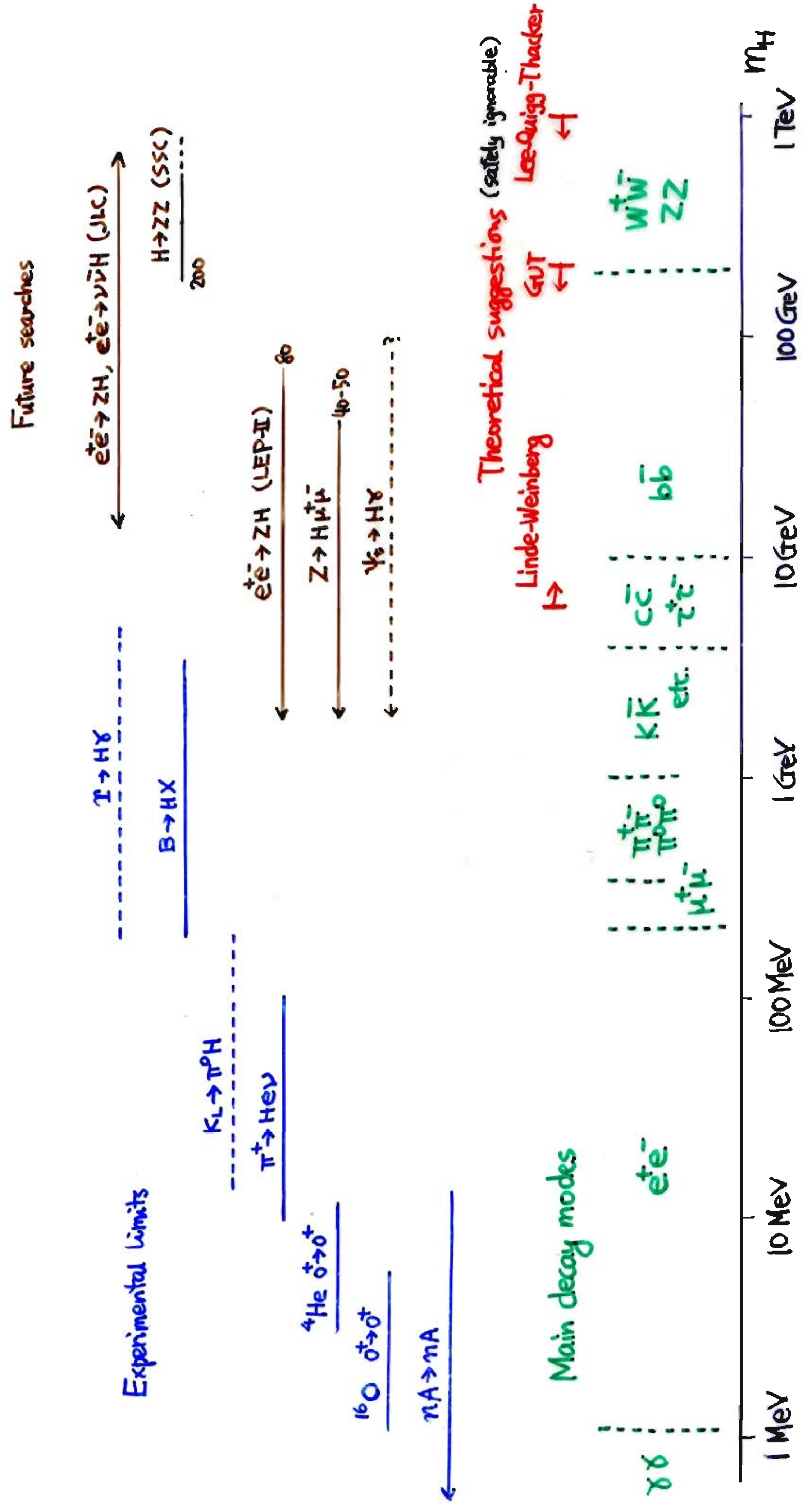


Realized as $Z \to H\ell\bar{\ell}$ or $e^+e^- \to ZH$



(on resonance)




Future searches
e+e- → ZH, e+e- → νν̄H (JLC)
H→ZZ (SSC)
200
e+e- → ZH (LEP-II)
80
Z→Hμ+μ-
40-50
Υ→Hγ
?
Υ→Hγ
B→HX
K_L→π0H
π+→Heν
4He 0+→0+
16O 0+→0+
nA→nA
Experimental limits
Theoretical suggestions (safely ignorable)
Linde-Weinberg
GUT
Lee-Quigg-Thacker
Main decay modes
γγ
e+e-
μ+μ-
π+π-
π0π0
KK̄
etc.
cc̄
τ+τ-
bb̄
W+W-
ZZ
1 MeV
10 MeV
100 MeV
1 GeV
10 GeV
100 GeV
1 TeV
m_H